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Premios Enrique Fuentes Quintana de Tesis Doctorales

ESSAYS ON FAMILIARITY AND CHOICE

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On the importance of theory:

“There are no facts, only interpretations.”

Friedrich W. Nietzsche

On the inevitability of habits:

“Habits gradually change the face of one’s life as time changes one’s physical face; one does not know it.”

Virgina Wolf

On the shortcomings of this thesis:

“Every man is guilty of all the good he didn’t do.”

Voltaire

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INTRODUCTION

This thesis studies the channels through which *familiar* experiences influence individual behavior through *automatic* psychological processes with the aim of getting a clearer understanding of some puzzling economic phenomena. In particular, the research focuses on understanding two main channels that are discussed in the next two sections: (i) how familiar experiences influence individual behavior through similarity comparisons and, (ii) how familiar experiences influence individual behavior through the effect of exposure on the perceived value of the alternatives.

FAMILIARITY AND SIMILARITY

Whenever we face decision problems that we perceive as similar enough with some others we have experienced, we tend to make *intuitive* decisions, that is, we tend to replicate past choices even if it is possible that new and better options are available. Thus, an important question arises. How does intuition affects individual behavior and, thus, market outcomes? In particular, two questions need to be addressed. What is the relationship between intuition and observed behavior? How does intuitive individual behavior affect aggregate market outcomes?

Chapter 1 answers the first question. In doing so it addresses a closely related question that is still open in the literature (Bernheim and Rangel, 2009; Rubinstein and Salant, 2012; Masatlioglu, Nakajima and Ozbay, 2012 and Apesteguia and Ballester, 2015) and that is extremely important for welfare analysis; how can we understand individual preferences from observed choices? If some decisions are intuitive and not driven by preferences the question is relevant and not trivial. If people make intuitive choices when they experience familiar decision problems, how can we understand their preferences? When we observe a decision maker sticking to past choices even when new alternatives are available, is it because he prefers what he chose in the past or is it because he chose intuitively thus disregarding new options he might prefer? To answer such questions it is important to study two issues: (i) how intuitive choices arise and (ii) how to distinguish conscious and intuitive choices from observed behavior.

In chapter 1 we address the first issue by providing a new model of decision making that is a simple formalization of an important theory in cognitive sciences known as Dual Process Theory (Evans and Frankish, 2009 and Kahneman, 2011) that still, to the best of our knowledge, has no tractable formalization in economics. Following such theory, individuals are described by the interaction of two systems. System 1 is associative and unconscious. System 2 is analytic, conscious and demanding of cognitive capacity. The model proposed summarizes such theory. System 1 uses a similarity function and a similarity threshold that represents the cognitive costs of activating the other system, to assess whether new experiences are similar enough with those past ones which behavior can be replicated. If the similarity is high enough, System 2 is not activated and past behavior is replicated. On the other hand, if the similarity is not high enough, then System 2 drives the decision process rationally, by maximizing a preference relation. In this formulation, intuitive choices arise because of System 1, thus, if we want to understand individual preferences and start to address the second issue, *i.e.* how to distinguish between conscious and intuitive choices, we need to understand which choices were made by System 2.

We propose a way of analyzing choice data that is based on the analogies System 1 makes and thus, given similarity comparisons are made with past experiences, it uses the sequence in which choices have been made. The intuition of the method is as follows. Whenever we know that in some moment in time preferences were maximized, we know that the decision problem in question was not similar enough with its past. Thus, all those decision problems that are even less similar with their past should be the outcome of maximization of preferences too. Using this simple principle we can find a set of choices that are informative for the elicitation of individual preferences. Furthermore, the method is applicable to find a set of intuitive choices. In fact, in a similar fashion than before, whenever we know some choices were intuitive, all those decision problems that are even more similar with their past must have been solved intuitively by the decision maker. Such information is useful to understand from choice data what is considered similar enough by the agent which is an important piece of information whenever we want to predict individual behavior. In particular it allows us to get information regarding the cognitive costs the decision maker incurs when making thoughtful, non-intuitive choices.

As an additional contribution, we provide an axiomatic characterization of the model that makes it falsifiable. In particular, we show that whenever the similarity function is known, a computationally simple weakening of the Strong Axiom of Revealed Preference is enough to characterize the entire model. The main intuition is that we should not observe inconsistent, *i.e.,* cyclic, choices among the ones that have to be explained by maximization of preferences. Moreover, whenever social data are available, even if the similarity function is unknown, the model can be characterized by two simple consistency requirements that allow for the perfect identification of individual preferences and similarity comparisons. Finally, we show that the theoretical results are not heavily affected by assuming imperfect memory of the decision maker or partial knowledge of the similarity function.

The simple model proposed in chapter 1 allows for the study of the implications of intuitive decisions for market outcomes, that is the second broad research question presented at the beginning of this section. Many different puzzling phenomena are observed in the markets that might be related to the kind of *sticky* behavior the model implies, but in chapter 2 we focus on the implications of intuitive decisions for financial markets. In particular, we study an economy similar to the one described in De Long *et al.,* 1990 where overlapping generations of traders which behavior is the outcome of the interaction between System 1 and System 2, live two periods and have to decide how much of two assets to buy, one risky and one riskless, in order to maximize last period consumption. We show that even if traders receive perfect information regarding first and second period dividends generated by the risky asset, due to the similarity between different market environments, they can fail to update their beliefs and thus behave *intuitively.* Hence prices in equilibrium can be far from their fundamental value because some decisions are not taking all available information into account. Moreover, prices depart from fundamentals in a predictable manner as some literature in the past years have suggested.

In particular, we model traders whose System 1 makes similarity comparisons between market environments, thus whenever the change in the environment is not enough to be perceived, traders do not update their beliefs and behavior is driven by System 1. They trade on *old* information that is irrelevant for the new market environment. Otherwise, decisions are driven by System 2 and traders behave rationally. In this sense, the model proposes a channel through which *noise trading* emerges endogenously in contrast to the exogenous proportion of noise or inattentive traders usually assumed in the literature (for example De Long *et al.,* 1990 and DellaVigna and Pollet, 2009). The paper then shows that in equilibrium, when noise trading emerges due to similarity between different market environments, prices should follow some patterns that have been observed in the data. To be more specific, prices should be more volatile than in the perfectly rational framework (for example Shiller, 1992), they should underreact in the short-run to new information (for example Cutler, Poterba, and Summers, 1991 and Bernard, 1993) while they might overreact to new information in the long-run (seminal contribution by De Bondt and Thaler, 1985).

The intuition behind these results is as follows. First, prices are more volatile because there are two sources of risk in the economy. One is standard and is due to the dividend generating process. The other one is novel and is due to the endogenous formation of noise traders in the economy. Even if traders know how noise traders emerge, they cannot predict without uncertainty what will be their relative weight in the future and so prices vary more than in an economy where such uncertainty is not present. Second, prices underreact to news in the short-run because of the behavioral model that describes traders behavior. If new information arrives in the market not all traders will use it because for some of them the change of the market environment is not enough to be perceived. Thus, some traders will not use all the information and so prices would be stickier than a rational framework would imply. Third, overreaction in the long-run can occur because of the interaction of underreaction and new information arriving in the market. Due to underreaction, information is gradually incorporated into prices from one period to the next *(momentum)*, thus when new information is available its effects on prices can sum up with old information being incorporated hence causing the movement in prices to be more pronounced than in a fully rational economy.

Finally, we conclude the chapter with an analysis of the implications of these results for the equity-premium puzzle (Mehra and Prescott, 1985). In particular, we discuss the fact that the equity-premium should be countercyclical in the economy described in the paper, something that is qualitatively in line with evidence presented in the last decades (Mehra and Prescott, 2003).

FAMILIARITY AND INERTIA

Since Samuelson and Zeckhauser, 1988 and Kahneman, Knetsch, and Thaler, 1991 a lot of evidence has been found pointing to the fact that individual decision making exhibits a strong kind of behavioral inertia, *i.e.,* the status-quo bias or the endowment effect, which source is not clear. Our choices are shaped by the experiences we had in a way that is more subtle that the mere acknowledgment of the fact that we learn through our experiences. Past choices become important for present ones because they become familiar and the effect of familiarity on choice is unconscious.

In chapter 3, we propose a possible cognitive foundation for this particular kind of inertia that is called *the mere exposure effect* that has been discovered by Zajonc, 1968 and that became a central process in cognitive sciences to understand the effects of familiarity on choices. The mere exposure effect can be described as the phenomenon by which people tend to develop a preference for things merely because they have been *exposed* to them, they are *familiar* with them. We use this idea to model a decision maker that chooses stochastically between alternatives and that the more he chose an alternative, *i.e.,* the he has been exposed to it, the higher the probability of choosing it.[1]

The model we propose can be seen as a more general kind of status-quo bias, thus helping explain the endowment effect, loss aversion and present bias in a dynamic framework. We formally discuss these possibilities and then we show how such framework not only highlights the importance of experiences on choices through its dynamic dimension but it can also help to substantiate this idea by providing a precise theoretical prediction of what kind of heterogeneity should emerge from a population of otherwise identical individuals that had different experiences. This implication of the model is of particular interest for the literature that analyzes the impact of the first years of life on successive social and economic outcomes through individual decisions, *e.g.,* Heckman, 2006. In particular this result makes possible to understand and quantify the effect of experiences on the kind of behavioral inertia that is the focus of the chapter.

Finally, we show how to falsify the model when we have data on choice probabilities of an homogeneous population that chooses from sequential choice problems. In particular, we propose four simple properties that fully characterize the model. The first of this properties is a generalization of the concept already introduced in Luce, 1959 of independence of irrelevant alternatives (IIA) which states that the relative probability of choosing an alternative over another should not depend on the other alternatives in the set. The only change we impose is that the property has to be satisfied for any given level of exposure. The second property says that the effect of exposure should not be alternative specific. The third property we propose is simply saying that exposure cannot decrease the probability of choosing an alternative. This is the key property capturing the exposure effect. Finally, the fourth and more technical property, imposes that the effect of exposure cannot be marginally increasing. We then discuss the possibility of considering a more general kind of exposure, *i.e.,* exposure to all the alternatives of the menu.

To ease the reading of the chapters all proofs have been moved to the appendix while, for the same reason, figures and tables have been kept in the main text.

[1] In the chapter we also discuss a more general interpretation of exposure.

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1 DUAL DECISION PROCESSES: RETRIEVING PREFERENCES WHEN SOME CHOICES ARE INTUITIVE

1.1. INTRODUCTION

Few of the choices we make every day are the result of a conscious decision making process. Automatic or intuitive decisions are ubiquitous. Think about daily routines, many times we just intuitively stick to decisions we have only consciously thought of once. This mechanism is true not only for marginal decisions. As highlighted by Simon (1987), managers take many intuitive decisions when deciding investment strategies for their firms. Moreover, according to Kahneman (2002), experts such as managers, doctors or policy makers often make intuitive decisions or follow their *hunches* due to their extensive experience. Thus, understanding automatic or intuitive decisions can be crucial to achieve a better understanding of individual behavior and in particular of market outcomes. Actual market equilibria might be far from the ones standard models predict if intuition plays a role in the decisions taken by the different economic agents. Hence, a question arises, what makes choices intuitive?

The dichotomy between fast and intuitive decisions versus slow and conscious ones has been formally studied in cognitive sciences at least since Schneider and Shiffrin, 1977. The seminal work by Evans (1977) and the successive models and findings in McAndrews, Glisky, and Schacter (1987), Evans (1989), Reber(1989) Epstein (1994) and Evans and Over (1996), stimulated the creation of a coherent theory of the individual mind as an interaction of two systems called Dual Process Theory that is described in Evans and Frankish (2009), and Kahneman (2011). System 1 is associative and unconscious. System 2 is analytic, conscious and demanding of cognitive capacity. [1] Using analogies, System 1, source of intuitive choices, draws from past behavior to influence decisions. System 2, source of conscious choices, is costly and hence is only activated to solve problems for which past experience cannot be used by System 1. The associative nature of System 1 makes analogies extremely important for individual behavior and it highlights the centrality of the timing of decision problems to understand choices.

Past behavior can influence present choices because intuitive decisions are driven by the analogies System 1 makes. Thus, if we want to understand the role of intuitive decisions in markets, we need first (i) to have a model of intuitive choices that takes into account the role of

[1] The names of the two systems appeared for the first time in Stanovich (1999). See Evans and Frankish (2009) to get a deeper description of the two different systems and of the historical development of the theory.

past choices on present ones and (ii) to analyze whether it is possible from observed behavior to distinguish between conscious and intuitive choices.[2] These are the research questions we address in this chapter.

To answer the first question we propose a simple formalization of Dual Process Theory. To the best of our knowledge there is no tractable formalization of such theory, thus the first contribution of the present chapter is to propose a formal model that can be used in standard economic analysis. In section 3.2, we model a decision maker composed by the two described systems. System 1 compares every decision environment with the ones that the decision maker has already faced according to some given similarity measure. Behavior is replicated from those past problems that are similar *enough* to the present one, *i.e.*, the similarity between the problems passes some threshold that represents the cost of activating the other system. If there are no such problems, System 2 chooses the best available option by maximizing a rational preference relation. Think for example of a consumer that buys a bundle of products from a shelf in the supermarket. The first time he faces the shelf, he tries to find the bundle that best fits his preferences. Afterwards, if the price and arrangement of products do not change *too much,* he will perceive the two decision problems as if they are the same and so he will intuitively stick to the past chosen bundle. If on the contrary, the change in price and arrangement of the products is evident to him, he will choose by maximizing his preferences again.

Even in such a simple framework, there is no trivial way to distinguish which choices are made intuitively and which ones are made consciously. Following the example, suppose our consumer faces again the same problem but this time a new bundle is available and he sticks with the old choice. Is it because the old bundle is preferred to the new one? Or is it because he is choosing intuitively? We show how to restore standard revealed preference analysis and thus answer the second research question by understanding in which decisions System 2 must have been active. Section 1.4 assumes that (i) the decision maker behaves according to our model and (ii) the similarity function is known while the threshold is not, *i.e.*, the cognitive costs of activating System 2 are unknown.[3] We then show that, for every sequence of decision problems, it is possible to identify a set of conscious observations and the interval in which the cognitive costs should lie. To the best of our knowledge this is the first work attempting to assess such costs from observed behavior. We do this by means of an algorithm that uses the analogies made by System 1. First notice that *new* observations, *i.e.*, those in which the choice is an alternative that had never been chosen before, must be generated by System 2. No past behavior could have been replicated. Starting from these observations, the algorithm iterates the following idea. If an observation is generated by System 2, any other *more novel* observation, that is any problem which is less similar to those decision problems that preceded it, must be also generated by System 2. Novelty depends on two factors: (i) how System 1 makes associations and (ii) the sequence of decision problems the decision maker has faced. These two are the factors the algorithm exploits to find conscious choices.

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[2] See Rubinstein (2007 and 2015) for a similar distinction between conscious and intuitive strategic choices for players participating in a game. See also the distinction in Cunningham and De Quidt (2015) between implicit and explicit attitudes.

[3] See sections 3.2 and 1.4 for a justification of the latter hypothesis.

Returning to our consumer, if we know that after a change in the price of the products on the shelf, the consumer chose consciously, then he must have done so also in all those periods where the change was even more evident.

The algorithm identifies a set of intuitive decisions, *i.e.,* those made by System 1, in a similar fashion, that is, first it highlights some observations that have to be intuitive and then uses this information to reveal other intuitive observations. In doing this, we consider *cyclical* datasets, *i.e.,* datasets in which the standard revealed preference relation is cyclic. Notice that any cycle must contain at least one intuitive observation, given that observations generated by System 2 cannot create cycles in the revealed preference. Then, a *least novel observation in a cycle, i.e.,* one that is more similar to its past among those forming a cycle, must be intuitive. Once we know that one observation is generated by System 1, so must be all observations not included in the cycle that are even *less novel.* Thus, the algorithm finds a set that contains only conscious observations and, in a dual way, another one that contains only intuitive ones. Interestingly enough, if the sequence meets some richness conditions, such sets contain all conscious and intuitive observations respectively. Obviously, knowing these two sets gives us crucial information regarding individual preferences and the degree of similarity that is needed to activate System 2. In fact, understanding if some decisions were made intuitively is very important to understand how analogies are made. Even if intuitive choices do not reveal individual preferences, they tell us what problems are considered similar enough by the decision maker. What is similar enough depends on the similarity threshold, thus intuitive choices provide crucial information regarding the unknown cognitive costs of activation of System 2.

It is important to notice that the preference revelation strategy we use in the chapter agrees with the one used in Bernheim and Rangel (2009). They analyze the same problem of eliciting individual preferences from behavioral datasets, and they do this in two stages. In a first stage they take as given the *welfare relevant domain,* that is the set of observations from which individual preferences can be revealed, and then in a second stage they analyze the welfare relevant observations and propose a criterion for the revelation of preferences that does not assume any particular choice procedure to make welfare judgments.[4] Even if similar, our approach differs in two important aspects. First, by modeling conscious and intuitive choices, we propose a particular method to find the welfare relevant domain, *i.e.* the algorithm highlighting a set of conscious choices. Second, by proposing a specific choice procedure, we use standard revealed preference analysis on the relevant domain, thus our method, by being behaviorally based, is less conservative for the elicitation of individual preferences. In this sense, our stance is also similar to the one proposed in Rubinstein and Salant (2012); Masatlioglu, Nakajima, and Ozbay (2012) and Manzini and Mariotti (2014) that make the case for welfare analysis based on the understanding of the behavioral process that generated the data. Thus, falsifiability of the model becomes a central concern.

In section 1.5 we propose a testable condition that is a weakening of the Strong Axiom of Revealed Preference that characterizes our model and thus renders it falsifiable. Moreover,

[4] Notice that Apesteguia and Ballester (2015) propose an approach to measure the welfare of an individual from a given dataset that is also choice-procedure free. They do so by providing a model-free method to measure how close actual behavior is to the preference that best reflects the choices in the dataset.

we show that if the data are rich enough, in particular if we observe choices made by an homogeneous population, two simple consistency requirements not only characterize the model but also allow us to uniquely identify individual preferences and how analogies between decision problems are made. Section 1.5 concludes with a discussion on what conscious and intuitive behavior can be in a more general context that relies on our framework in section 1.5.1 and with the formal analysis of how to estimate the similarity function from an heterogeneous population of individuals sharing it in section 1.5.2.

Section 1.6 discusses some possible extensions of the base model. First, we analyze the possibility of a decision maker with imperfect memory, that is, that recalls only the m most recent decision problems. We show that such extension does not hinder our algorithmic analysis. In fact, not only the analysis can be reproduced but also, with rich enough data, it is possible to identify the preferences and how similarity comparisons are made by analyzing just one sequence of observations, that is, without recurring to social data. Second, we study the impact of a weaker assumption regarding the similarity of different problems. We allow for the possibility of having only ordinal and partial information on it. Interestingly enough, we find the logic behind the algorithm to be robust to this extension. Section 3.5 discusses further implications of the model for the understanding of *sticky behavior*. In particular the model can be seen as a general framework capable of formalizing the idea of behavioral inertia.[5] The appendix contains all proofs.

1.2. RELATED LITERATURE

In our model the presence of similarity comparisons makes behavior more sticky, that is, if two environments are similar enough then behavior is replicated. This is a different approach with respect to the theory for decisions under uncertainty proposed in Gilboa and Schmeidler (1995) and summarized in Gilboa and Schmeidler, 2001. In case-based decision theory, as in our model, a decision maker uses a similarity function in order to assess how much alike are the problem he is facing and the ones he has in his memory. In that model the decision maker tends to choose the action that performed better in past similar cases. There are two main differences with the approach we propose here. First, from a conceptual standpoint, our model relies on the idea of two systems interacting during the decision making process. Second, from a technical point of view, our model uses the similarity in combination with a threshold to determine whether the individual replicates past behavior or maximizes preferences while in Gilboa and Schmeidler (1995) preferences are always maximized. Nevertheless, the two models agree on the importance of experiences embedded in the memory of the decision maker in shaping observed behavior.

Finally, we would like to stress that even if the behavioral model we propose is new and it is a first formalization of Dual Process Theory, nonetheless the idea that observed behavior can be the outcome of the interaction between two different selves is not new and it dates back at least to Strotz (1955). Strotz kind of models, such as Gul and Pesendorfer (2001) or Fudenberg and Levine (2006), are different from the behavioral model we introduce here,

[5] See Chetty (2015) for a discussion of the importance of understanding behavioral inertia for public policy.

since they represent the two selves as two agents with different and conflicting preferences, usually *long-run* vs *short-run* preferences.[6] In our approach however, the two systems are *inherently* different one from the other. One uses analogies to deal with the environment in which the decision maker acts, while the other one uses a preference relation to consciously choose among the alternatives available to the decision maker. Furthermore, which system is activated in a particular decision problem depends on the problems that have been experienced and how similar they are with the present one and thus, it does not depend on whether the decision is affecting the present or the future.[7]

1.3. DUAL DECISION PROCESSES

Let X and E be two finite sets. The decision maker (DM) faces at every moment in time t a *decision problem* (A_t, e_t) with At $\subseteq X$ and $e_t \in E$. The set of alternatives A_t that is available at time t, and from which the DM has to make a choice, is usually called the *menu*. An alternative is any element of choice like consumption bundles, lotteries or even streams of consumption. The *environment* e_t is a description of the possible characteristics of the problem that the DM faces at time t.[8] We simply denote by $a_t \in A_t$ the chosen alternative at time t. With little abuse of the notation, we refer to the couple formed by the decision problem (A_t, e_t) and the chosen alternative a_t as *observation t*. We denote the collection of observations in the sequence $\{(A_t, e_t, a_t)\}_{t=1}^{T}$ as D, *i.e.*, $D = \{1, ..., T\}$.

The DM is composed by two systems, System 1 (S1) and System 2 (S2) and the chosen alternative is determined by either one of them. S1 is constantly operating and uses *analogies* to relate the decision environment the DM is facing with past ones. If a past environment is similar enough to the one the DM is facing, then S1 replicates the choice made in the past whenever available. If no past environment is similar enough or replication is not possible, S2 is activated. S2 uses a *preference* relation to compare the alternatives in the menu and chooses the best one.[9] We call this behavioral model a *dual decision*(DD) process.

Formally, let $\sigma: E \times E \rightarrow [0, 1]$ be the *similarity function*. The value $\sigma(e, e')$ measures how similar environment e is with respect to e'. S1 is endowed with a *similarity threshold* $\alpha \in [0, 1]$ that delimits which pairs of environments are similar enough. Whenever $\sigma(e, e') > \alpha$ the individual considers e to be similar enough to e'. At time t and facing the decision problem (A_t, e_t), S1 executes a choice if it can replicate the choice of a previous period $s < t$ such that $\sigma(e_t, e_s) > \alpha$. The choice is the alternative a_s chosen in one such period. That is, if the DM

6 In some models the difference between the two selves comes from the fact that they have different information. See for example Cunningham (2015) that proposes a model of decision making where the two selves hierarchically aggregate information before choosing an alternative.

7 Nevertheless, we do not exclude the possibility that the fact that a decision affects the present or the future has some kind of influence on how analogies are made.

8 See below for some examples of environments.

9 The idea that conscious behavior comes from the maximization of a preference relation is a simplification we use to focus the analysis on the main novelties of the framework presented in this work. For a more detailed discussion regarding this point, see section 1.5.1.

faces a decision environment e_t that is similar enough to a decision environment e_s he has already faced and the alternative chosen in s is present in t, the DM chooses it again in t.[10] S2 is endowed with a *preference relation* over the set of alternatives.[11] At time t, if S2 is activated, the most preferred available alternative is chosen, that is, S2 chooses the alternative a_t that maximizes $\succ$ in A_t. Summarizing:

$$a_t = \begin{cases} a_s \textit{ for some } s - t < \text{ such that } \sigma(e_t, e_s) > \alpha \text{ and } a_s \in A_t, \\ \text{the maximal element in } A_t \text{ with respect to } \succ, \text{ otherwise} \end{cases}$$

Two remarks are useful here. First, notice that intuitive and conscious decisions are separated by the behavioral parameter α. In some sense α is summarizing the cost of using the cognitive demanding system, *i.e.,* the *laziness* of S2 as it is described in Kahneman (2011). The higher the cost, the lower the threshold. Thus, parameter α captures individual heterogeneity on S1. In fact, following the common interpretation in cognitive sciences, we take the similarity function as given, *i.e.,* as an innate component of all individuals, while the similarity threshold, by representing cognitive costs, is what makes analogy comparisons different at the individual level.[12] Notice that while the similarity function has been widely studied in cognitive sciences, *e.g.,* Tversky (1977), Medin, Goldstone, and Gentner (1993) and Hahn (2014), the cognitive costs of activating S2 are still an unknown, thus the method we propose in section 1.4 can be seen as a first attempt of identifying from observed behavior the interval in which such costs should lie, given the similarity function.

As a second remark, notice that we are describing a class of models be-cause we do not impose any particular structure on the replicating behavior. We do not specify which alternative would be chosen when more than one past choice can be replicated. Many different behaviors can be part of this class, *e.g.,* choosing the alternative that was chosen in the most similar past environment or choosing the alternative that maximizes the preference relation over the ones chosen in similar enough past environments, or choosing the alternative chosen in the most recent similar enough environment etc.[13] All the analysis that follows is valid for the class as a whole.

As a final remark, given the centrality of the similarity function for the model, we propose here some examples of environments that are relevant for economic applications and a possible similarity function that can be used in such cases.

Environments as Menus: *In many economic applications it seems sensible to see the whole menu of alternatives,* e.g., *the budget set, as the main driver of analogies. That is, E could be the set of all possible menus and two decision problems are perceived as similar as their menus are. In this framework,* $E = 2^X$.

[10] Notice that we assume that the DM has perfect memory. There is evidence in favor of perfect memory for choices that are unconscious. See Duhigg (2012) for an informal discussion of how *consciously* forgotten psychological cues can still affect our behavior and decisions. However, as shown in section 1.6, this assumption does not weaken the analysis.

[11] For ease of exposition, we assume that >- is a strict order, *i.e.,* an asymmetric, transitive and complete binary relation, defined over X.

[12] In section 1.5.2 we show how it is possible to recover the similarity function when different individuals share the same similarity function.

[13] A formal analysis of these possibilities is available upon request.

Environments as Attributes: *Decision makers many times face alternatives that are bundles of attributes. In those contexts, it is reasonable to assume that the at¬tributes of the available alternatives determine the decision environment. If A is the set containing all possible attributes, then* $E = 2^A$.

Environments as Frames: *We can think of the set E as the set offrames or an-cillary conditions as described in Salant and Rubinstein (2008) and Bernheim and Rangel (2009). Frames are observable information that is irrelevant for the rational assessment of alternatives, for example how the products are disposed on a shelf. Every frame can be seen as a set of irrelevant features of the decision environment. Thus, if the set containing all possible irrelevant features is F, we have* $E = 2^F$.

In all the previous examples it is natural to assume that the similarity func-tion relates different environments depending on their commonalities and differences. For example, $ó(e,e')=\frac{|e\cap e'|}{|e\cup e'|}$, that is, two environments are more similar the more characteristics they share relative to all the charac-teristics they have.[14] Although, it is sometimes not possible to have all the information regarding the similarity function, a case we analyze in section 1.6, from now on we take E and a as given.[15]

We now provide an example to illustrate the behavioral process we are modeling.

Example 1. *Let* $X = \{1, 2,3, ..., 10\}$. *We assume that environments are menus, i.e., E is the set of all subsets of X and we assume that* $\sigma\,(A,A')=\frac{|A\cap A'|}{|A\cup A'|}$. *Suppose that S1 is described by* $\alpha = .55$ *and that the preference* $1 \succ 2 \succ 3 \succ \bullet\bullet\bullet \succ 10$ *describes S2. We now explain how our DM makes choices from the following list of ordered menus:*

t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8
3,4,5,6	1,2,3,4,5,6	1,3,4,7	2,4,7	1,3,6	1,2,3,4	2,4,8	2,4,8,9,10

In the first period, given the DM has no prior experiences, S2 is going to be active. Thus, the choice comes from the maximization of preferences, that is, $a_1 = 3$. *Then, in the following period, given that we have* $\sigma(A_2, A_1) = 4/6 > .55$, *S1 is active and so we would observe a replication of past choices, that is,* $a_2 = 3$. *Now, in period 3, notice that the similarity between* A_3 *and* A_2 *or* A_1 *is always below the similarity threshold and this makes S2 to be active. The preference relation is maximized and so* $a_3 = 1$. *A similar reasoning can be applied for the fourth and fifth periods to see that* $a_4 = 2$ *and* $a_5 = 1$. *Then, in period six, S1 is active given that* $\sigma(A_6, A_3) = 3/5 > .55$, *leading to* $a_6 = 1$. *Notice that* A_6 *is also similar to* A_2. *Given we are analyzing the class of DD processes as a whole, replication of the choice in 2 or 3 are both possible. The behavior we assume here could be an example of the model where S1 replicates behavior of the most recent similar enough decision problem. In period seven, given no past environment is similar enough, S2 is*

[14] Such function is just a symmetric specification of the more general class considered in Tversky (1977).

[15] In those cases where the similarity function cannot be completely known, weaker as-sumptions can be made. For example, one could think that the similarity function respects the symmetric difference between sets. That is environment e and environment e0 are more similar than g and g' if $e\cup e'\setminus e\cap e'\subseteq g\cup g'\setminus g\cap g'$.

active and so $a_7 = 2$. *Finally, in the last period S1 is active again given that* $\sigma(A_8, A_7) = 3/5 > .55$ *and so behavior will be replicated, i.e.,* $a_8 = 2$.

One may wonder what an external observer would understand from this choice behavior. How would the observer determine which choices were done by S1 or S2 and hence which observations are informative on our DM's preferences? Is it possible to retrieve the similarity threshold? In section 1.4 we propose an algorithm that allows us to disregard the prefer-ential information coming from the choices in periods two, six and seven, as it should be, while maintaining all the preferential information coming from the remaining choices.

1.4. THE REVEALED PREFERENCE ANALYSIS OF DUAL DECISIONS

In this section, we discuss how to recognize which observations were generated by either S1 or S2 in a DD process. This information is crucial to elicit the unobservables in the model that are the sources of individual heterogeneity, that is the preference relation and the similarity threshold. As we previously discussed in section 3.2, we take the similarity function, common across individuals, as given, while we want to understand from observed behavior the cognitive costs of activating S2, that is the similarity threshold α.[16]

It is easy to recognize a set of observations that is undoubtedly generated by S2. Notice that all those observations in which the chosen alternative was never chosen in the past must belong to this category. This is so because, as no past behavior has been replicated, S1 could not be active. We call these observations *new* observations.

In order to identify a first set of observations generated by S1, notice that S2 being rational, it cannot generate cycles of revealed preference.[17] Clearly, for every cycle there must be at least one observation that is generated by S1. Intuitively, the one corresponding to the most familiar environment should be a decision mimicking a past behavior. The *unconditional familiarity* of observation t is

$$f(t) = \max_{s<t, a_s \in A_t} \sigma(e_t, e_s).^{18}$$

That is, unconditional familiarity measures how similar observation t is to past observations from which behavior *could be* replicated, *i.e.,* those past decision problems for which the chosen alternative is present at t. Then, we say that observation t is a *least novel in a cycle* if it is part of a cycle of observations, and within it, it maximizes the value of the unconditional familiarity.

The major challenge is to relate pairs of observations in a way that allows to transfer the knowledge of which system generated one of them to the other. In order to do so, we

[16] Notice that, as discussed in section 1.5, the similarity function and the similarity thresh¬old define a binary similarity function that is individual specific. Thus, by separating the similarity function from the similarity threshold we are able to associate individual hetero-geneity to a parameter related with individual cognitive costs without having too many degrees of freedom to properly run the technical analysis.

[17] As it is standard, a set of observations $t_1, t_2, \ldots, t_k$ forms a *cycle* if $a_{t_{i+1}} \in A_{t_i}, i = 1, \ldots, k-1$ and $a_{t_1} \in A_{t_k}$, where all chosen alternatives are different.

[18] W.l.o.g., whenever there is no $s < t$ such that $a_s \in A_t$, we say $f(t) = 0$.

introduce a second measure of familiarity of an observation t, that we call *conditional familiarity.* Formally,

$$f(t \backslash a_t) = \max_{s<t, a_s \in a_t} \sigma(e_t, e_s).$$ [19]

That is, conditional familiarity measures how similar observation t is with past observations from which behavior *could have been* replicated, *i.e.*, those past decision problems for which the choice is the same as the one at t. The main difference between $f(t)$ and $f(t|a_t)$ is that the first one is an *ex ante* concept, *i.e.*, before considering the choice, while the second one is an *ex post* concept, *i.e.*, after considering the choice. Our key definition uses these two measures of familiarity to relate pairs of observations.

Definition 1 (Linked Observations). *We say that observation t is* linked *to observation s, and we write $t \in L(s)$, whenever $f(t|a_t) \leq f(s)$. We say that observa-tion t is* indirectly linked *to observation s if there exists a sequence of observations $t_1,...,t_k$ such that $t = t_1$, $t_k = s$ and $t_i \in L(t_{i+1})$ for every $i = 1, 2,..., k - 1$.*

Denote by D^N the set of all observations that are indirectly linked to new observations and by D^C the set of all observations to which least novel observations in a cycle are indirectly linked.[20] We are ready to present the main result of this section. It establishes that observations in D^N are generated by S2, while observations in D^C are generated by S1. As a consequence, it guarantees that the revealed preference of observations in D^N, *i.e.*, $R(D^N)$, is useful information regarding the preferences of the individual.[21] Moreover, an interval in which the similarity threshold has to lie is identified. Such interval provides bounds for the individual specific cognitive costs of activating S2.

Proposition 1. *For every collection of observations D generated by a D^D process:*

1. *all observations in D^N are generated by S2 while all observations in D^C are generated by S1,*
2. *if x is revealed preferred to y for the set of observations D^N, then x y,*
3. $\max_{t \in D^N} f(t) \leq \alpha < \min_{t \in D^C} f(t|a_t)$.

To understand the reasoning behind Proposition 1, consider first an observation t that we know is new, and hence generated by S2. We have learnt that its corresponding environment is not similar enough to any other previous environment. In other words, $f(t) \leq \alpha$. Then, any observation s for which the conditional familiarity is less than $f(t)$ must be generated by S2 too. In fact, $f(s|a_s) \leq f(t) \leq \alpha$ implies that no past behavior that could have been replicated in s comes from an environment that is similar enough to the one in s. Thus, any observation linked with a new observation must be generated by S2. It is easy to see that this reasoning can be iterated, in fact, any observation linked with an observation generated by S2 must be generated by S2 too.

[19] W.l.o.g., whenever there is no $s < t$ such that $a_s = a_t$, we say $f(t|a_t) = 0$.

[20] The binary relation determined by the concept of linked observations is clearly reflexive, thus, new observations and least novel observations in a cycle are contained in D^N and D^C respectively.

[21] We say that x is revealed preferred to y in a set of observations O, and write $xR(O)y$, if there is a sequence of different alternatives $x_1, x_2, \ldots, x_k$ such that $x_1 = x$, $x_k = y$ and for every $i = 1, 2, \ldots, k - 1 \in O$, it is $x_i = a_t$ and $x_{i+1} \in A_t$ for some t.

Similarly, consider a least novel observation in a cycle *t*, that we know is generated by S1. Any observation *s* for which the unconditional familiarity is greater than the conditional familiarity of *t* must be generated by S1 too. In fact, we know that $\alpha < f(t|a_t)$ because *t* is generated by S1. Then, any observation *s* to which *t* is linked has an unconditional familiarity above α, which implies that some past behavior *could be* replicated by S1, and so such observation must be generated by S1 too. Again, the reasoning can be iterated. Thus, we can start from a small subset of observations undoubtedly generated by either S1 or S2, inferring from there which other observations are of the same type.

We now use Example 1 to illustrate the algorithm. In doing so, we show that it is possible to see our algorithmic analysis in terms of graph theory. In fact, when two observations are linked we can think of them as connected by an undirected edge. Then, we can clearly see that D^N and D^C have to be the sets containing all those nodes that belong to the connected components of new and least novel observations in a cycle respectively.

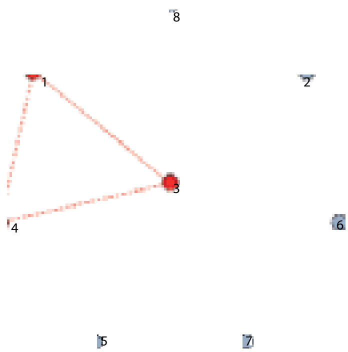
Graphical Intuition: Example 1

Suppose that we observe the decisions made by the DM in Example 1, without any knowledge on his preferences $\succ$ *or similarity threshold α. The following table summarizes the different observations.*

t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8
3,4,5,6	1,2,3,4,5,6	1,3,4,7	2,4,7	1,3,6	1,2,3,4	2,4,8	2,4,8,9,10
a_1=3	a_2=3	a_3=1	a_4=2	a_5=1	a_6=1	a_7=2	a_8=2

We can easily see that the only new observations are observations 1, 3 *and* 4, *and hence we can directly infer that S2 was active in making the corresponding choices. It is immediate to see that new observations are always linked to each other and hence, observations* 1, 3 *and* 4 *are connected by undirected edges as in the graph of Figure 1.1.*[22]

Figure 1.1
New observations are linked with each other

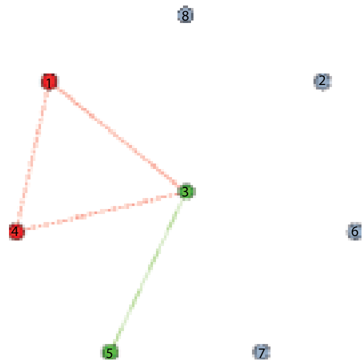


Source: Own elaboration.

[22] Notice that for any new observation t, $f(t|a_t) = 0$.

We can go one step further and consider observation 5. *From observed behavior we cannot understand whether the choice comes from maximization of preferences or the replication of past behavior in period* 3. *Nevertheless, S2 was active in period* 3 *and one can easily see that* $f(5 \mid a_5) = 2/5 \leq 3/7 = f(3)$, *making observation* 5 *linked with observation* 3 *and according to Proposition 1, making it generated by S2 too. This is represented in Figure 1.2.*

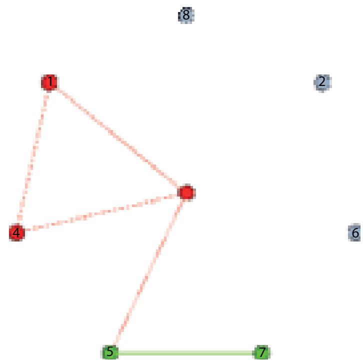
Figure 1.2
5 is linked with 3



Source: Own elaboration.

Consider now observation 7. *We cannot directly link observation* 7 *to either observations* 1, 3 *or* 4, *because* $f(7|a_7) = 1/2 > \max\{f(1), f(3), f(4)\}$. *However, we can indirectly link observation 7 to observation 3 through observation 5, because* $f(7|a_7) = 1/2 \leq 1/2 = f(5)$, *thus making* 7 *an element of* D^N. *See Figure 1.3 for a graphical description. No other observation is indirectly linked to observations* 1, 3 *or* 4 *and hence,* $D^N = \{1, 3, 4, 5, 7\}$. *The method rightfully excludes all S1 observations from* D^N.

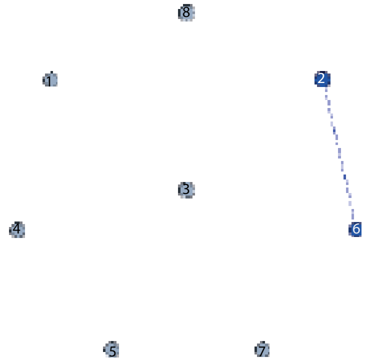
Figure 1.3
7 is (indirectly) linked with (3) 5



Source: Own elaboration.

The example presents inconsistencies in the revealed preference. Observation 3 *and* 6 *are both in conflict with observation* 2*. That is, observations* 2 *and* 3 *and* 2 *and* 6 *form cycles. Then, noticing that* max $\{f(2), f(3)\} = f(2)$ *and that* max $\{f(2), f(6)\} = f(2) = f(6)$ *we can say that observations* 2 *and* 6 *are generated by S1 thanks to Proposition 1, given they are least novel in a cycle. It is immediate again to see that* 2 *and* 6 *are connected by an undirected edge. See Figure 1.4 for a graphical description.*

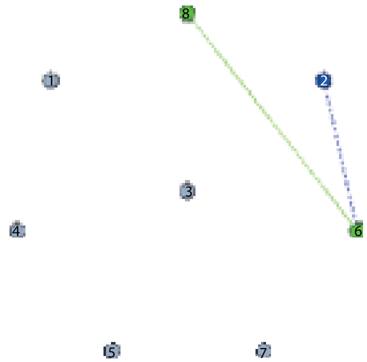
Figure 1.4
Least novel observations in a cycle



Source: Own elaboration.

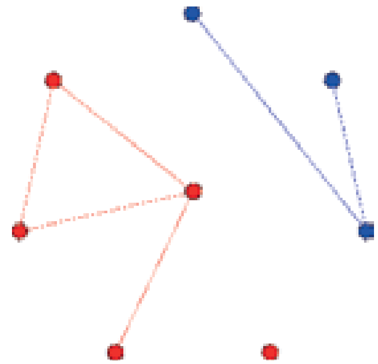
But then, notice that observation 6 *is linked to observation* 8 *given that* $f(6|a_6) = 3/5 \leq f(8) = 3/5$ *revealing that the latter must have been generated by S1 too. Figure 1.5 shows this idea graphically. Thus, we get* $D^C = \{2, 6, 8\}$ *that were the observations rightfully excluded from* D^N*. No decision made by S2 has been cataloged as intuitive. Thanks to the algorithm, we found the two connected components as shown in Figure 1.6.*

Figure 1.5
6 is linked with 8



Source: Own elaboration.

Figure 1.6
D^N and D^C



Source: Own elaboration.

The modified revealed preference exercise helps us determine that alternative 1 *is better than any alternative from* 2 *to* 7*, alternative* 3 *is better than any alternative from* 4 *to* 6*, and alternative* 2 *is better than alternatives* 4*,* 7 *and* 8 *as it is indeed the case. The value of the similarity threshold a by Proposition 1 can be correctly determined to be in the interval* [0.5, 0.6) *thanks to the information retrieved from observations* 7 *and* 8 *respectively.*

Notice that in general D^N and D^C will be proper subsets of the whole S2 and S1 sets of observations, respectively. The set D^N is built upon the set of new observations and those indirectly linked to them.[23] It may be the case that some S2 observations are not linked to other S2 observations.[24] Nonetheless, if the observations are *rich enough,* it is possible to guarantee that D^N and D^C coincide with the sets of conscious and intuitive decisions. In the following section we analyze a rich dataset, where richness is achieved through social data, that allows for the identification of the two sets of conscious and intuitive observations.[25] More importantly, notice that Proposition 1 relies on one important assumption, that is, the collection of observations is generated by a DD process. The following section addresses this issue.

1.5. A CHARACTERIZATION OF DUAL DECISION PROCESSES

In section 1.4 we showed how to elicit the preferences and the similarity threshold of an individual that follows a DD process. Here, building upon the results of that section, we provide a necessary and sufficient condition for a set of observations to be characterized as a DD process with a known similarity function. In other words, we provide a condition that

[23] D^N is never empty because it always contains the first observation.

[24] For this reason, nothing guarantees that $D \backslash D^N$ are intuitive observations and hence, Proposition 1 needs to show how to dually construct a set of intuitive decisions D^C.

[25] In section 1.6 we study the case of a forgetful decision maker and we show how in such case it is possible to identify the two sets when one sequence of observations is rich enough.

can be used to falsify our model. Finally, we provide an alternative characterization of the model whenever the similarity function is also unknown and observations generated by an homogeneous population are available. This alternative characterization allows us to uniquely identify the preferences of the DM and also how similarity comparisons are made.

From the construction of the set D^N, we understand that a necessary condition for a dataset to be generated by a DD process is that the indirect revealed preference we obtain from observations in D^N, *i.e.*, $R(D^N)$, must be asymmetric. It turns out that this condition is not only necessary but also sufficient to represent a sequence of decision problems as if generated by a DD process. One simple postulate of choice characterizes the whole class of DD processes. Interestingly enough though, it is possible to characterize such class with a condition that is computationally easier to test.

Axiom 1 (Link-Consistency). *A sequence of observations* $\{(A_t, e_t, a_t)\}_{t=1}^{T}$ *satisfies Link-Consistency if, for every* $t \in D^N$, $xR(t)y$ *implies not* $yR(L(t))x$.

This is a weakening of the Strong Axiom of Revealed Preference. In fact it imposes that no cycle can be observed when looking at an observation t in D^N and those directly linked to it. This condition is easy to test compu-tationally because it only asks to check for acyclicity between linked obser-vations. The next theorem shows that such condition is indeed necessary and sufficient to characterize DD processes with known similarity.

Theorem 1. *A sequence of observations* $\{(A_t, e_t, a_t)\}_{t=1}^{T}$ *satisfies Link-Consistency if and only if there exist a preference relation* $\succ$ *and a similarity threshold* α *that characterize a DD process.*

The theorem is saying that Link-Consistency makes possible to deter-mine whether the DM is following a DD process or not. In particular, when the property is satisfied, we can characterize the preferences of the DM with a completion of $R(D^N)$ which is asymmetric thanks to Link-Consistency and use the lower bound of α as described in Proposition 1 to characterize the similarity threshold. In fact, by construction, for any observation t outside D^N it is possible to find a preceding observation that can be replicated, *i.e.*, the one defining $f(t|a_t)$. Clearly the familiarity of an observation can be defined only if the similarity function is known. In the final part of this section we show how to identify such function from a population of heterogeneous individuals following a DD process. Here we propose an alternative approach, that also uses social data, that allows us to jointly determine preferences and similarity comparisons.

Notice that we do not assume any particular structure for the sequence of observations we use as data and hence, the characterization of preferences does not have to be unique, even when the similarity is known. One way to uniquely characterize the preferences of the DM that allow us also to determine how similarity comparisons are made, is to get different sequences of observations generated by an homogeneous population. That is, suppose we observe many and different sequences of decision problems and choices generated by a population of individuals sharing the same preferences, similarity threshold and similarity function.[26]

[26] It is possible to argue that individuals that share the same socioeconomic characteristics and have similar cognitive capabilities are likely to be described by the same DD process.

Let $\mathcal{D}$ be the set containing such sequences and $D \in \mathcal{D}$ be the collection of observations composing one of them. Then, if $\mathcal{D}$ is rich enough we can perfectly identify not only the preference relation but also, for every decision environment, which other environments are considered similar enough and which are not. Notice that this completely identifies the model. In fact, the similarity threshold and the similarity function define a binary similarity function, *i.e.,* a Boolean similarity function. That is, given any environment, the combination of similarity function and similarity threshold partitions the set of environments in two sets of similar and dissimilar environments. Thus, knowing for every decision environment which other environments are considered similar enough and which are not, completely identifies such function. To simplify the exposition, from now on, we consider the case in which environments are independent of menus, *e.g.,* frames.[27]

We say that $\mathcal{D}$ is rich if:

- For any $x,y \in X$ there exists a $D \in \mathcal{D}$ such that there is some $t \in D$ where $x, y \neq a_s$ for $s < t$ and $A_t = \{x, y\}$.
- For any $x, y, z \in X$ there exists a $D \in \mathcal{D}$ such that there is some $t \in D$ where $x, y, z \neq a_s$ for $s < t$ and $A_t = \{x, y, z\}$.
- For any $e, e' \in E$ and any $x, y \in X$, there exists a $D \in \mathcal{D}$ such that there is some $t \in D$ where $x, y \neq a_s$ for $s < t - 1$ and $A_{t-1} = \{y\}$, $e_{t-1} = e'$, $A_t = \{x, y\}$ and $e_t = e$.

The first two requirements impose that for every pair and triple of alter-natives, there is some collection D in which until some moment in time t, they have never been chosen.[28] That is, for any pair and triples of alternatives there is a sequence of decision problems in which for some moment in time t they were part of a new observation. The third requirement imposes that for any pair of environments and any pair of alternatives there is a sequence of observations such that the environments are part of two consecutive decision problems in which the two alternatives have never been chosen before. Thus, the choice in t can be either new or the same as in $t-1$. These conditions allow us to perfectly identify the preferences of the homogeneous population and how analogies are made whenever the observed choices satisfy some consistency requirements.

Before stating such requirements, it is useful to define a set for any environment $e \in E$ that contains all those environments that would be considered similar enough to e by a DM following a DD process. Suppose, without loss of generality, that for some $D \in \mathcal{D}$ there exists $t \in D$ such that $A_t = \{x, y\}$ and $a_t = x$ with $x, y \neq a_s$ for $s < t$. If the observations are generated by a DM following a DD process, then t would be a new observation and x would be revealed preferred to y. Then let $S(e)$ be defined as follows:

$$S(e) = \{e' \in E \mid \exists D \in \mathcal{D} \text{ such that, for some } t \in D,\ t-1 = (\{y\}, e', y) \text{ is new and } t = (\{x, y\}, e, y)\}.$$

[27] The reasoning and conditions exposed below would follow on in the general case but the notation would be more cumbersome. Material is available upon request.

[28] Notice that the first two conditions play the same role of the Universal Domain assumption in standard choice theory.

For the same reasoning developed before, if the observations are generated by individuals following a DD process, $S(e)$ would contain only environments considered similar enough to e because the observed inversion of preferences in t is possible only when past behavior is replicated. In fact, if y is chosen over x in t, it must be because of replication of behavior in $t-1$. This is a concept similar to the one of revealed preferences. Environment e' is revealed similar to e whenever such inversion of preferences occurs. Notice that by richness, $S(e)$ would contain all those environments that are considered similar enough to e.

Finally, for any collection $D \in \mathcal{D}$ define for all observations $t \in D$ the following set:

$$I(t) = \{x \in X \mid x = a_s, \text{ for some } s < t \text{ such that } a_s \in A_t \text{ and } e_s \in S(e_t)\}.$$

If the observations are generated by DD processes, $I(t)$ would contain all those past choices that could be replicated in t. Now we have all the ingredients to state the consistency requirements that characterize the whole class of DD processes for a rich dataset $\mathcal{D}$. The following axioms are intended for $D, D' \in \mathcal{D}$.

The first axiom requires that conscious choices are consistent. That is, there do not exist two observations in which x is consciously chosen over y in one of them and y over x in the other. This is a weakening of the Weak Axiom of Revealed Preference (WARP).

Axiom 2 (Conscious Consistency (CC)). *For any $t \in D$ and $t' \in D'$ such that $I(t) = I(t') = \emptyset$, if $x, y \in A_t \cap A_{t'}$ and $x = a_t$ then $y \neq a_{t'}$.*

The second axiom requires that intuitive choices come from replication of past behavior.

Axiom 3 (Intuitive Consistency (IC)). *For any $t \in D$ such that $I(t) \neq \emptyset$, $a_t \in I(t)$.*

Then we can state the following theorem.

Theorem 2. *A rich dataset $\mathcal{D}$ satisfies CC and IC if and only if there exist a preference relation $\succ$, a similarity function σ and a similarity threshold α that characterize a DD process. Moreover, the preference relation $\succ$ and the binary similarity function defined by σ and α are uniquely identified.*

Intuitively, the first two requirements of a rich dataset plus CC assure that the revealed preference relation constructed from the observations that would have to be explained as conscious choices is complete and transitive. Then, IC assures that those choices that should be explained as intuitive, replicate past behavior. Uniqueness comes from the fact that every pairwise comparison between alternatives and between environments is observable thanks to richness.

1.5.1. On Conscious and Intuitive Behavior

Throughout all the chapter we have assumed that conscious behavior is the maximization of a given preference relation. In some cases this assumption can be too strong. The framework we propose does not depend on the particular conscious behavior that is assumed. In fact, as the previous analysis should clarify, for any given decision environment analogies determine a partition with two components, one containing those problems that are similar enough to the reference environment and another one containing those that are not. Such partition is based only on one and simple assumption, intuitive behavior must come from the replication of past behavior.

Alternative conscious behaviors are possible, the only element of the formal analysis we conducted that has to be changed is what kind of consistency requirement to test on those problems that fall in the part of the partition that has to be assigned to conscious behavior.[29] Thus we can think about preferences that depend on the decision environment or preferences that satisfy only quasi-transitivity or less consistent choice behaviors that are falsifiable and we would still be able to run the same kind of analysis.

The purpose of this kind of flexible framework is to provide a theoretical benchmark to analyze in a structured way what conscious and intuitive behavior are. The model we propose here is just a first step into this new direction and hence it is simplified to keep the analysis focused on the novel aspect of the problem we address, that is, (i) the relationship between intuitive and conscious choices and (ii) the structure observed behavior should have once we consider these two sources of individual behavior.

Finally, notice that the assumptions that intuitive behavior comes from the replication of past behavior is much less restrictive than what would appear at a first glance. We have considered only the case of replicating past choices made by *the same* DM because we are considering a fictitious and quite restrictive decision theoretic environment. It would be easy to incorporate social considerations by changing what experiences and decision problems form part of the DM's memory. For example, we can consider cases where the DM stores in his memory not only his experiences, but also his parents', siblings' or friends' experiences thus making the concept more general than what the literal interpretation of the model would suggest.

1.5.2. Estimation of the Similarity Function

The similarity function is a key component of a DD process and for the sake of exposition it is assumed to be known in the main part of the chapter. Nonetheless, we discuss here how to identify it by studying the choice behavior of a group of individuals sharing it.

Consider a continuous population of individuals sharing the similarity function σ, with a continuous and independent distribution of the similarity threshold over [0, 1]. Sequences of decision problems and preferences are independently distributed.

Consider a pair of alternatives x, y $\in X$ such that each of them is considered better than the other for a non-negligible part of the population. For every pair of environments $e, e' \in E$, assume there exists a non-negligible subpopulation for which there is an observation t as follows:

- $x \notin A_t$, $A_{t+1} = A_t \cup \{x\}$ and no alternative in A_{t+1} was chosen before t,

[29] Obviously, dually, to find intuitive choices we should analyze violations of such requirements.

- $e_t = e'$ and $e_{t+1} = e$ and
- $a_t = y$.

The main result of this section shows that we can compare the similar-ity of two different pairs of environments by considering the correspond¬ing aforementioned subpopulations and sampling them. Formally, denote by $v(e, e')$ the average relative number of randomly sampled individuals sticking to y at $t + 1$. That is, for any pair of environments (e, e') we take a sample of finite magnitude n from the aforementioned subpopulations and we compute the average of the relative number of individuals that stick to y in such sample. Such average is $v(e, e')$.

Proposition 2 (Eliciting the Similarity). *For every two pairs of environments (e, e') and (g, g'),* $Pr(v(e, e') \geq v(g, g') | \sigma(g, g') > \sigma(e, e')) \xrightarrow{p} 0$. *That is, the probability of having* $v(e, e') \geq v(g, g')$ *when* $\sigma(g, g') > \sigma(e, e')$ *probabilistically converges to zero.*

Given our assumptions, each sample we take to calculate $v(e, e')$ is an independent estimation of the relative number of individuals that sticks to y. Thus, we are getting a consistent estimate of the relative number of individuals that would stick to y in the whole subpopulation. Then, comparing the average relative number of individuals that stick to alternative y in $t + 1$ for different pairs of environments, gives the required information on the similarity function. The main intuition of Proposition 2 is the following. There are two reasons that force the average relative number of people sticking to y with the pair of environments (e, e') to be bigger than the average relative number of people sticking to y with another pair of environments (g, g'). First, the pair of environments (e, e') is more similar than the pair of environments (g, g'). This implies that S1 is active in $t + 1$ for a larger number of individuals on average, which leads to replication of the choice in t, *i.e.*, alternative y.[30] Second, y is preferred to x by a larger number of sampled people among those using S2 with the pair of environments (e, e'). The law of large numbers makes the second concern disappear as the number of samples taken to measure v grows, revealing the similarity of different pairs of environments.

1.6. EXTENSIONS

The analysis developed in the previous sections is based on two assump-tions that we relax here. First, we have assumed that the DM has perfect memory. In this section we show that such an assumption is not needed to perform the algorithmic analysis. Moreover, if some richness conditions are satisfied by the sequence of decision problems under study, we show that by considering imperfect memory we can completely identify the preferences of the DM and determine which system generated every single ob-servation by studying just one sequence of observations. The second assumption we relax concerns the similarity function. We show that the analysis of the previous sections is perfectly valid, even if we have only partial information regarding the similarity function.

[30] Notice that our assumptions imply that only the choice in t can be replicated if S1 is active.

1.6.1. A Forgetful Decision Maker

So far, we have assumed that the DM has perfect memory, *i.e.*, intuitive decisions can come from the replication of *any* past choice. In this section, we depart from such assumption and analyze the possibility of a DM that *forgets* older choices.

Suppose the DM can remember up until $m \geq 1$ periods of past choices. In a DD-m process, the chosen action in period t is:

$$a_t = \begin{cases} a_s \textit{ for some } t - m \geq s < t \text{ such that } \sigma(e_t, e_s) > \alpha \text{ and } a_s \in A_t, \\ \text{the maximal element in } A_t \text{ with respect to } \succ \text{, otherwise} \end{cases}$$

Notice that we have just changed the periods that are considered by S1 for the replication of behavior, the structure of the process is otherwise unchanged. Thus, if we take this new assumption into account, we should be able to directly apply the logic behind the algorithm to this new framework. This is indeed the case.

We say that an observation is *new with imperfect memory* whenever $a_t \neq a_s$ for all $t - m \leq s < t$. That is, the choice in t was never chosen in the previous m periods. Such observations must be generated by S2 for the same logic explained in section 1.4. In a similar fashion, let *unconditional* and *conditional familiarity with imperfect memory* be as follows:

$$\bar{f}\ (t) = \max_{t\text{-}m \leq s < t, a_s \in A_t} \sigma\ (et,\ es).$$

$$\bar{f}\ (t \backslash a_t) = \max_{t\text{-}m \leq s < t, a_s = a_t} \sigma\ (et,\ es).$$

Again, the only change is that now, for any observation t, only the preceding m periods are important for the replication of behavior, and so they are the only ones considered when defining the two concepts of familiarity. Then, we say that an observation is *the least novel in a cycle with imperfect memory* whenever it maximizes the unconditional familiarity with imperfect memory among those observations in the cycle. For the same logic we used before, any least novel observation in a cycle must be generated by S1.

Finally, we say that observation t is linked to observation s whenever $\bar{f}\ (t|a_t) \leq \bar{f}\ (s)$, and indirectly linked observations are defined in an analogous way. Thus, considering imperfect memory only changes the key defi-nitions on which the algorithm is based, not the logic behind it. Again, any observation linked to an observation generated by S2 must be generated by S2 too. Symmetrically, any observation to which an observation generated by S1 is linked, must be generated by S1 too. Denote with $D^{\bar{N}}$ the set containing all observations that are indirectly linked to new observations with imperfect memory and with $D^{\bar{C}}$ the set containing all observations to which a least novel in the cycle with imperfect memory is linked. Then we can state the parallel version of Proposition 1. The proof is omitted.

Proposition 3. *For every collection of observations D generated by a DD-m process:*

1. *All observations in* $D^{\bar{N}}$ *are generated by S2 while all observations in* $D^{\bar{C}}$ *are generated by S1,*

2. *if x is revealed preferred to y for the set of observations* $D^{\bar{N}}$, *then* $x \succ y$,

3. $\max_{t \in D^{\bar{N}}} \bar{f}(t) \leq \alpha < \min_{t \in D^{\bar{C}}} \bar{f}(t \backslash a_t)$.

Similarly, let Link-Consistency* be the parallel version of Link-Consistency defined over $D^{\bar{N}}$. We can state the analogous version of Theorem 1. Again, the proof is omitted.

Theorem 3. *A sequence of observations* $\{(A_t, e_t, a_t)\}_{t=1}^{T}$ *satisfies Link-Consistency* if and only if there exist a preference relation* $\succ$ *and a similarity threshold* α *that characterize a DD-m process.*

The analysis of a forgetful individual can provide additional insights if we impose some *richness* conditions on the sequence of decision problems the DM faces. A sequence of observations $\{(A_t, e_t, a_t)\}_{t=1}^{T}$ is *m-rich* for an individual following a $DD - m$ process whenever:

- For any $x, y \in X$ there exists a t such that $x, y \neq a_s$ for $t - m \leq s < t$ and $A_t = \{x, y\}$.
- For any $x, y, z \in X$ there exists a t such that $x, y, z \neq a_s$ for $t - m \leq s < t$ and $A_t = \{x, y, z\}$.
- For any $e, e' \in E$ and any $x, y \in X$, there exists a t such that $x, y \leq a_s$ for $t - m - 1 \leq s < t - 1$ and $A_{t-1} = \{y\}$, $e_t\text{-}1 = e'$, $A_t = \{x, y\}$ and $e_t = e$.

These conditions are similar to the ones we studied in section 1.5 so we do not analyze them further. One important thing to notice is that in this case richness is imposed on *one* sequence of observations not on a *collection* of sequences. Then, we can state the following:

Proposition 4. *For every m−rich sequence of observations* $\{(A_t, e_t, a_t)\}_{t=1}^{T}$ *generated by a* $DD - m$ *process:*

1. $D^{\bar{N}}$ *contains all the decisions generated by S2 and* $D^{\bar{C}}$ *contains all the decisions generated by S1, that is* $D^{\bar{N}} \cup D^{\bar{C}} = D$,

2. *x is revealed preferred to y for the set of observations* $D^{\bar{N}}$ *if and only if* $x \succ y$.

Thus, Proposition 4 highlights the fact that preferences and similarity com-parisons of a forgetful decision maker can be recovered entirely without the need of observing social data whenever the sequence of observations is rich enough.

Furthermore, for the same reasoning developed in Section 1.5, an m−rich sequence of observations allows for the characterization of a DD−m process and the identification of the preferences and analogies of the DM. Given the analysis would be almost identical, in fact only the definitions of $S(e)$ and $I(t)$ would slightly change, we omit it to avoid repetitions.[31]

1.6.2. Revealing S1 and S2 with Partial Information on the Similarity

In this section we show that our algorithmic analysis is robust to weaker assumptions concerning the knowledge of the similarity function. In particular, we study the case in which

[31] Material is available upon request.

only a partial preorder over pairs of environments is known, denoted by $\succeq$.[32] Such extension can be relevant in many contexts where it is not possible to estimate the similarity function. In such cases, it is sensible to assume that at least some binary comparisons between pairs of environments are known. Coming back to the example of the introduction, we might not know how the DM compares different prices and dispositions of the products on the shelf, but we might know that for any combination of prices, a small change in just one price, results in a more similar environment than a big change in all prices.

We show here that, even if the information regarding similarity compar¬isons is partial, it is still possible to construct two sets that contain only S1 and S2 observations respectively, and that one consistency requirement of the data characterizes all DD processes. In order to do so, we assume that, if the individual follows a DD process, the similarity σ cardinally represents a completion of such partial order. Thus, for any $e, e', g, g' \in E$, $(e, e') \succeq (g, g')$ implies $\sigma(e, e') \geq \sigma(g, g')$ and we say that (e, e') dominates (g, g'). As with the analysis of a forgetful DM, we first adapt the key concepts on which the algorithmic analysis is based in order to encompass this new assumption.

The two concepts of familiarity need to be adapted. In particular, given that it is not always possible to define the *most* familiar past environment, the new familiarity definitions will be sets containing undominated pairs of environments. Let $F(t)$ and $F(t|a_t)$ be defined as follows:

$F(t) = \{(e_t, e_s)|s < t, a_s \in A_t$ and there is now $w< \mathrm{t}$ such that $(e_t, e_w) \succeq (e_t, e_s)$ and $aw \in A_t\}$,

$F(t|a_t) = \{(e_t, e_s)|s < t, a_s = a_t$ and there is no $w < t$ such that $(e_t, e_w) \succeq (e_t, e_s)$ and $a_w \in A_t\}$.

That is, $F(t)$ and $F(t|a_t)$ generalize the idea behind $f(t)$ and $f(t|a_t)$, respectively. In fact, $F(t)$ contains all those undominated pairs of environments where e_t is compared with past observations which choice *could be* replicated. Similarly, $F(t|a_t)$ contains all those undominated pairs of environments where et is compared with past observations which choice *could have been* replicated. We can easily redefine the concept of link. We say that observation t is *linked to the set* of observations O whenever either $F(t|a_t) = \emptyset$ or for all $(e_t, e) \in F(t|a_t)$ there exists $s \in O$ such that $(e_s, e') \succeq (e_t, e)$, for some $(e_s, e') \in F(s)$. Two things are worth underlining. First, notice that $F(t|a_t) = \emptyset$ only if t is new, thus, as in the main analysis, new observations are linked with any other observation. Second notice that this time we defined the link between an observation t and a *set* of observations O. This helps understand whether an observation is generated by S2 once we know that another observation is. If all observations in O are generated by S2 and for each one of them there exists a pair of environments that dominates a pair in $F(t|a_t)$ then it must be that S2 generated t too. This is because for all observation s in O, the similarity of *all* pairs of environments contained in $F(s)$ must be below the similarity threshold.

Then, we say that observation t is S2-*indirectly linked to* the set of observations O if there exists a sequence of observations $t_1, \ldots, t_k$ such that $t = t_1$, t_k is linked to O and t_i is

[32] A partial preorder is a reflexive and transitive binary relation. The Symmetric Difference between sets satisfies this assumption.

linked to $\{t_{i+1}, t_{i+2}, ..., t_k\} \cup O$ for every $i = 1, 2, \ldots, k - 1$. Define $D^{\hat{N}}$ as the set containing all new observations and all those observations indirectly linked to the set of new observations. Proposition 5 shows that $D^{\hat{N}}$ contains only S2 observations.

What about S1? As in section 1.4, whenever a cycle is present in the data, we know that at least one of the observations in the cycle must be generated by S1. This time, given that we assume only a partial knowledge of the similarity comparisons, it is not always possible to define a least novel observation in a cycle.[33] Nevertheless, notice that whenever an observation is inconsistent with the revealed preference constructed from $D^{\hat{N}}$, it must be that such observation is generated by S1. Thus, say that observation t is *cloned* if it is either a least novel in a cycle or $xR(t)$ y while $yR(D^{\hat{N}})x$.

Say that observation t is *S1-indirectly linked to* observation s if there exists a sequence of observations $t1, \ldots, t_k$ such that $t = t_1$, $t_k = s$ and t_i is linked to t_{i+1} for every $i = 1, 2, ..., k - 1$. Whenever we know that observation t is generated by S1, we can infer that observation s is generated by S1 too, only if for all pairs of environments in $F(t|A_t)$ there exists a pair of environments in $F(s)$ that dominates it. In fact, in general, only the similarity of *some* pairs of environments contained in $F(t|a_t)$ is above the similarity threshold. As before, let $D^{\hat{C}}$ be the set containing all cloned observations and the observations to which they are indirectly linked. Proposition 5 below shows that $D^{\hat{C}}$ contains only S1 observations.

Proposition 5. *For every collection of observations D generated by a dual decision process where only a partial preorder over pairs of environments is known:*

1. *all decisions in $D^{\hat{N}}$ are generated by S2 and all decisions in $D^{\hat{C}}$ are generated by S1,*
2. *if x is revealed preferred to y for the set of observations $D^{\hat{N}}$, then $x \succ y$.*

Thus, we see that knowing only a partial preorder does not heavily affect the structure of the algorithm and the main logical steps behind it. What is of interest is that even with this assumption it is possible to characterize a DD process with just one single condition, that is $D^{\hat{N}}$-Consistency.

Axiom 4 ($D^{\hat{N}}$-Consistency). *A sequence of observations $\{(A_t, e_t, a_t)\}_{t=1}^{T}$ satisfies $D^{\hat{N}}$-Consistency whenever $xR(D^{\hat{N}})y$ implies not $yR(D^{\hat{N}})x$.*

$D^{\hat{N}}$-Consistency imposes asymmetry on the revealed preference obtained from $D^{\hat{N}}$. If a sequence of decision problems satisfies $D^{\hat{N}}$-Consistency when only a partial preorder is known, then we are able to characterize the preferences of the individual, the similarity threshold and, more impor-tantly, a similarity function that respects such preorder. This is what the next theorem states. Notice that $\succeq$ is assumed to be known.

Theorem 4. *A sequence of observations $\{(A_t, e_t, a_t)\}_{t=1}^{T}$ satisfies $D^{\hat{N}}$-Consistency if and only if there exist a preference relation $\succ$, a similarity function σ representing $\succeq$ and a similarity threshold α that characterize a DD process.*

[33] Obviously, in this context, a least novel observation in a cycle would be an observation t belonging to a cycle such that for any other observation s in the cycle, $F(t)$ dominates $F(s)$. That is, for any $(e_s, e) \in F(s)$ there exists $(e_t, e') \in F(t)$ such that $(e_t, e') \succeq (e_s, e)$.

Intuitively, the observations in $D^{\hat{N}}$ are used to construct the preference relation of the individual. The similarity function represents a possible extension of the partial preorder that respects the absence of links between observations in $D^{\hat{N}}$ and the ones outside that set. This is possible thanks to how $D^{\hat{N}}$ has been constructed and it allows for the definition of the similarity threshold in a similar fashion as before.

1.7. FINAL REMARKS

Cognitive sciences have highlighted the fact that choices can be divided into two categories. Conscious choices and intuitive ones. This hinders greatly the use of standard economic models that generally do not take into account automatic or intuitive choices.

In this chapter, we make two main contributions. First, we propose a new behavioral model that incorporates the ideas coming from cognitive sciences where individual behavior is seen as the result of the interaction of two systems, one conscious and rule based, the other one unconscious and analogy based, being the latter the source of more intuitive choices. Intuitive choices are the result of replication of past behavior in those decision problems that are perceived as *familiar*, that is those problems that are similar enough with other ones that have been already experienced. Second, we study the implications of the model for observed behavior and we propose an algorithm that allows to understand which choices are truly informative regarding individual preferences and to identify an interval in which the cognitive costs of activating the conscious system should lie. Finally, we provide an axiomatic characterization of the model and we analyze some possible extensions.

The model of decision making we propose can be seen as a possible explanation of different phenomena and puzzles that are observed in different contexts. In particular it can be seen as a source of *stickiness, i.e.,* inertia, in individual behavior. Whenever past behavior can be replicated, analogies between different decision problems make individual choices less responsive to changes in the quality and number of available options.[34] There is a lot of empirical evidence showing that individual behavior does not adapt immediately to changes in the economic environment. Consumption tends to be sticky as Carroll, Slacalek, and Sommer (2011) show. Traders tend to show under-reaction to news and trading behavior is less responsive to market conditions, see for example Chan, Jegadeesh, and Lakonishok (1996). Finally, doctors tend to stick to suboptimal treatments even when no other rational explanation can explain this behavior, *e.g.,* see Hellerstein (1998). The model we propose allows for a formal analysis of these different frameworks in a simple and tractable environment. Take a doctor for example. If analogies are made between patients' symptoms, different patients can be treated with the same drug because they have similar enough symptoms for the doctor, even if such a treatment is suboptimal for the patients. If this is the case, older doctors, by having more experience, should be more prone to make this kind of mistakes, in line with the evidence in Hellerstein (1998).[35]

[34] Notice in fact that whenever past choice can be replicated a necessary condition for behavior to adapt to changes in the menu is that the environment changes sufficiently enough. Clearly this decreases the responsiveness of choices to changes in menus.

[35] See King *et al.* (2013) and Norman and Eva (2010) for a general discussion of the problems in health care and the need of understanding how dual processes influence doctors behavior.

The model provides a novel way of understanding sticky behavior that does not depart too much from the standard framework used in economic modeling and that provides a natural benchmark to formalize heterogeneity in behavior. In fact, the model allows in a same population for adaptive or conscious behavior and for sticky or intuitive one. In fact, for a given population, the aggregate choices will be less responsive to changes in the environment because a part of the population will make intuitive decisions but still there will be a reaction because of the individuals choosing consciously. Individuals with a low enough similarity threshold will stick to past behavior disregarding new possibilities. On the other hand, the rest of the population will react to the change in the environment adapting their choices. As a result, the aggregate choices adapt in a slower manner than what the standard framework would imply while individual choices vary from individual to individual. This kind of dynamics can be crucial for the efficacy of the implementation of a policy, *e.g.*, change in the interest rate, and can lead to wrong predictions if they are not taken into account.

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APPENDIX A

Appendix to Chapter 1

Proof of Proposition 1. We start by proving the statement regarding conscious observations. Trivially, new observations must be generated by S2 since they cannot replicate any past behavior. Consider an observation $t \in D^N$. By definition, there exists a sequence of observations $t_1, t_2, \ldots, t_n$ with $t_1 = t$, $f(t_i|a_{t_i}) \leq f(t_{i+1})$ for all $i = 1,2, \ldots, n-1$ and t_n being new. We prove that t is generated by S2 recursively. We know that t_n is generated by S2. Now assume that t_k is generated by S2 and suppose by contradiction that t_k-1 is generated by S1. From the assumption on t_k, we know that $f(t_k) \leq \alpha$. From the assumption on t_{k-1}, we know that $f(t_{k-1}|a_{t_{k-1}}) > \alpha$, which implies $f(t_{k-1}|a_{t_{k-1}}) > f(t_k)$, a contradiction with the hypothesis. Hence, t_{k-1} is also generated by S2, and the recursive analysis proves that observation t is generated by S2.

We now prove the statement regarding intuitive observations. Consider first an observation t which is a least novel in a cycle and assume by contradiction that it is generated by S2. Then, $f(t) \leq \alpha$. By definition of least novel in a cycle, it must be $f(s) \leq \alpha$ for every s in the cycle, making all decisions in the cycle being generated by S2. This is a contradiction with the maximization of a preference relation. Consider now an observation $t \in D^C$. By definition, there exists a sequence of observations $t1, t2, \ldots, t_n$ with $t_n = t$, $f(t_i|a_{t_i}) \leq f(t_{i+1})$ for all $i = 1,2,\ldots, n-1$ and t_1 being a least novel in a cycle. We proceed recursively again. Since t_1 is generated by S1, we have $f(t_1|a_{t_1}) > \alpha$. Now assume that t_k is generated by S1 and suppose by contradiction that t_{k+1} is generated by S2. We then know that $f(t_k|a_{t_k}) > \alpha \geq f(t_{k+1})$, which is a contradiction concluding the recursive argument.

For the revelation of preferences part, since D^N can only contain observations generated by S2, it is straightforward to see that the revealed information from such a set must respond to the preferences of the DM. Regarding α, notice that since observations in D^N are generated by S2, we know that $\max_{t\in D^N} f(t) \leq \alpha$ and also that, since observations in D^C are generated by S1, we know that $\alpha < \min_{t\in D^C} f(t|a_t)$, which concludes the proof.

Proof of Theorem 1. Necessity: If D is generated by a DD process, then it satisfies Link-Consistency as explained in the text.

Sufficiency: Now suppose that D satisfies Link-Consistency. We need to show that it can be explained as if generated by a DD process. We first show that Link-Consistency implies that the revealed preference relation defined over D^N, *i.e.*, $R(D^N)$, is asymmetric. Asymmetry of $R(D^N)$ means that it is not possible to construct cycles composed by observations in D^N. Suppose by contradiction that we have a cycle in D^N. That is, there is a set of observations

$C = \{t_1, t_2, ..., t_k\} \subseteq D^N$ such that $a_{t_{i+1}} \in A_{t_i}$, $i = 1,...,k-1$ and $a_{t_1} \in A_{t_k}$. Take the observation in the cycle with the highest unconditional familiarity. Denote it with t_{i^*}. Then all the other observations in the cycle are linked to t_{i^*}, that is, $C \subseteq L(t_{i^*})$, contradicting Link-Consistency. Thus, $R(D^N)$ must be asymmetric. By standard mathematical results, we can find a transitive completion of $R(D^N)$, call it $\succ$. By construction, all decisions in D^N can be seen as the result of maximizing $\succ$ over the corresponding menu.

Define $\alpha = \max_{t \in D^N} f(t)$. Notice that by definition of D^N, there is no observation $s \notin D$ such that $f(s|a_s) \leq f(t)$ for some $t \in D^N$. This implies that for all $s \notin D^N$, $f(s|a_s) > \alpha$, so, for all of them, it is possible to find a preceding observation they would seem to replicate. In particular, the one defining $f(s|a_s)$.

Thus, we can represent the choices as if generated by an individual with preference relation $\succ$ and similarity threshold α.

Proof of Theorem 2. Necessity: As said in the text, if all DM in the population follow a DD process with common preferences, similarity function and similarity threshold, then $I(t)$ would contain only those choices that can be replicated at t because they come from some preceding period which environment is similar enough. Then, by definition of a DD process CC and IC must be satisfied.

Sufficiency: Suppose that $\mathcal{D}$ is rich and satisfies CC and IC. We prove that $\mathcal{D}$ can be represented as if generated by a DD process by steps. First we characterize the preference relation, then we characterize the similarity function and similarity threshold and finally we show that the preference relation and the binary similarity function defined by the combination of similarity function and similarity threshold are unique.

As a first step, let P be a revealed preference relation defined as follows. For any $D \in \mathcal{D}$, let xPy if and only if, for some $t \in \mathcal{D}$ such that $I(t) = \emptyset$ and $A_t = \{x, y\}$, $x = a_t$. It is easy to see that CC implies that P is irreflexive and asymmetric. Furthermore, richness of $\mathcal{D}$ implies that the relation is also complete. To see that P is transitive, suppose that xPy, yPz but zPx for some $x, y, z \in X$. By $\mathcal{D}$ being rich, for some $D \in \mathcal{D}$ there exists an observation $t \in D$ such that $x, y, z \neq a_s$ for $s < t$ and $A_t = \{x, y, z\}$. Clearly, given that x, y and z have never been chosen before, $I(t) = \emptyset$. W.l.o.g. suppose that $a_t = x$. By zPx we know that $z = a_{t'}$ for some $t' \in D' \in \mathcal{D}$ such that $A_{t'} = \{x, z\}$ and $I(t') = \emptyset$. But then, by CC we cannot have $a_t = x$ and so we reached a contradiction. Hence P must be transitive. Let $P \succ$.

As a second step we need to characterize the similarity threshold α and the similarity function σ. Let $\alpha = 0$ and the similarity function be as follows. For any $e \in E$:

- $\sigma(e, e') = 1$ whenever $e' \in S(e)$.
- $\sigma(e, e') = 0$ whenever $e' \notin S(e)$.

The third requirement of a rich dataset assures that if $e' \notin S(e)$ then no inversion of preferences is observed due to replication of behavior associated with e' when e is the reference environment.

We now show that constructing the preference relation and the similarity function in this way allows to explain all choices. In fact, the definition of the similarity function and threshold implies that any observation t such that $I(t) = \emptyset$ must be explained as the outcome of maximization of preferences and any other observation t' such that $I(t') \neq \emptyset$ must be the outcome of replication of past similar behavior. To analyze the first point, take any observation t such that $I(t) = \emptyset$. Suppose that x maximizes $\succ$ in the menu A_t, but $y = a_t$ for some $t \in D \in \mathcal{D}$ and $y \neq x$. By definition of P, x being the maximal element in A_t implies that for any $y \in A_t \setminus \{x\}$, there exists $t' \in D_, \in \mathcal{D}$ such that $A_{t'} = \{x, y\}$ and $a_{t'} = x$. Thus, by CC, we cannot have $a_t = y$ and so we reached a contradiction. Thus, we can identify the preference relation with P. Regarding the second point, take any observation t' such that $I(t') \neq \emptyset$. Then, there is some $s < t'$ such that $a_s \in A_{t'}$ and $\sigma(e_{t'}, e_s) = 1 > \alpha$ and then IC assures that the choice in t' is the same than the choice in one of such past problems as a DD process would require.

Finally, we need to show that the preference relation and the binary similarity function are uniquely identified. This is equivalent to show that P and $S(e)$ are uniquely determined. First, suppose that we can determine two different relations P and P'. This implies that there exist some $x, y \in X$ such that xPy and $yP'x$. Given that by richness P and P' must be complete, the previous binary relations imply that there are two observations t and t' such that $I(t) = \emptyset$ and $A_t = \{x, y\}$, $x = a_t$ and $I(t') = \emptyset$ and $A_{t'} = \{x, y\}$, $y = a_{t'}$ which contradicts CC, so P is unique. Second, suppose that for some $e \in E$, $S(e)$ is not uniquely determined. That is, suppose we can determine two different sets $S(e)$ and $S'(e)$. This implies that there is an $e' \in E$ such that $e' \in S(e)$ and $e' \notin S'(e)$. By definition of $S(e)$ this is not possible. In fact, $e' \in S(e)$, given richness, can only imply that there are two alternatives $x, y \in X$ such that xPy and that there is some observation t where $x, y \neq a_s$ for $s < t - 1$ and $t - 1 = \{\{y\}, e', y\}$ and $t = \{\{x, y\}, e, y\}$, then e' must be in $S'(e)$ too.

Proof of Proposition 2. Let $\mu(e, e')$ be the relative number of individuals that would choose y *in the whole non-negligible subpopulation* from which the sample defining $v(e, e')$ is taken. First, notice that whenever $\mu(e, e') \geq \mu(g, g')$ it implies that environments (e, e') are more similar than environments (g, g'). In fact, given that preferences and decision problems are independently distributed in the population, any of the subpopulations we consider is representative of the whole population. Then, given that in the whole population no alternative is preferred over the other by every individual, if the actual relative numbers are different it implies that one pair of environments is more similar than the other. Moreover, given α is continuously and independently distributed in the population we can compare any pair of environments with any other pair and find how they are related by the similarity function. That is, for any $e, e', g, g' \in E$ such that $\sigma(e, e') < \sigma(g, g')$ it must be $\mu(e, e') < \mu(g, g')$ given that there always exists a non-negligible part of the whole population with similarity threshold α in the interval $(\sigma(e, e'), \sigma(g, g'))$ and again, the subpopulations are representative. Then, notice that given that every subpopulation is infinite, any sample that defines $v(e, e')$ for any $e, e' \in E$ can be considered independent. Thus, the law of large numbers applies in this context and we get, and the result follows.

$$Pr(|v(e, e') - \mu(e, e')| > \varepsilon) \xrightarrow{p} 0,$$

Proof of Proposition 4. Given Proposition 3, we just need to show the following:

1. If t is generated by S2 then it is contained in $D^{\bar{N}}$.

2. If t is generated by S1 then it is contained in $D^{\bar{C}}$.

3. If $x \succ y$ then $xR(D^{\bar{N}})y$.

We start with point 3. Notice that, by m-richness, for any pair of alternatives $x, y \in X$ there exists a t such that $x, y \neq a_s$ for $t-m \leq s < t$ and $A_t = \{x, y\}$. Thus, for any pair of alternative there is a new observation that reveals the preferences of the DM and the result follows.

Now we analyze point 1. First notice that for any DD-m process there exists a pair of environments (e^*, e^{**}) for which there is no $(e, e') \in E \times E$ such that $\sigma(e^*, e^{**}) < \sigma(e, e') \leq \alpha$. That is, e^* and e^{**} maximize the value of the similarity function among those pairs of environments that that are considered *dissimilar enough* by the DM. Take $x, y \in X$. By the proof of point 3, we know that for any pair of alternatives we can determine the preference of the DM, thus we can assume w.l.o.g. that $x \succ y$. Then, by m-richness there exists a t such that $x, y \neq a_s$ for $t-1-m \leq s < t-1$ and $A_{t-1} = \{y\}$, $e_{t-1} = e^{**}$, $A_t = \{x, y\}$ and $e_t = e^*$. Given $\sigma(e^*, e^{**}) \leq \alpha$, it must be $a_t = x$ making t a new observation and so $t \in D^{\bar{N}}$. Hence, given that only behavior in $t-1$ could be replicated in t, we must have $\underline{\bar{f}}(t) = \sigma(\underline{e}^*, e^{**})$ and so, by definition, for any observation s generated by S2 we must have $\bar{f}(s|a_s) \leq \bar{f}(t) = \sigma(e^*, e^{**})$. Thus, all S2 observations are linked to t, and hence any S2 observation must be in $D^{\bar{N}}$. By a dual argument point 2 can be shown to be valid. This concludes the proof.

Proof of Proposition 5. First we show that any observation t linked to a set O of S2 observations must be generated by S2 too. In fact, notice that for any $s \in O$ we know that $\sigma(es, e') \leq \alpha$ for all $(es, e') \leq F(s)$. Then, given t is linked to O we know that for any $(e_t, e) \in F(t|a_t)$ there exists $s \in O$ such that $(e_s, e') \succeq (e_t, e)$, for some $(e_s, e') \in F(s)$. Now, given the definition of $F(t|a_t)$ this implies that $\sigma(e_t, e_w) \leq \alpha$ for all $w < t$ such that $a_w = a_t$ and the result follows. Then, by Proposition 1 we know that new observations are generated by S2 and applying the previous reasoning iteratively it is shown that $D^{\hat{N}}$ must contain only S2 observations.

As a second step, we show that any observation s to which an observation t generated by S1 is linked, must be generated by S1 too. Given t is generated by S1 it means that there exists a $w < t$ such that $\sigma(e_t, e_w) > \alpha$ and $a_w = a_t$. Then, either $(e_t, e_w) \in F(t|a_t)$ or $(e_t, e_w) \notin/ F(t|a_t)$.

- Let $(e_t, e_w) \in F(t|a_t)$. Then, given t is linked to s, there exists a pair $(e_s, e') \in F(s)$ such that $(e_s, e') \succeq (e_t, e_w)$. This implies $\sigma(e_s, e') \geq \sigma(e_t, e_w) > \alpha$, and the result follows.

- Let $(e_t, e_w) \notin/ F(t|a_t)$. Then, there exists a $w' < t$ such that $(e_t, e_{w'}) \succeq (e_t, e_w)$ and $(e_t, ew' \in F(t|a_t)$. This implies that $\sigma(e_t, e_{w'}) > \sigma(e_t, e_w) > \alpha$. Then, given $_t$ is linked to s we know that for all $(e_t, e) \in F(t|a_t)$, there exists a $(e_s, e') \in F(s)$ such that $(e_s, e') \succeq (e_t, e)$. In particular, there exists a $(e_s, e') \in F(s)$ such that $(e_s, e') \succeq (e_t, e_{w'})$. This implies $\sigma(e_s, e') \geq \sigma(e_t, e_{w'}) > \alpha$, and the result follows.

Then, given that cloned observations are generated by S1, we can apply the previous reasoning iteratively to show that $D^{\hat{C}}$ must contain only S1 observations.

Finally, by a reasoning similar to the one developed in the proof of Proposition 1, given all observations in $D^{\hat{N}}$ must be generated by S2, $R(D^{\hat{N}})$ reveals the preference of the DM.

Proof of Theorem 4. Necessity: Suppose that the sequence $\{(A_t, e_t, a_t)\}_{t=1}^{T}$ is generated by a DD process. Then it satisfies $D^{\hat{N}}$ -Consistency given that, according to Proposition 5, $D^{\hat{N}}$ contains only S2 observations and $\succ$ is a linear order.

Sufficiency: Suppose that the sequence $\{(At, et, at)\}_{t=1}^{T}$ satisfies $D^{\hat{N}}$ -Consistency. We need to show that it can be explained as if generated by a DD process. Notice that $D^{\hat{N}}$ -Consistency implies that the revealed preference relation defined over $D^{\hat{N}}$, *i.e.* $R(D^{\hat{N}})$, is asymmetric. Thus, by standard mathematical results, we can find a transitive completion of $R(D^{\hat{N}})$, call it $\succ$. By construction, all decisions in $D^{\hat{C}}$ can be seen as the result of maximizing $\succ$ over the corresponding menu.

We now define σ. We first complete $\succeq$. Notice that by construction of $D^{\hat{N}}$, for all $t \notin D^{\hat{N}}$ there exists a pair $(e_t, e) \in F(t|a_t)$ such that there is no $s \in D^{\hat{N}}$ for which $(e_s, e') \succeq (e_t, e)$, for some $(e_s, e') \in F(s)$. That is, for all observations not in $D^{\hat{N}}$ there exists a pair of environments that is not dominated by any pair of environments of observations in $D^{\hat{N}}$, a pair that we call *undominated*. Then, let $\succeq'$ be the following reflexive binary relation. For any undominated pair $(e_t, e) \in F(t|a_t)$ with $t \notin D^{\hat{N}}$, let for all $s \in D^{\hat{N}}$ and for all $(e_s, e') \in F(s)$, $(e_t, e) \succeq' (e_s, e')$ and not $(e_s, e') \succeq' (e_t, e)$. Let $\succeq''$ be the transitive closure of $\succeq \cup \succeq'$. Notice that $\succeq''$ is an extension of $\succeq$ that preserves its reflexivity and transitivity. Thus we can find a completion $\succeq^*$ of $\succeq''$ and a similarity function $\sigma : E \times E \rightarrow [0, 1]$ that represents $\succeq^*$.

Finally, we can define α. For any observation t, let $f^*(t)$ be as follows:

$$f^*(t) = \max_{s<t, a_s \in A_t} \sigma(e_t, e_s),$$

Then let $\alpha = \max_{t \in D^{\hat{N}}} f^*(t)$. Notice that by construction of σ for all $t \notin D^{\hat{N}}$ there exists a pair of environments $(e_t, e) \in F(t|a_t)$ such that for all $s \in D^{\hat{N}}$, $\sigma(e_t, e) > f^*(s)$, hence $\sigma(e_t, e) > \alpha$. So, for every observation not in $D^{\hat{N}}$ we can find a preceding observation to imitate.

Thus, we can represent the choices as if generated by an individual with preference relation $\succeq$, similarity function σ and similarity threshold α.

2 DUAL DECISION PROCESSES AND NOISE TRADING

2.1. INTRODUCTION

Price movements depend on traders beliefs and how they use the information they have regarding different assets. Since Grossman and Stiglitz (1980), it has been debated in the literature whether markets can be fully informationally efficient, that is, are agents fully informed? If this is the case, their demand functions should include all the available information and thus, any variation in prices should be the consequence of unexpected noise as summarized in Fama (1970). Here we propose a behavioral model that presents a channel through which markets might fail to be informationally efficient.

The hypothesis that economic agents in financial markets can be described by the rational model of decision making is in contrast with some of the evidence that has been documented in the last decades in the literature.[1] Asset prices move through time in ways that cannot be fully explained by movements in their fundamental values.

In this chapter we model an economy in which markets are not fully efficient because of the presence of some traders, noise traders, that use *bad information*.[2] This idea is not new in the literature and it is present at least since the seminal work by De Long *et al.* (1990). The main and crucial difference is that we consider a particular cognitive process that makes the presence of noise traders to emerge endogenously due to the changes in information that agents face hence, in contrast with De Long *et al.* (1990), we explicitly model how noise traders form their beliefs. Having a clear model of beliefs formation allows us to show that the endogenous formation of noise traders helps explaining the emergence of underreaction and momentum and also gives some interesting insights regarding overreaction and the equity premium puzzle. These phenomena are at odds with the efficient markets hypothesis because they imply that price movements can be partially predicted.

In Section 3.2 we build on the model presented in chapter 1. Traders here receive information regarding present and future dividends generated by a risky asset and they have to decide how much of their income to invest in such asset. Whenever the information they receive is familiar, that is, present and future dividends are similar enough for them, where enough depends on a threshold, they do not update their beliefs. Thus, the intuitive self is

[1] See Shiller (1990, 2003 and 2014).

[2] See DeLong (2005) for a discussion regarding agents that trade on bad information.

active and they trade using old information. Otherwise, the rational self is active and they consciously consider all the information present in the market and decide how much of the asset to buy.

Noise traders arise because for some individuals, the new information is *indistinguishable* from past one, hence they do not update their beliefs. By doing so they use outdated information that is not relevant in the new market situation. On the other hand, individuals that are able to perceive the change in information behave rationally. Thus, in any moment in time, there are two types of traders, noise traders and fully rational ones. Their proportions will depend on the magnitude of the variation in information and on how the similarity threshold, that determines what is *indistinguishable*, is distributed in the population. In general, if for an individual past and new information are similar enough, all traders with an even lower threshold will consider the two pieces of information *indistinguishable* too while all others will behave rationally.

In Section 2.3, in an overlapping generations model in which traders live two periods and that behave following the process we explained earlier, we show that in equilibrium prices of risky assets are not at their fundamental value and that, thanks to the endogenous formation of noise traders, their movements qualitatively reproduce empirical facts that have been documented in the literature. First, prices are more volatile because the presence of noise traders increases the overall risk of the economy. Prices vary due to changes in fundamentals in a direct way, as in the standard rational model, but also in an indirect way due to the change in the fraction of noise traders that such changes in fundamentals imply. These changes in noise trading are subject to uncertainty making rational traders unwilling to fully take advantage of arbitrage opportunities and thus prices vary more than what would happen in a rational framework. Second, prices underreact to changes in information because of the fact that at any moment in time there is always a fraction of traders, noise traders, that form their demand functions using *old* information, and thus do not react to the *new* information. Finally, prices can overreact to changes in information in the lung-run. Information gets gradually incorporated into prices due to underreaction, thus, under some circumstances, the effect of old information being incorporated in prices and new information becoming available can sum up and amplify the movement of prices thus causing overreaction.

Finally, in Section 2.4.1 we discuss how non-deliberative thinking can shed new light on the equity-premium puzzle. Since Mehra and Prescott (1985), there is evidence that investment in risky assets is too low given their returns with respect to riskless bonds and such behavior can be accommodated in the standard rational framework only by assuming that traders are extremely risk averse. As we previously explained, the kind of noise trading we model here has two effects on financial markets. First, it increases the risk and second it might cause prices to overreact in the long-run. These two effects imply that risky returns can often be higher than normal due to overreaction while on the other hand the increase in market risk implies that the risky asset is less demanded than in a rational economy, thus creating an intuitive channel for the puzzle to arise. Moreover, our model has some further implications that align with some of the empirical evidence presented in Mehra and Prescott (2003). In particular we discuss how the equity premium should move through time and we argue that, if automatic thinking plays the role we describe, the equity premium should be expected to be counter-cyclical.

The remainder of the chapter is organized as follows. In Section 3.2 we present the model. Section 2.3 defines the equilibrium of the economy and provides the pricing function that describes the pricing of the risky asset for any moment in time. Finally, Section 2.4 studies some implications of the model and 3.5 concludes. All proofs are in the Appendix.

2.1.1. Related Literature

As previously said, the structure of the model is mainly based on two papers. The economy resembles closely the one presented in the seminal paper by De Long *et al.* (1990) while the behavioral model describing traders is based on the same ideas presented in Cerigioni (2017). Having a clear model of belief formation allows us to generalize the structure of De Long *et al.* (1990) in two ways. First, we can endogenize the proportion of noise traders present in the economy and their beliefs and second, we can show that underreaction, momentum and overreaction naturally arise in such economy.

The emergence of underreaction, momentum and overreaction in simple economy is not new in the literature. In fact, Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999) can explain similar phenomena. The three papers provide different models that encompass underreaction and overreaction of prices. Barberis, Shleifer, and Vishny (1998) assume that a representative investor, not knowing the true process generating the earnings of an asset, has wrong beliefs about their movements. In particular they assume that the representative trader thinks that either earnings are mean reverting or they trend. Similarly, Daniel, Hirshleifer, and Subrahmanyam (1998) assume that traders are overconfident about the precision of private information and their confidence moves asymmetrically because of biased self-attribution of investment outcomes. Finally, Hong and Stein (1999) assume that the economy is composed by two types of traders, news watchers that receive different pieces of information regarding the assets that slowly diffuse in the market, and momentum traders that trade based on past price changes. While all these papers can explain underreaction and overreaction, they all do so by abandoning the rational framework, that is, there is no space for fully rational traders in their economies. On the other hand in our model both rational and less than rational traders coexist making our model closer to the standard framework while at the same time proposing a new way of thinking about automatic reasoning and its impact on trading behavior. This allows for a model that has the flexibility and strengths of the standard rational framework while still being able to qualitatively reproduce puzzling price movements empirically observed.

2.2. THE ECONOMY: OLG AND DUAL PROCESSES

The basic structure of our model is taken from De Long *et al.* (1990). As in their formalization, we consider an overlapping generations model with two-period lived traders with no first period consumption, no labor supply decision, no bequests and resources to invest are exogenous. Similarly the economy contains only two assets. Asset s is riskless, it pays a dividend r in every t and its supply is perfectly elastic: a unit can be created out of, and a unit can be turned back into, a unit of consumption good in every period. The price of this asset, given consumption is the numeraire, is fixed at 1. Asset *u* is risky, it pays a dividend θ_t in every *t*, with θ_t defined as a random walk, that is:

$$\theta_t = \theta_{t\text{-}1} + \epsilon_t \text{ with } \epsilon_t \sim N(0, \sigma^2).$$

Its supply is inelastic, *i.e.,* it is in fixed and unchangeable quantity normalized at one. The price of *u* in every t is denoted by p_t. Notice that this is a first difference with respect to De Long *et al.* (1990). In their model in fact there is no uncertainty concerning the dividends of the risky asset, riskiness of the asset is due to the fact that an *exogenous* proportion of traders holds wrong beliefs regarding the dividend that such asset can produce. As it will be clear from the description of the timing of our economy, also in our framework traders will have no uncertainty regarding dividends that are realized during their lives but uncertainty plays a role for two reasons. First, dividends that will be faced by future generations are uncertain. Second, and more important, the *endogenous* proportion of noise traders is uncertain.

At every moment in time *t* a continuum of traders of mass 1 is born. Every generation lives two periods. In the first period traders perceive an exogenous labor income and they decide how many assets to buy in order to maximize their utility in the final period they live. Traders maximize the following mean-variance utility function:

$$\mathbb{E}_t[\omega] - \rho\mathbb{E}_t\left[\sigma^2_\omega\right]$$

Where $\mathbb{E}_t$ is the expectation operator at time *t, ω* is the wealth in the final period, *ρ* is the parameter measuring the absolute risk aversion and σ^2_ω, is the variance of wealth in the final period.[3] As we explained in the introduction, traders are described by a dual self model. Each trader *i'* s is endowed with a similarity function $\sigma : \mathbb{R} \times \mathbb{R} \rightarrow [0,1]$ and a similarity threshold $\alpha_i \in [0,1]$. Every time traders face the problem of deciding how much to invest in the risky asset, they can either use old information because the market environment is unconsciously perceived as similar enough with the past, or they consciously and rationally adapt their behavior to the new environment. Finally, we assume that the distribution of the different similarity thresholds for every generation follows a continuous and time invariant distribution F() with support [0,1] and density function *f*().[4] One can think of *f*() as a simple way of describing the different types of traders present in the market.

The timing of the economy is as follows. For every *t*:

- A generation of traders is born.
- The dividend θ_t is realized and publicly observed.
- A public and perfect signal of the dividend of asset *u* in period *t*+1 is drawn, θ_{t+1}. This implies that there is no uncertainty regarding dividends in period *t* and *t*+1.

[3] With normally distributed returns to holding a unit of the risky asset, maximizing the previous function is equivalent to maximizing expected utility with a CARA utility function: $U = -e^{-(2\rho)\omega}$ or a quadratic utility function.

[4] Notice that the analysis that follows would not be heavily affected by assuming that *ρ* is distributed in the population and that such distribution is time invariant and independent form the distribution of the similarity threshold *α*. We do not analyze this more general case because it would only make the exposition less clear.

- Every trader i maximizes his expected utility in period $t+1$. The only heterogeneity among traders comes from the information they consciously or unconsciously use to *forecast* the dividend of asset u in period $t+1$.

  In fact:

  – If $\sigma(\theta_{t+1}, \theta_t) > \alpha_i$ trader i does not consciously perceive the signal and θ_t is used as if it was the true one.

  – If $\sigma(\theta_{t+1}, \theta_t) \leq \alpha_i$ then trader i consciously perceives the signal and the utility is maximized with the correct information.

- Asset u is bought at a price p_t that clears the market.[5]

- The old generation of traders consumes all its wealth and dies.

Thus, traders are boundedly rational. Whenever the realized dividend and the signal are *similar enough* a fraction of traders do not update their beliefs and do not take new information into account.[6] This is a way of describing a process in which traders tend to think as if past trends will continue in the future. This is in line with the evidence shown in Greenwood and Shleifer, 2014 and Barberis *et al.*, 2015 where traders seem to rely heavily on past performances of assets to predict their future profitability. Moreover, as said in the introduction, this mechanism allows for the endogenization of the fraction of *noise traders*. In fact, in every period t, given the realized dividend and signal, a fraction μ_t of traders will use past information to decide how much of the risky asset to buy. Such fraction will depend on how the similarity threshold is distributed in the population. In fact:

$$\mu_t = \int_0^{\alpha^*} f(\alpha)d\alpha = F(\alpha^*)$$

That is, in every t, a proportion μ_t of traders finds the realized dividend and the signal to be similar enough, *i.e.*, indistinguishable, and thus, they do not update their beliefs.

As previously said, we call traders that use the old information, *i.e.*, those traders which decision is automatic, *noise traders*, while the remaining part of the population will be composed by *rational traders, i.e.*, those traders which decision is deliberate. Then the individual maximization problems are as follows. A noise trader in t maximizes the following:[7]

$$\max_{\lambda_t^N} \mathbb{E}_t^N[\omega] - \rho \mathbb{E}_t^N[\sigma_\omega^2]$$

That is:

$$\max_{\lambda_t^N} c_0 + \lambda_t^N[\theta_t + \mathbb{E}_t^N(p_{t+1}) - p_t(1+r)] - \rho(\lambda_t^N)^2 \mathbb{E}_t^N(\sigma_{p_{t+1}}^2)$$

Where c_0 is a function of the exogenous labor income, $\mathbb{E}_t^N$ stands for the expectation operator at time t given the information noise traders use and λ_t^N is the quantity of asset u demanded in t by noise traders. This is a concave problem with the following necessary and FOC:

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[5] Following De Long *et al.* (1990) we allow quantities and prices to be negative.

[6] This is simplification of a much more general model of behavior. The reader should see this model as a way of formalizing the whole market situation where traders receive many and different pieces of information the understanding and usage of which might be cognitively overwhelming.

[7] Note that N stand for noise traders while R for rational traders.

$$\theta_t + \mathbb{E}_t^N(p_{t+1}) - p_t(1+r) - 2\rho\mathbb{E}_t^N(\sigma_{p_{t+1}}^2) = 0$$

That gives the following demand function:

$$\lambda_t^N = \frac{\theta_t + \mathbb{E}_t^N(p_{t+1}) - p_t(1+r)}{2\rho\mathbb{E}_t^N(\sigma_{p_{t+1}}^2)}$$

On the other hand, a rational trader in t maximizes the following:

$$\max_{\lambda_t^R} \mathbb{E}_t^R[\omega] - \rho\mathbb{E}_t^R[\sigma_\omega^2]$$

That is:

$$\max_{\lambda_t^R} c_0 + \lambda_t^R[\theta_t + \mathbb{E}_t^R(p_{t+1}) - p_t(1+r)] - \rho(\lambda_t^R)^2\mathbb{E}_t^R(\sigma_{p_{t+1}}^2)$$

The necessary and sufficient FOC is the following:

$$\theta_{t+1} + \mathbb{E}_t^R(p_{t+1}) - p_t(1+r)] - 2\rho\lambda_t^R\mathbb{E}_t^R(\sigma_{p_{t+1}}^2) = 0$$

That leads to:

$$\lambda_t^R = \frac{\theta_{t+1} + \mathbb{E}_t^R(p_{t+1}) - p_t(1+r)}{2\rho\mathbb{E}_t^R(\sigma_{p_{t+1}}^2)}$$

Notice that the two types of traders face very similar problems except for the fact that they use different information. Noise traders use the realization of the *dividend* today to forecast dividends tomorrow, on the other hand, rational traders use the realization of the *signal* to decide how much of the risky asset to buy.

2.3. THE EQUILIBRIUM

In equilibrium the demand of the risky asset has to be equal to its supply. That is, formally, it must be, for every t:

$$\mu_t\lambda_t^N + (1-\mu_t)\lambda_t^R = 1$$

Thus, from market clearing we get:

$$\mu_t = \frac{\theta_t + \mathbb{E}_t^N(p_{t+1}) - p_t(1+r)}{2\rho\mathbb{E}_t^N(\sigma_{p_{t+1}}^2)} + (1-\mu_t)\frac{\theta_{t+1} + \mathbb{E}_t^R(p_{t+1}) - p_t(1+r)}{2\rho\mathbb{E}_t^R(\sigma_{p_{t+1}}^2)} = 1$$

That gives the following pricing function:

$$p_t = \frac{\mu_t\left(\theta_t + \mathbb{E}_t^N(p_{t+1}) - 2\rho\mathbb{E}_t^N(\sigma_{pt+1}^2)\right)2\rho\mathbb{E}_t^R(\sigma_{pt+1}^2)}{(1+r)\mu_t 2\rho\mathbb{E}_t^R(\sigma_{pt+1}^2)} + \frac{(1-\mu_t)\left(\theta_{t+1} + \mathbb{E}_t^R(p_{t+1}) - 2\rho\mathbb{E}_t^R(\sigma_{pt+1}^2)\right)2\rho\mathbb{E}_t^N(\sigma_{pt+1}^2)}{1(\mu_t)2\rho\mathbb{E}_t^N(\sigma_{pt+1}^2)}$$

Now, to ease the reading, we use the following notation that stresses the information the different traders are using:

- $\mathbb{E}_t^N(p_{t+1}) = P_{t+1}(\theta_t)$.
- $\mathbb{E}_t^R(p_{t+1}) = P_{t+1}(\theta_{t+1})$.
- $\mathbb{E}_t^N(\sigma_{p_{t+1}}^2) = V_{t+1}(\theta_t)$.
- $\mathbb{E}_t^R(\sigma_{p_{t+1}}^2) = V_{t+1}(\theta_{t+1})$.

Thus, the previous pricing function becomes as follows:

$$p_t = \frac{\mu_t 2\rho V_{t+1}(\theta_{t+1})(\theta_t + P_{t+1}(\theta_t) - 2\rho V_{t+1}(\theta_{t+1}))}{(1+r)[\mu_t 2\rho V_{t+1}(\theta_{t+1}) +} + \frac{(1-\mu_t)2\rho V_{t+1}(\theta_t)(\theta_{t+1} + P_{t+1}(\theta_{t+1}) - 2\rho V_{t+1}(\theta_{t+1}))}{(1-\mu_t)2\rho V_{t+1}(\theta_t)]}$$

If we define the relative variance of the price due to rational traders' expectations as follows:

$$\beta_t = \frac{\mu_t 2\rho V_{t+1}(\theta_{t+1})}{\mu_t 2\rho V_{t+1}(\theta_{t+1}) + (1-\mu_t)2\rho V_{t+1}(\theta_t)}$$

We can write the previous expression as:

$$p_t = \frac{1}{1+r}\left[\beta_t(\theta_t + P_{t+1}(\theta_t) - 2\rho V_{t+1}(\theta_t)) + (1-\beta_t)(\theta_{t+1} + P_{t+1}(\theta_{t+1}) - 2\rho V_{t+1}(\theta_{t+1}))\right] \quad [2.1]$$

The price in t is a convex combination of the utilities the two types of traders expect to get in t + 1 from holding a unit of the risky asset. The utility a trader gets from holding the asset in t is the dividend he believes the asset will pay, plus its selling value, that is the expected price in t + 1, minus the expected variance of the price that negatively affects traders' utility. Then, such utility is weighted by the relative frequency of the type in the market multiplied by the variance of the price the other type expects for t+ 1. Thus, in a way, a certain trader type's expectation is more important in driving the price today, the higher is the measure of that type in the economy and the less variant his prediction is with respect to the other type.

2.3.1. A Consistent Pricing Function

Equation [2.1] highlights the importance of understanding traders' expectations to obtain a closed-form solution of the model. We assume that traders form their expectation rationally except for the fact that they use different informations. That is, they form expectations knowing that there can be two types of traders in the economy, that their proportions depend on the realized dividend and signal, but they are boundedly rational in the sense that they do not use such information to understand from the price in t whether their decisions are conscious or automatic. This assumption, admittedly strong with just two assets, tries to capture the complexity of cognitively processing a real economy with many different assets and sources of information that affect automatic thinking but might not be consciously perceived.

To obtain the pricing function then we first need to define how $P_{t+1}(\theta_t)$, $P_{t+1}(\theta_{t+1})$, $V_{t+1}(\theta_t)$ and $V_{t+1}(\theta_{t+1})$ are formed. First notice that, in general, given the information in t the two types of traders will have different expectations regarding the price in the next period, that is:

$$P_{t+1}(\theta_t) \neq P_{t+1}(\theta_{t+1})$$

On the other hand, given the process generating the dividends, the expected variance of prices should not depend on the reference the different traders use. In fact, the price can vary due to the signal in $t + 1$ and the fraction of noise traders in $t + 1$. These two factors only depend on the noise term that realizes with the signal in $t + 1$ that is independent of the reference. Hence:

$$V_{t+1}(\theta_t) = V_{t+1}(\theta_{t+1}) = V_{t+1}$$

Given this, we can rewrite the pricing function as follows:

$$p_t = \frac{1}{1+r}\left[\mu_t\left(\theta_t + P_{t+1}(\theta_t)\right) + (1-\mu_t)\left(\theta_{t+1} + P_{t+1}(\theta_{t+1})\right) - 2\rho V_{t+1}\right]$$

Then, given that traders use the actual pricing function to form their expectations as we said in the beginning of this section, we get:

$$P_{t+1}(\theta_t) = \mathbb{E}_t\left[\frac{1}{1+r}\left[\mu_{t+1}\left(\theta_t + P_{t+2}(\theta_{t+1})\right) + (1-\mu_{t+1})\left(\theta_t + \varepsilon_{t+2} + P_{t+2}(\theta_t + \varepsilon_{t+2})\right) - 2\rho V_{t+2}\right]\right]$$

$$P_{t+1}(\theta_{t+1}) = \mathbb{E}_t\left[\frac{1}{1+r}\left[\mu_{t+1}\left(\theta_{t+1} + P_{t+2}(\theta_{t+1})\right) + (1-\mu_{t+1})\left(\theta_{t+1} + \varepsilon_{t+2} + P_{t+2}(\theta_{t+1} + \varepsilon_{t+2})\right) - 2\rho V_{t+2}\right]\right]$$

Now notice that a particular trader will form in t the same expectation for all future periods prices and variances given that dividends follow a random walk. Thus, expectations have a closed form solution and so it is possible to show the following.

Proposition 6. *The price of the risky asset at time t is defined by the following equation:*

$$p_t = \frac{1}{r}\left[\theta_{t+1} - \mu_t\varepsilon_t + 1 - \frac{2\rho}{(1+r)^2}\left(\sigma_\varepsilon^2 + \sigma_{\varepsilon\mu}^2 - 2\gamma_{\varepsilon^2\mu}\right)\right] - \frac{1}{(1+r)}\gamma_{\varepsilon\mu}, \qquad [2.2]$$

where $\sigma_{\varepsilon\mu}^2$ is the variance of the product between the noise and the fraction of noise traders in the market, $\gamma_{\varepsilon^2\mu}$ is the covariance between the distance between two consecutive dividends and the proportion of noise traders and, similarly, $\gamma_{\varepsilon\mu}$ is the covariance between the noise and the fraction of noise traders in the market.

Thus, the price today is a function of the signal of future dividends, θ_{t+1}, the fraction of noise traders in the economy that do not perceive the signal, *i.e.*, μ_t, the variance due to the random walk of dividends, i.e., σ_ε^2, the variance due to noise trading, *i.e.*, $\sigma_{\varepsilon\mu}^2$, and finally the covariance between the noise ε and the fraction of noise traders μ, that is $\gamma_{\varepsilon\mu}$. Notice that whenever the similarity function is symmetric, *e.g.*, when it is the inverse of a distance function, $\gamma_{\varepsilon\mu}$ must be zero. In fact, if the similarity function is symmetric, negative or positive realizations of the noise ε will have the same effect on the similarity comparisons, only their absolute value is of importance. Hence, a symmetric similarity implies that positive or negative noises of the same magnitude have the same effect on μ, thus making the covariance equal to zero.

We leave the interpretation and discussion of equation [2.2] to the next section. We will just consider the case of a symmetric similarity function for two reasons. First, it is the most sensible assumption on the similarity function in our framework and second, it increases the clarity of exposition.

2.4. DISCUSSION: UNDERREACTION AND OVERREACTION

First, it is useful to stress an evident characteristic of the model.

Remark 1. *Prices are not at their fundamental value.*

This is immediate to see once we consider a fully rational economy. That is, if traders were fully rational we should have the following pricing function:

$$p_t^* = \frac{1}{r}\left[\theta_{t+1} - \frac{2\rho}{(1+r)^2}\sigma_\varepsilon^2\right] \qquad [2.3]$$

The interpretation of such equation is straightforward and quite standard. The price today is the present value of future utility gains from holding the asset. That is, it is the present value of the difference between the expected dividends and the variance of the dividends due to the noise in the dividend generating process.

Less trivial is to understand whether prices can be greater than their fundamental value. When we take the difference between equation [2.2] and equation [2.3] we get:

$$\frac{1}{r}\left[-\mu_t\varepsilon_{t+1} - \frac{2\rho}{(1+r)^2}\left(\sigma_{\varepsilon\mu}^2 - 2\gamma_{\varepsilon^2\mu}\right)\right]$$

This difference can be non-negative only if:

$$-\varepsilon_{t+1} \geq \frac{1}{\mu_t}\left(\frac{2\rho}{(1+r)^2}\left(\sigma_{\varepsilon\mu}^2 - 2\gamma_{\varepsilon^2\mu}\right)\right)$$

First, a necessary condition for the previous inequality to be satisfied is that ε_{t+1} is non-positive. This is in line with the intuition of the model. That is, a bubble can only appear whenever noise traders do not realize that intrinsic value of the asset due to future dividends is lower than the one they use to form their demand functions. Bubbles emerge when noise traders are bullish. They think good past dividend realizations will continue in the future and they do not notice that the market environment is worse than what they think. In a way, anytime the noise is negative, we should expect noise traders to be too optimistic about the future. It is a stylized way of representing the idea of *animal spirits* that have been extensively analyzed in the literature, see for example Shiller (2003), Akerlof and Shiller (2010) and Shiller (2015). Second the difference between the dividend and the signal has to be big enough to offset the depressive effect noise traders have on the economy. This is not a trivial trade-off. The farther away are the dividend and the signal, the smaller is the proportion of noise traders in the economy and thus, the bigger is the right-hand side. Which effect is preponderant depends on the distribution function of the similarity threshold in the economy and so, given the generality of the framework we analyze here, we do not study it in further detail. The next corollary highlights why the presence of noise traders can be depressive for prices.

Corollary 1. *Noise traders increase the risk in the economy.*

Imagine our simple economy without noise traders. As previously said, the price would be the discounted utility gain of holding the risky asset and the risk traders would bear would depend only on the variance of such price due to the random walk dividends follow. On the other hand, in the kind of economy we model here the presence of noise traders, in line with the literature, increases the overall risk of the market. Future demand for the risky asset is even more uncertain because the distance between the realizations of dividends and signals make the proportion of noise traders to vary. Such added uncertainty means that prices are more volatile than what the rational benchmark would imply. This is in line with a lot of evidence that have been documented in the literature in particular by Shiller, see for example Shiller (1992). The evidence shows that prices in stock markets are too volatile to be explained by a rational asset pricing model and that the movement in prices can somehow be predicted in contrast with the idea that, if markets are rational, prices should follow a random walk. Here we suggest that such volatility is due to noise trading as in De Long *et al.* (1990) but with one important difference. In De Long *et al.* (1990) noise traders increase the risk of the economy because their beliefs are random. Here, we provide a mechanism through which noise trading emerges and beliefs are formed. Noise traders beliefs are not random. Nevertheless, noise traders increase the risk of the economy because their proportion varies with the realization of the signals. Depending on the information available in the economy and how it compares to past one, noise traders can be more or less present in the economy. Thus, their proportion is *ex ante* uncertain. Moreover, given the endogenous formation of noise traders, our model has different implications regarding how prices move. As we show in the following corollaries and in Section 2.4.1, the fact that noise traders are present due to similarity between market environments has some testable implications that are novel and worth studying further. The first one concerns underreaction of prices to news, that is, new information is only partially incorporated into prices.

Corollary 2. *Prices underreact to changes in information in the short-run.*[8]

Thus, the signal influences the price but in a *milder* way with respect to what should happen in a fully rational economy. The signal is not fully incorporated in the price. This is in line with evidence that has been shown in the literature, see for example the seminal paper by Cutler, Poterba, and Summers (1991) and Bernard (1993). We here propose a novel explanation of underreaction as a consequence of non-deliberative thinking. Due to their bounded rationality traders can misperceive the changes in the market environment thus they might have demand functions based on *old* information, causing the prices to be sticky. Prices do not adapt immediately because a portion of traders use past realizations of the dividends to forecast future ones due to the fact that they do not perceive the change in the market

[8] Throughout the chapter we will refer to *short-run* when analyzing an intra period result and *long-run* when analyzing an inter-period result.

environment. Another way to see this is that prices show *momentum*. In fact, momentum in the literature is often defined as the slow incorporation of information into prices. That is, given the information in *t*, the price change between $t + 1$ and *t* can be partially predicted beacuse the price in $t + 1$ will incorporate also the information that was not considered in t due to underreaction.

Corollary 3. *Prices show momentum.*

Here we are in line with the literature. Prices show momentum because of underreaction. Again, the novelty is that this is due to the presence of non-deliberative thinking. When noise traders buy the asset they use the *wrong* belief that past performance will repeat in the future. Once they have to sell the asset to the new generation, the price of the asset will correct due to the fact that the actual realized dividend is different than the one that was used to form their beliefs. To put it differently, due to underreaction in the short-run, prices do not incorporate all the information present in the market, thus, they readjust once the information realizes. That is, prices incorporate fundamental value slowly through time. This is in line with the evidence in Jegadeesh and Titman (1993) and Chan, Jegadeesh, and Lakonishok (1996) among others, that shows that prices have a *momentum pattern,* that is they slowly drift toward the fundamental value. Thus, price changes can be partially predicted. What is of interest here is that the interplay between momentum and underreaction has an additional implication for the predictability of price movements in the long-run. In our economy prices can overreact in the long-run.

Overreaction since De Bondt and Thaler (1985) is defined as the negative covariance between returns in the long-run. In our framework that would translate into prices that overreact to the information present in the market in the long-run and then they adjust toward the fundamental value, creating the negative covariance. It turns out that it is indeed the case in our model.

Corollary 4. *Prices overreact to changes in information in the long-run.*

To understand why this is the case, it is useful to study the difference between two prices in successive periods in our model and in the rational benchmark respectively. That is:

$$\Delta_t(p) = p_{t+1} - p_t = \frac{1}{r}\left[\varepsilon_{t+2} + \left(\mu_t\varepsilon_{t+1} - \mu_{t+1}\varepsilon_{t+2}\right)\right]$$

$$\Delta_t\left(p^*\right) = p^*_{t+1} - p^*_t = \frac{1}{r}\varepsilon_{t+2}$$

The term in parenthesis is the one that can cause overreaction, a phenomenon studied and documented at least since De Bondt and Thaler (1985).[9] It is interesting to first analyze the two components separately. The first term depends on the underreaction of the price in t to news in t. It is highlighting the fact that past news, *i.e.,* ε_{t+1}, affect future prices. By Corollary 3, prices incorporate slowly past information. Thus, the change in price from period *t* to period $t + 1$ can be greater than the one we would have in a rational framework because past

9 See Lee and Swaminathan (2000) for empirical evidence of prices underreacting in the short-run and overreacting in the long-run.

information can make the shift in price due to new information even more accentuated. Thus, prices can overreact, in the sense that the change in price from period $_t$ to period $_t$+1 can be higher than what would be justified by the change in information. Obviously, overreaction depends on the second term in parenthesis that describes underreaction in $t + 1$ due to news in $t + 1$. Notice that underreaction in $t + 1$ is key for the understanding of Corollary 4. The less prices underreact to a change in information in $t + 1$, the higher the returns, due to the fact that momentum and new information play together amplifying the movement in prices. But then, the less the underreaction in $t + 1$, the less the effect of momentum on the successive price change and so the lower the returns. This creates negative covariance between price changes in two successive periods. Notice that these insights will be important to understand how our model can shed some light on the equioty premium puzzle as discussed in the next section.

2.4.1. Equity-Premium Puzzle and Non-Deliberative Thinking

In the previous subsection we have discussed some of the implications of non-deliberative thinking in financial markets. Here we go further, and study the implications of such cognitive process for the equity premium puzzle. Since De Long *et al.* (1990), noise traders risk has been seen as a possible source of the equity premium puzzle. In our framework, the same argument that has been used in the literature can be applied, we do not try to provide a new explanation for such puzzle. Nonetheless, we think that understanding the process that makes noise trading emerge can help having a better grasp of how and when we should expect the equity-premium puzzle to arise and be stronger. In fact, we here discuss how the mechanism we propose can shed new light on this phenomenon and on its empirical regularities.

If the increase of the overall risk of the economy due to noise traders is big enough, risk averse traders will demand less of the risky asset for any given dividend than what they would demand if the economy was fully rational. This implies that the riskless asset is overdemanded with respect to the rational benchmark. This is a stylized way of understanding the equity premium puzzle. Since Mehra and Prescott (1985), the literature has tried to understand why risky assets are less demanded than what rational models would imply. All the different explanations that have been proposed to solve the puzzle had to depart from the standard rational framework by assuming for example habits in consumption as in Constantinides (1990). What we propose here is that the inherent risk that noise traders represent for the economy might be enough to offset the potential gains rational traders can obtain due to the erroneous beliefs noise traders have. Noise traders make the demand of the risky asset vary *too much* due to the fact that their proportion varies with the economy, thus amplifying the inherent risk of the market and making non-profitable for rational traders to bet too much on the risky asset. The *animal spirits* present in financial markets make them too risky, thus depressing the demand. Thus, a higher variability of the price implies two things. First it can make returns higher, as explained in the analysis of Corollary 4, but second it depresses the demand of risk averse traders, thus giving an alternative way of understanding the equity-premium puzzle.

What is of interest here is analyzing the impact that the introduction of automatic reasoning has on the understanding of the equity premium. In order to to this, it is interesting to look again at a price change in *t*:

$$\Delta_t(p) = p_{t+1} - p_t = \frac{1}{r}\left[\varepsilon_{t+2} + \left(\mu_t\varepsilon_{t+1} - \mu_{t+1}\varepsilon_{t+2}\right)\right]$$

As we said, overreaction depends on the term in parenthesis. Thus, to get overreaction of prices we need two conditions to be met; (1) News in *t* and in *t* + 1 have the same sign, that is they are both positive noises or negative noises, and (2) in absolute terms, the effect of underreaction in t was bigger than the effect of underreaction in *t* + 1, taking into account the marginal impact on the fraction of noise traders. The first implication is in line with empirical evidence, see for example Kaestner (2006). In fact, prices seem to overreact more after periods of news of the same sign, that is positive or negative. Thus we study the case of two consecutive signals of the same sign. The second implication is equivalent to the following:

$$|\mu_t\varepsilon_{t+1}| \geq |\mu_{t+1}\varepsilon_{t+2}|.$$

We analyze the case when both ε_{t+1} and $\varepsilon t{+}2$ are positive, the other possibility is symmetric. Clearly the satisfaction of the previous condition depends on the interplay between the change in information, *i.e.,* ε, and the change in the fraction of noise traders, *i.e.,* μ. Thus, it depends on the particular distribution of the similarity threshold in the population. Nonetheless, we are still able to say something for any general distribution that satisfies our assumptions. Notice in fact that the previous inequality is equivalent to the following:

$$\frac{\varepsilon_{t+1}}{\varepsilon_{t+2}} \geq \frac{\mu_{t+1}}{\mu_t}$$

Whenever ε_{t+2} is small enough, prices overreact. In fact, the limit of the left hand side when ε_{t+2} goes to zero is infinite, while the right hand side converges to $\frac{1}{\mu_t}$ given that, due to automatic reasoning, the closer are the dividend and the signal, the more people will trade on noisy information. Thus, we should expect prices to overreact in the lung-run in those situations where past news were relatively more *surprising* than new ones. This can explain even more clearly how bubbles can arise in the long-run when we take non-deliberative thinking into consideration. Periods of high performances of the market should make prices overreact in the long-run creating the possibility of bubbles. This happens because traders do not perceive that the market environment is changing and is less performing than what they believe. On the other hand, when ε_{t+2} is big relative to ε_{t+1} the implications of the model are less clear and depend on the distribution of the similarity threshold in the population. Nevertheless, in the extreme case when ε_{t+2} tends to infinity, both sides of the inequality converge to zero, which means that in the limit there is no overpricing.

Thus prices overreact when market are bullish, so, in those circumstances, the equity premium should be expected to be higher. This happens when the price of the asset is overreacting and a relative downturn should be expected. Even if this intuition is counter-intuitive following the standard economic analysis, it is in line with empirical evidence, as in Mehra and Prescott (2003), where the equity premium seems to be counter-cyclical. In our simplified framework, we can interpret the dividends of the risky asset as the performance of the economy overall. Then, our result regarding overreaction of prices of the asset, is saying that returns should be expected to be high when the economy is expected to slow down and can be low when the opposite happens. That is, whenever ε_{t+2} is small enough relative to ε_{t+1}, returns should be high and thus, given the depressing effect of noise traders on the demand of the risky asset, the equity premium should be larger. On the other hand, when the opposite happens, that is ε_{t+2} is big enough with respect to ε_{t+}, returns should be lower and so the equity premium smaller.

2.5. FINAL REMARKS

In this chapter we formalize the interaction between conscious, deliberative thinking and non-deliberative, automatic one to describe trading behavior in an overlapping generations economy. Whenever present and future dividend realizations are similar enough, traders decisions are automatic, they do not update their beliefs and use old information to decide how much of a risky asset to buy. Otherwise, if dividend are sufficiently dissimilar, traders update their beliefs thus behaving rationally. We show that, whenever the similarity threshold that defines what is similar enough for our traders is continuously distributed in the population, the equilibrium pricing function of such economy can accommodate many empirical regularities by allowing prices to be far from their fundamental value. First, prices are more volatile than what would be normal in a rational framework, second they underreact in the short-run to changes in information and third they overreact in the long-run. Finally, we show how such implications can shed new light on the equity-premium puzzle.

Clearly the model is an oversimplified version of the markets it is supposed to study. It would be interesting to expand the model to the possibility of having more than one risky asset and thus simulate the behavior of the economy to see whether it can reproduce some of the regularities that the empirical distributions of asset prices show. We leave such possibilities for future research.

To conclude, even if stylized, the model we propose gives interesting insights on some empirical facts. In particular we have showed that considering non-deliberative thinking can be useful to understand different puzzling phenomena in a coherent theory of human behavior where *rational* and *less than rational* behaviors can coexist.

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APPENDIX B

Appendix to Chapter 2

Proof of Proposition 6. First notice that, given dividends follow a random walk, the information in t does not make it possible for traders in t to distinguish between the expected prices of future generations, that is:

$$P_{t+2}(\theta_t) = P_{t+2}(\theta_t + \epsilon_{t+2})$$
$$P_{t+2}(\theta_{t+1}) = P_{t+2}(\theta_{t+1} + \epsilon_{t+2})$$

Similarly, we must have:

$$P_{t+1}(\theta_t) = P_{t+2}(\theta_t) = \ldots = P_{t+n}(\theta_t) = P(\theta_t) \text{ for any n} > 1$$
$$P_{t+1}(\theta_{t+1}) = P_{t+2}(\theta_{t+1}) = \ldots = P_{t+n}(\theta_{t+1}) = P(\theta_{t+1}) \text{ for any } n > 1$$
$$V_{t+1} = V_{t+2} = \ldots = V_{t+n} = V \text{ for any } n > 1$$

So, solving recursively the previous equations we get:

$$P(\theta_t) = \frac{1}{r}(\theta_t - 2\rho V - \gamma_{\epsilon\mu}) \qquad [B.1]$$

$$P(\theta_{t+1}) = \frac{1}{r}(\theta_{t+1} - 2\rho V - \gamma_{\epsilon\mu}) \qquad [B.2]$$

Given all these results, we can solve for the expected variance of price. That

$$V = \mathbb{E}_t\left[\left(\frac{1}{1+r}(\theta_{t+} P(\theta_t) + \epsilon_{t+2} - \epsilon_{t+2}\mu_{t+1} - 2\rho V - \theta_t - P(\theta_t) + \gamma_{\epsilon\mu} + 2\rho V)\right)^2\right]$$

Thus, we get:

$$V = \mathbb{E}_t\left[\left(\frac{1}{1+r}((\epsilon_{t+2} - 0) + (\sigma_{\epsilon\mu} - \epsilon_{t+2}\mu_{t+1}))\right)^2\right]$$

Which gives the following result:

$$V = \frac{1}{(1+r)^2}\left(\sigma_\varepsilon^2 + \sigma_{\varepsilon\mu}^2 - 2\gamma_{\varepsilon^2\mu}\right) \qquad [B.3]$$

We can use all the previous result to get the final pricing function:

$$p_t = \frac{1}{r}\left[\theta_{t+1} + \mu_t(\theta_t - \theta_{t+1}) - \frac{2\rho}{(1+r)^2}\left(\sigma_\varepsilon^2 + \sigma_{\varepsilon\mu}^2 - 2\gamma_{\varepsilon^2\mu}\right)\right] - \frac{1}{(1+r)r}\gamma_{\varepsilon\mu}$$

Notice that equation [2.2] given the random walk assumption is equivalent to the following:

$$p_t = \frac{1}{r}\left[\theta_{t+1} + \mu_t \epsilon_{t+1} - \frac{2\rho}{(1+r)^2}\left(\sigma_\varepsilon^2 + \sigma_{\varepsilon\mu}^2 - 2\gamma_{\varepsilon^2\mu}\right)\right] - \frac{1}{(1+r)r}\gamma_{\varepsilon\mu}$$

and the result follows.

Proof of Corollary 1. Notice that the variance of prices in a rational economy is:

$$V^* = \frac{1}{(1+r)2}\sigma_\varepsilon^2$$

On the other hand, the variance in the economy described in the chapter is:

$$V = \frac{1}{(1+r)^2}\left(\sigma_\varepsilon^2 + \sigma_{\varepsilon\mu}^2 - 2\gamma_{\varepsilon^2\mu}\right)$$

Thus:

$$V - V^* = \frac{1}{(1+r)^2}\left(\sigma_{\varepsilon\mu}^2 - 2\gamma_{\varepsilon^2\mu}\right)$$

which is positive given that $\sigma_{\varepsilon\mu}^2$ is positive by definition and $\gamma_{\epsilon^2\mu}$ must be negative. In fact, the higher the distance between dividend and signal, the lower the proportion of noise traders in the economy. Thus the result follows.

Proof of Corollary 2. This is easy to see when we study the marginal impact of a change in information in *t*, *ceteris paribus*. Clearly in our simple model a change in information in *t* is represented by a change in the signal. Any change in the signal is immediately reflected in the price of the asset in the rational benchmark. On the other hand, in our model, given *f*() is a continuous density function with full support, there will always be noise traders in the market, making the price less responsive to changes in information. This is even clearer if we rewrite equation [2.2] as follows:

$$p_t = \frac{1}{r}\left[\theta_t + (1-\mu_t)\epsilon_{t+1} - \frac{2\rho}{(1+r)^2}\left(\sigma_\varepsilon^2 + \sigma_{\varepsilon\mu}^2 - 2\gamma_{\varepsilon^2\mu}\right)\right] - \frac{1}{(1+r)r}\gamma_{\varepsilon\mu}$$

Proof of Corollary 3. Define the price change from *t* to *T* + 1 as follows:

$$\Delta_t(p) = p_{t+1} - p_t = \epsilon_{t+2} + (\mu_t\epsilon_{t+1} - \mu_{t+1}\epsilon_{t+2})$$

Then, the expected price change at *t* is:

$$\mathbb{E}_t(\Delta_t(p)) = \mathbb{E}_t(\epsilon_{t+2} + (\mu_t\epsilon_{t+1} - \mu_{t+1}\epsilon_{t+2})) = \mu_t\epsilon_{t+1},$$

and the result follows.

Proof of Corollary 4. We show that the corollary is true by showing that the covariance between two successive price changes conditional on the information in *t* is negative. First notice that:

$$Cov(\Delta_t(p), \Delta_{t+1}(p)) = \mathbb{E}_t(\Delta_t(p)\Delta_{t+1}(p)) - \mathbb{E}_t(\Delta_t(p)\,\mathbb{E}_t(\Delta_{t+1}(p))$$

and that:

$$\mathbb{E}_t\,(\Delta_{t+1}\,(p)) = \mathbb{E}_t\,\frac{1}{r}\,[\epsilon_{t+3} + (\mu_{t+1}\epsilon_{t+2} - \mu_{t+2}\epsilon_{t+3})] = 0.$$

Thus:

$$Cov\,(\Delta_t\,(p), \Delta_{t+1}(p)) = \mathbb{E}_t\,(\Delta_t\,(p)\Delta_{t+1}(p)).$$

That is:

$$Cov\,(\Delta_t\,(p), \Delta_{t+1}(p)) = \mathbb{E}_t\,\frac{1}{r^2}\,(\mu_{t+1}\,\varepsilon_{t+2}^2 - \mu_{t+1}^2\varepsilon_{t+2}^2)),$$

which implies

$$Cov(\Delta_t\,(p), \Delta_{t+1}(p)) = \frac{1}{r^2}\,(\gamma_{\epsilon^2\mu} - \gamma_{\epsilon^2\mu^2}).$$

Clearly $\gamma_{\epsilon^2\mu}$ and $\gamma_{\epsilon^2\mu^2}$ are negative, and moreover, given μ is smaller than one, it must be that $|\gamma_{\epsilon^2\mu}| > |\gamma_{\epsilon^2\mu^2}|$ and the result follows.

3 STOCHASTIC CHOICE AND FAMILIARITY: INERTIA AND THE MERE EXPOSURE EFFECT

3.1. INTRODUCTION

The opportunities we had and the choices we took shape our decisions today. This influence is much deeper and much more subtle than the simple acknowledgment that through experiences we learn. Past choices shape our decisions today because they become routinary, they become familiar. Since the seminal papers by Samuelson and Zeckhauser (1988) and by Kahneman, Knetsch, and Thaler (1991) a lot of evidence has been accumulated highlighting the fact that people tend to stick to the *status quo*.[1] *Inertia*, in the form of *status quo* bias, endowment effect, present bias, etc., is a key component of human decision making and it has important economic effect that have been documented and studied. Nevertheless, it is still not clear what its source could be and thus, what are its implications trough time. What is clear though, is that inertia can have important dynamic effects. For example, Strahilevitz and Loewenstein (1998) find that the endowment effect is increasing with respect to the time of endowment. In this chapter we provide a first dynamic model of behavioral inertia that can explain those findings and we use it to identify the kind of heterogeneity in behavior we should expect to emerge once differences in experiences and opportunities are taken into account.

We propose a possible cognitive source for such inertia known as *the mere exposure effect* that was firstly discovered by Zajonc (1968) and that has been recognized through many successive studies as an important behavioral regularity.[2] The mere exposure effect is the phenomenon by which people tend to develop a preference for things merely because they have been exposed to them, they are *familiar* with them. Although the concept of exposure might be more general, we adopt the simplified view that an individual is exposed to a product or an alternative if he has chosen it.[3] The main idea is that the more an individual is exposed to an alternative, *i.e.,* the more he is familiar with it, the higher is the probability that he chooses such alternative due to the particular cognitive bias we are analyzing. That is, we propose a more general and dynamic notion of *status quo* bias that allows us (i) to give a first explanation for the evidence presented in Strahilevitz and Loewenstein (1998), (ii) to obtain the endowment effect, loss aversion and present bias as byproducts, and (iii) to quantify the

[1] See for example Harbaugh, Krause, and Vesterlund (2001); Kempf and Ruenzi (2008) and Sprenger (2015).

[2] See for example Pliner (1982); Gordon and Holyoak (1983); Bornstein and D'Agostino (1992); Monahan, Murphy, and Zajonc (2000); Harmon-Jones and Allen (2001); Zajonc (2001) and Huang and Hsieh (2013).

[3] In Section 3.4.1 we discuss the possibility of having a more general interpretation of exposure.

behavioral inertia that affects choices. This last feature in particular makes possible to give clear-cut predictions about the dynamics of this kind of inertia.

Section 3.2 presents a decision maker that chooses stochastically between the available alternatives at a given moment in time. In our framework, choice probabilities are given by a model similar to the one presented in Luce (1959). The key difference is that the utility of an alternative is not static. The utility of an alternative at any moment in time can be decomposed into two factors, the basic utility of the alternative and the effect of exposure. That is, as previously explained, there is a dynamic relationship between choices and choice probabilities due to the mere exposure effect. In Section 3.3 we show that the model can have important economic implications. First, we show how this more general kind of *status quo* bias implies the endowment effect, loss aversion and present bias in a dynamic framework. Second, we show that, in line with empirical evidence, the exposure effect implies that the endowment effect increases with exposure. Finally, as the main result of the section, we show that the model not only predicts the emergence of heterogeneous behavior from an homogeneous population due to the different choice paths followed by the different individuals, but it does so in a structured way. That is, it is possible to identify the distribution of choice probabilities that characterizes the heterogeneous behavior of the population at the limit. Interestingly enough, such distribution depends only on the basic utilities of the different alternatives and the distribution of menus. This implication of the model is of particular interest for the literature that analyzes the impact of the first years of life on successive social and economic outcomes through individual decisions, *e.g.*, Heckman (2006), but also, potentially, to understand phenomena like the *home bias* in trade and finance. More generally, the result makes possible to comprehend and quantify the effect of experiences on individual decision making through the kind of behavioral inertia that is the focus of this chapter.

All theoretical results of Section 3.3 depend on the knowledge of the process generating the observed choices. Thus, falsifiability of the model becomes an issue. This is why in Section 3.4 we propose four simple properties that fully characterize the model when we have data coming from an homogeneous population. The first of these properties is a generalization of the concept already introduced in Luce (1959) of independence of irrelevant alternatives (IIA) which states that the relative probability of choosing an alternative over another should not depend on the other alternatives in the set. The only change we impose is that the property has to be satisfied for any given level of exposure. The second property says that the effect of exposure should not be alternative specific. The third property we propose imposes that exposure cannot decrease the probability of choosing an alternative. This is the key property capturing the exposure effect. Finally, the fourth and more technical property, imposes that the effect of exposure cannot be marginally increasing. Section 3.4.1 then discusses the more general idea of exposure to the whole set of alternatives in a given menu, not just the ones chosen.

Section 3.5 concludes. All proofs are in the Appendix.

3.1.1. Related Literature

As previously said, this chapter proposes a new and dynamic specification of the *status quo* bias, based on the mere exposure effect. This differs from the literature in one important aspect, that is, the dynamic relationship between experiences, choices and inertia. In the main papers present in the literature that analyze the *status quo* bias from a decision theoretic perspective, that is Masatlioglu and Ok (2005); Apesteguia and Ballester (2009); Masatlioglu and Ok (2014) and Ok, Ortoleva, and Riella (2015), the analysis is static, the possible dynamics of the *status quo* bias are not taken into account. Thus, our model can be seen as complementary with respect to the ones presented in the previously mentioned papers because we present a dynamic model of *status quo* bias that allows us to explain some empirical regularities, like the fact that the endowment effect increases with the time of ownership as shown in Strahilevitz and Loewenstein, 1998, that need a dynamic structure to be fully understood.

Moreover, from a more conceptual standpoint, the work presented here complements the others mentioned above in two other respects. First, as explained in Section 3.4.1, the exposure effect is something more general than the *status quo* bias. That is, it can be seen as a bias that affects not only the *status quo* but also all the alternatives which a decision maker has been exposed to. Second, our work is the first, to the best of our knowledge, to consider this kind of behavioral inertia in a stochastic choice model thus allowing for a formal analysis of these concepts in a probabilistic framework.[4]

Finally, the dynamic relationship between choices and inertia is something that is studied also in models of habit formation as, for example, the ones presented in the seminal papers by Pollak (1970) and by Becker and Murphy (1988). The main difference between our approach and the one used in the mentioned papers is that habits are not used as references. In fact, the usual models of habit formation use a utility function that depends on the distance between present consumption and last period consumption. That is, all alternative consumption plans are evaluated relative to the habit. If a consumption plan falls short of the habit utility is negatively affected. In our framework, habits are not references. They become prominent through exposure, that is, the probability of choosing them increases with exposure, they are not references that are used to evaluate other alternatives. Moreover, once we take into account the possibility of exposure to all the alternatives in the menu, as we do in Section 3.4.1, the conceptual difference between the two approaches becomes even more evident. In fact, when considering menu exposure, we abandon the idea that habits can emerge only through choices as the models in habit formation do.

3.2. STOCHASTIC CHOICE AND EXPOSURE EFFECT

Let X be a finite set of alternatives. The decision maker (DM) chooses at every moment in time t from a menu (A_t) with $A \subseteq X$. That is, A_t is the set of alternatives that is available at time t and from which the DM has to make a choice. An alternative is any element of choice like consumption bundles, lotteries or even streams of consumption. We denote by $a_t \in A_t$ the chosen alternative at time t. With little abuse of the notation, we refer to the couple formed by the menu A_t and the chosen alternative a_t as *observation t*. We denote the collection of observations in the sequence $\{(A_t, a_t)\}_{t=1}^{T}$ as D, *i.e.*, $D = \{1,\ldots,T\}$.

[4] See Chew, Shen, and Zhong (2015) for empirical evidence of reference dependent behavior in stochastic choice.

We model a DM that chooses randomly among alternatives in a menu. The probability of choosing an alternative depends on how relatively preferred is the alternative to the other ones in the menu but also on how much the DM has been exposed to the alternative. In particular, *ceteris paribus*, we model a DM that, by experiencing exposure bias, is more likely to choose an alternative the more he chose it in the past. Formally, the value of alternative $x \in X$ for a DM at time t is the sum of two components. One component, $u\ X \to \mathbb{R}_{++}$, represents the utility the alternative x gives to the DM and we call $u(x)$ the basic utility or simply the utility of alternative x. The second component, $f: \mathbb{R}_+ \to \mathbb{R}_+$ with $f(0) = 0$, captures the exposure effect and we call it the *exposure function*. That is, the value of an alternative x at time t after having been chosen n_x times will be equal to $u(x) + f(n_x)$. We assume that f is non-decreasing and concave. This assumption is justified by some of the evidence in Zajonc (1968). Then we can formally define our model of stochastic choice that is based on the classical model presented in Luce, 1959.

Definition 2. [Exposure Biased Luce Model (EBLM)]. *p is an Exposure Biased Luce Model if there exist a basic utility u and an exposure function f such that* $\forall A \subseteq X$:

$$p_t(x|A) = \frac{u(x)+f(n_x)}{\sum_{y\in A}\left(u(y)+f(n_y)\right)}$$

where n_x *and* n_y *are the number of times that alternative x and alternative y have been chosen up until t.*

Notice that an Exposure Biased Luce Model can be the standard Luce model whenever $f(r) = 0$ for all $r \in \mathbb{R}$.

An Exposure Biased Luce Model implies that the probability of choosing an alternative is increasing in the utility the alternative provides, but also in how much the DM has been exposed to it. This is a dynamic model. As underlined in the introduction, past experiences influence present behavior in a way that has clear economic relevance. This is what we analyze in the next section.

3.3. EXPOSURE EFFECT, INERTIA AND HETEROGENEITY

In this section we discuss the implications of the model proposed. It is immediate to see that, as discussed in the introduction, the model provides a possible explanation for the kind of behavioral inertia studied since the seminal papers by Samuelson and Zeckhauser (1988) and by Kahneman, Knetsch, and Thaler (1991). Through exposure, past choices become *more attractive* for our DM. That is, after having chosen something, it becomes more difficult for our DM to change his behavior. This can have very important economic and social consequences. Think for example of product fidelity or drugs. Our model predicts that the more people indulge in the habit of choosing a product or consuming drugs, the less likely it is they will change their habits. Experiences create routines that then are hard to overcome. To see some of these points more clearly, we propose some remarks that are direct consequences of the model. In order for the analysis to be more interesting, we consider the case in which f is strictly increasing. That is, we exclude the standard Luce Model.

To simplify the presentation of some of the remarks, we assume that the DM is endowed with some money $m \in \mathbb{R}$, and that his overall utility of consuming an alternative x having been chosen n times up until t while disposing of m units of money is equal to $u(x) + f(n) + m$. That is, the overall utility is quasilinear in money. Furthermore, we assume there exists an alternative $0 \in X$ that does not incur in the exposure effect and such that $u(0) < u(x)$, $\forall x \in X \backslash \{0\}$. This alternative can be interpreted as *having nothing, having no alternative*. Given these assumptions, we can easily define the willingness to pay at time t (WTP_t^x) for an alternative x chosen n times up to t, as the amount of money for which a DM endowed only with m units of money will be indifferent between paying WTP_t^x to obtain the good, and paying nothing not obtaining the good; that is, $u(0) + m = u(x) + f(n) + m - WTP_t^x$, or $WTP_t^x = u(x) + f(n) - u(0)$. Similarly, we can define the willingness to accept at time t (WTP_t^x) for an alternative x that has been chosen n times up to t, as the amount of money for which a DM endowed with the good x and m units of money would be indifferent between getting WTP_t^x in order to give up the good, and staying with the good without receiving any amount of money; that is, $u(x)+f(n)+m = \mathrm{u}(0)+\mathrm{m}+ WTP_t^x$, or $WTP_t^x = u(x) + f(n) - u(0)$. Notice that the argument of the exposure function can be different in the two expressions. Then, it is easy to see that, due to the exposure effect, there should be a gap between WTA and WTP of a good. In particular, the DM should experience the endowment effect as the next remark highlights.

Remark 2. *If a DM follows a EBLM then he experiences the endowment effect.*

The endowment effect, as previously stated, is usually defined as the difference between the willingness to accept once endowed with a good and the willingness to pay for such good if not endowed with it. Clearly, the exposure effect implies that once a DM is endowed with an alternative, such alternative becomes more relevant to him due to exposure. That is, after the DM is endowed with the alternative, it is more difficult for him to *leave it*. There is a plethora of evidence that people incur in the endowment effect, *e.g.*, Harbaugh, Krause, and Vesterlund (2001) and Kempf and Ruenzi (2006), here we provide a possible cognitive foundation for such behavioral bias. The exposure effect might be what drives the endowment effect. The second direct implication is more interesting, even if still straightforward, and it is related to the dynamic aspect of the model.

Remark 3. *If a DM follows a EBLM then the endowment effect increases with exposure.*

This result is a direct consequence the exposure effect increasing in the number of times an alternative has been chosen. Remarks 2 and 3 point to the findings of Strahilevitz and Loewenstein (1998). These authors empirically show that the endowment effect is an increasing function of the time an alternative is owned. That is, the gap between WTA and WTP for a good should be increasing in the time of ownership. In particular, they find that the endowment effect is not only a function of current ownership, as Remark 2 underlines, but also previous ownership can increase the valuation of an object, as stated in Remark 3. These implications can have interesting economic consequences. They state that the more an alternative is *familiar*, the more a DM has been exposed to it, the more it becomes *routinary* and so the more difficult it is for the DM to change his habits. These results suggest that, whenever from a social point of view it would be better to change some habit a particular individual has, *the sooner we intervene, the better*. This conclusion is in line with the literature developed by Heckman, regarding the best timing for policy interventions in order to improve the social and economic outcomes of the individuals that compose a society.[5] It can also be important for the understanding of the *home bias* in trade and finance. In fact, people might tend to choose more products or assets of their home country only because they have been more exposed to them.

The fact that the value of an alternative increases the more it is chosen, or in this context the longer the DM is endowed with it or experiences it, has clear implications also for the *status quo* bias. In fact, the existence of the endowment effect can be seen, as it is usual in the literature, as the driving force of the *status quo* bias. An interesting and straightforward implication of the exposure effect regards the dynamic behavior of the bias as the next remark underlines. To properly understand our contribution, the reader should refer to the ideas presented in Masatlioglu and Ok (2005); Apesteguia and Ballester (2009); Masatlioglu and Ok (2014) and Ok, Ortoleva, and Riella (2015). One of the key features of these works is that they assume the existence of some alternatives that dominate the *status quo*, that are the only alternatives for which a DM would leave the *status quo* if he could choose. A consequence of the model we present here is that the set cannot increase the more a DM experiences the *status quo*. The longer the period of time the DM stays with the *status quo*, the more the value of the *status quo* will have shifted upwards due to the exposure effect, hence the number of alternatives that dominate it cannot increase in time.

Remark 4. *If a DM follows a EBLM, the number of alternatives dominating a particular alternative x at least weakly decreases every time alternative x is chosen.*

Another phenomenon that have always been connected to the endowment effect and the *status quo* bias, is loss aversion. In particular, it has been used as a plausible explanation for this kind of biases. As we said, we here propose a different channel that might be driving this kind of inertia. We think that the exposure effect is a better explanation for the empirical evidence that has been gathered, for two main reasons. First, as underlined by Remark 3 it can explain some of the evidence that has been found in different experiments regarding the dynamics of the endowment effect, while loss aversion, being a static concept, cannot explain those findings without additional assumptions. Second, as Remark 5 highlights, the exposure effect can be seen as a primitive of loss aversion whenever the DM is unaware of the fact that he experiences it.[6] A DM described by a EBLM evaluates gains and losses differently due to the exposure effect. If we ask our DM whether he would prefer to gain alternative $x \in X$ or, once we endowed him with it, not lose it, *ceteris paribus*, his answer will be influenced by the presence of the exposure bias. Similarly to the driving mechanism behind Remark 2, once having the alternative, the DM experiences an exposure effect that makes the value of the alternative higher than its *ex ante* value. Losses are evaluated with the value of the alternative after experiencing exposure while gains before experiencing it and without forecasting it. This difference is the driving force behind Remark 5. The DM will be more willing to avoid the loss than to obtain the gain.

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5 For a general survey of the literature, see Cunha, Heckman, and Lochner (2006). See also Sen (1997b) for a discussion over opportunities and economic inequality.

6 This assumption is in line with the evidence presented, among others, in Zajonc (1968); Zajonc (2001); and Hansen and Wänke (2009) that see the mere exposure effect as a consequence of an unconscious process.

Remark 5. *A naïve DM that follows a EBLM experiences loss aversion.*

As a final and more abstract remark, we would like to underline that, due to a reasoning similar to the one behind the previous remarks, a naïve DM described by a EBLM, should experience present bias as described in O'Donoghue and Rabin (1999), that is, the difference between the utility of today's and tomorrow's consumptions should be higher when the DM is experiencing today's consumption than when the DM is still not experiencing it. It is immediate to see that a DM affected by exposure effect who is not aware of it, should consider differently consumption at time t *before* being at t with respect to consumption at time t *once* being at t. The reasoning is similar to the one behind Remark 5. When evaluating consumption at time t and time $t + 1$ from s, with $s < t$, a naïve DM, not forecasting the exposure effect, will provide a different evaluation than the one given at time t, once experiencing the exposure effect.

Remark 6. *A naïve DM that follows a EBLM experiences present bias.*

It is worthwhile to highlight the fact that all previous remarks do not depend on the stochasticity of choices assumed in the model. In fact, all remarks depend on the basic specification of the utility of an alternative when the exposure effect is taken into account. Thus, they would be valid also in a deterministic choice model considering the same issues with the same basic structure.

We conclude the section with its main result. We said that experiences can have an effect on present choices that runs deeper than mere learning. The exposure effect is a powerful tool to substantiate this claim not only because it is well documented in cognitive sciences, as previously explained, but also because the structure that it implies can be extremely helpful to predict individual behavior and hence to have better policy designs. In fact, if the social objective is to eradicate some damaging behavior or to reinforce a positive habit, it is crucial to know the cognitive process that make habits and routines to emerge. Moreover, thanks to its simplicity, it is able to give sharp predictions. This tractability is evident in a EBLM. In particular we focus on a particular specification of the model for which the exposure effect is linear.

Definition 3 (Linear Exposure Biased Luce Model (Lin-EBLM)). *p is a LinEBLM if it is a EBLM and the exposure function f is defined as follows for any $n \in$ R:*

$$f(n) = kn,$$

with $k\ E\ \mathbb{R}_{+}$.

Now, suppose we have an homogeneous population at $t = 0$, homogeneous in the sense that all the individuals composing it share the same basic utilities and exposure function. Imagine we observe the individuals choosing from the grand set X, what kind of heterogeneity should we expect to emerge in the behavior of the individuals composing the population? The answer to this question is given in the next proposition.[7]

[7] The result can be generalized thanks to classical results in statistics to an homogeneous population facing a time invariant distribution of menus. We do not pursue this route because it would not change the main message of the proposition.

Proposition 7. *An homogeneous population choosing from X following a LinEBLM will show heterogeneous behavior as t goes to infinity. The limiting distribution of choice probabilities will be a Dirichlet distribution with parameters equal to the utilities of the different alternatives divided by k. That is, for any alternative* $x \in X$*, the probability* $p_t(x|X)$ *as t goes to infinity, will be distributed following a Beta distribution with parameters equal to* $u(x)/k$ *and* $\sum_{y \neq x} u(y)/k$.

This proposition gives a clear cut answer to the question we previously posed that is crucial for the strand of research analyzing the impact of early life experience on economic decisions and thus outcomes.[8] The result is saying that if we want to intervene and reduce the possibility of a bad habit to endure, we have two possible routes to follow. One, that can be costly, is to change the alternatives to which the different individuals are exposed. That is, we should change the menus the different individuals face. This is in line with the literature on freedom of choice that analyzes the impact of menus on individual freedom, see for example Pattanaik and Xu (1990); Sen (1991 and 1997a), Sudgen (1998), Barbera, Bossert, and Pattanaik (2004); Ballester and De Miguel (2006) and Savaglio and Vannucci (2007). Our results add to this debate a dynamic implication of opportunities. Early life opportunities might have a big weight on our decisions, limiting our possibilities of evading bad habits. The second route is more difficult to follow in many circumstances, *e.g.*, drugs, but it can sometimes be achieved with proper disincentives. That is, it is possible to intervene on the basic utilities directly. Anything that affects the basic utilities has an impact on the limiting distribution of behavior we should observe.

All the results in this section rely on the fact that the DM is described by a EBLM, but is it possible to falsify the model? If yes, what are the properties that describe it? These questions are addressed in the following section.

3.4. CHARACTERIZATION

What properties should a process generated by a EBLM satisfy? This question is the focus of the present section. We assume we observe the choice probabilities of a population sharing the same preferences over the alternatives in X. That is, suppose we observe many and different sequences of decision problems and choices generated by a continuous population of individuals sharing the same preferences. Let $\mathcal{D}$ be the set containing such sequences and $D \in \mathcal{D}$ be the collection of observations composing one of them. Let any $D \in \mathcal{D}$ have length $T > 2$. We assume that for any $D \in \mathcal{D}$ and for any $t \in D$, we observe the probabilities of choosing the different alternatives in A_t. Moreover, we assume that $\mathcal{D}$ contains the following sequences:

- For any $x, y \in X$ and any $n_x, n_y < T$, there exists a $D \in \mathcal{D}$ such that there is some $t \in D$ where x and y have been chosen n_x and n_y times before respectively and $A_t = \{x, y\}$.
- For any $x, y, z \in X$ and any $n_x, n_y, n_z < T$ there exists a $D \in \mathcal{D}$ such that there is some $t \in D$ where x, y and z have been chosen n_x, n_y and n_z times before respectively and $A_t = \{x, y, z\}$.

[8] See Heckman (2006).

These requirements in our framework are equivalent to the standard assumption in decision theory that asks to observe choices from all menus of cardinality two and three. The main difference is that here we ask to observe such menus after any kind of history of choices. This is an important assumption that allows us to identify the values of the different alternatives from observed behavior. Notice however that the property is not as strong as it seems at first glance. In fact, it is equivalent to observing an infinite population choosing deterministically and the stochastic choice would be representing the frequency of choices. Now we are ready to state the four properties that characterize the whole model. The properties are intended for any D, $D' \in \mathcal{D}$ and any n_x, n_y, n, $n' \in \mathbb{Z}_+$.

Axiom 5 [Exposure IIA (EIIA)]. *For any $x, y \in X$ having been chosen n_x and n_y times up until $t \in D$ and $t' \in D'$ respectively and for any $A \subseteq X$:*

$$\frac{p_t(x|\{x,y\})}{p_{t'}(y|\{x,y\})} = \frac{p_t(x|A)}{p_{t'}(y|A)}$$

This is just a more general version of the classical IIA presented in Luce (1959). It imposes that the relative likelihood of choosing an alternative over the other, given the number of times the alternatives have been chosen, should not be influenced by the other alternatives present in the set. Clearly a EBLM satisfies this axiom given that the relative likelihood of choosing an alternative over the other, given the bias, depends only on the ratio of their utilities and biases.

Axiom 6 [Anonymous Bias (AB)]. *For any $x, y, \in X$ having been chosen n times up until $t \in D$ and $t' \in D'$ respectively, and for any $z \in X$ having been chosen n' times up until t and t', if $x = a_t$ and $y = a_{t'}$ then:*

$$\frac{p_t(x|\{x,z\})}{p_t(z|\{x,z\})} - \frac{p_{t+1}(x|\{x,z\})}{p_{t+1}(z|\{x,z\})} = \frac{p_{t'}(y|\{y,z\})}{p_{t'}(z|\{y,z\})} - \frac{p_{t'+1}(y|\{y,z\})}{p_{t'+1}(z|\{y,z\})}$$

This axiom imposes that the effect that choosing an alternative has on the relative likelihood of choosing it again cannot be alternative specific and does not depend on the particular sequence under study. Given the exposure function does not depend on the alternative that has been chosen or on the sequence, a EBLM clearly satisfies it.

Axiom 7 [Exposure Bias (EB)]. *For any $x, y \in X$ and for any $t \in D$, if $x = a_t$ then:*

$$p_t(x|\{x, y\}) \leq p_{t+1}(x|\{x, y\})$$

This axiom is the key one capturing the idea behind the cognitive bias we are analyzing. The probability of choosing an alternative cannot be negatively affected by the fact that the alternative has been chosen in the past. Notice that this axiom excludes the possibility of becoming satiated of a good. Clearly, given the exposure function is non-decreasing, the axiom has to be satisfied by a EBLM.

Axiom 8 [Marginally Decreasing Bias (MDB)]. *For any $x \in X$ having been chosen n times up until $t \in D$ and such that $x = a_t$:*

$$|p_t(x|\{x,y\}) - p_{t+1}(x|\{x,y\})|$$

is non-increasing in n.

This is a more technical axiom which imposes that the effect of choosing an alternative has on the likelihood of choosing it again is marginally non-increasing. Given the exposure function is concave a EBLM trivially satisfies such property.[9]

Theorem 5. *A dataset $\mathcal{D}$ satisfies EIIA, EB, AB and MDB if and only if there exist a function $u\ X \to \mathbb{R}_{++}$ and a non-decreasing concave function $f : \mathbb{R}_+ \to \mathbb{R}_+$ with $f(0) = 0$ that define an Exposure Biased Luce Model.*

3.4.1. Discussion: Menu Exposure

In the introduction we briefly discussed the possibility of considering a more general kind of exposure, that is, the exposure effect affects all the alternatives in the menu, not only the chosen one. There is in fact evidence that the exposure effect is something more general and that has to do with the unconscious processing of environmental stimuli, thus making the idea of extending the effect to the whole set of alternatives in the menu a sensible one. This can be extremely important, for example, for marketing strategies that try to increase the exposure of a product in the market.

The framework we propose here can easily accommodate a more general exposure effect. This is in fact quite trivial. Only two changes need to be implemented. First, we need to change the interpretation of the exposure function as the following definition highlights.

Definition 4 [Exposure Biased Luce Model* (EBLM*)]. *p is an Exposure Biased Luce Model* if there exist a function $u : X \to \mathbb{R}_{++}$ and a function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\forall A \subseteq X$:*

$$p_t(x \mid A) = \frac{u(x) + f(n_x)}{\sum_{y \in A}\left(u(y) + f(n_y)\right)}$$

where n_x and n_y are the number of times that alternative x and alternative y have been present in a menu up until t and f non-decreasing and concave.

It is immediate to notice that the only difference between a EBLM and a EBLM* is in the argument of the exposure function. While a EBLM considers only the exposure a chosen alternative gets, a EBLM* generalizes the idea to all the alternatives of a menu. This implies that the characterization of a EBLM* would be almost identical to the one of a EBLM except for a second change that needs to be done. The specification of the data has to change in order to properly state the new axioms. Let the new dataset $\mathcal{D}^*$ be as follows:

- For any $x, y \in X$ and any $n_x, n_y < T$, there exists a $D \in \mathcal{D}^*$ such that there is some $t \in D$ where x and y have been present in the menu n_x and n_y times before respectively and $A_t = \{x, y\}$.

[9] Notice that the properties are independent. In fact, it is easy to think about procedures that satisfy all properties except one. For example, think about a EBLM in which the exposure effect is negative. Such procedure would satisfy all properties except EB. Another example can be an EBLM where the basic utility value of an alternative *x*, *i.e.*, *u(x)*, depends on the other alternatives in the menu. Such procedure would satisfy all properties except EIIA. Similarly an EBLM with a convex exposure effect would satisfy all properties except MDB. Finally, an EBLM with an alternative specific exposure function would satisfy all properties expect AB.

- For any $x, y, z \in X$ and any $n_x, n_y, n_z < T$ there exists a $D \in \mathcal{D}^*$ such that there is some $t \in D$ where x, y and z been present in the menu n_x, n_y and n_z times before respectively and $A_t = \{x, y, z\}$.

Now we are ready to present the parallel version of the axioms we previously presented. For all of them, the only thing that changes is that this time we consider menu exposure, not only choice exposure. Again, the properties are intended for any $D, D' \in \mathcal{D}^*$ and any n_x, n_y, $n, n' \in \mathbb{Z}_+$.

Axiom 9 [Exposure IIA* (EIIA*)]. *For any $x, y \in X$ having been present in a menu n_x and n_y times up until $t \in D$ and $t' \in D'$ respectively, and for any $A \subseteq X$:*

$$\frac{p_t(x|\{x,y\})}{p_{t'}(y|\{x,y\})} = \frac{p_t(x|A)}{p_{t'}(y|A)}$$

Axiom 10 [Anonymous Bias* (AB*)]. *For any $x, y, \in X$ having been present in the menu n times up until $t \in D$ and $t' \in D'$ respectively, and for any $z \in X$ having been present in the menu n' times up until t and t', if $x = a_t$ and $y = a_{t'}$ then:*

$$\frac{p_t(x|\{x,z\})}{p_t(z|\{x,z\})} - \frac{p_{t+1}(x|\{x,z\})}{p_{t+1}(z|\{x,z\})} = \frac{p_{t'}(y|\{y,z\})}{p_{t'}(z|\{y,z\})} - \frac{p_{t'+1}(y|\{y,z\})}{p_{t'+1}(z|\{y,z\})}$$

Axiom 11 [Exposure Bias* (EB*)]. *For any $x, y \in X$ and for any $t \in D$, if $x = a_t$ then:*

$$p_t(x|\{x, y\}) \leq p_{t+1}(x|\{x, y\})$$

Axiom 12 [Marginally Decreasing Bias* (MDB*)]. *For any $x \in X$ having been present in a menu n times up until t and such that $x = a_t$:*

$$|p_t(x|\{x,y\}) - p_{t+1}(x|\{x,y\})|$$

is non-increasing in n.

This means that the parallel version of Theorem 5 can be stated also for this new specification of the model. The proof is omitted.

Theorem 6. *A dataset $\mathcal{D}$ satisfies EIIA*, EB*, AB* and MDB* if and only if there exist a function $u : X \to \mathbb{R}_{++}$ and a non-decreasing concave function $f : \mathbb{R}_+ \to \mathbb{R}_+$ that define an Exposure Biased Luce Model*.*

3.5. FINAL REMARKS

Experiences play a major role in shaping our decisions. In particular, routinary choices have a very prominent role in our day to day decision making. Our decisions tend to show inertia that seems difficult to overcome. The economic effects of this kind of inertia can be catastrophic if it pushes individuals to reiterate suboptimal decisions. Thus, understanding the source of inertial behavior is crucial to design better economic policies.

In this chapter, we make two main contributions. First we propose a more general and dynamic cognitive specification of the well-known *status quo* bias. We adopt the mere exposure effect, a cognitive phenomenon first documented in Zajonc (1968), to model a process in which the probability of an alternative being chosen cannot decrease the more it is chosen. We show that it implies the emergence of phenomena such as the endowment effect, loss aversion and present bias and we use it to build a tractable model that can be implemented in standard economic analysis. Second we show that with our model it is possible to substantiate the idea that experiences are important in decision making for a deeper reason than mere learning. In fact, the model allows us to predict the distribution of heterogeneous behaviors we should observe from otherwise homogeneous individuals that had different experiences during their lifetime, and thus, through the exposure effect, end up choosing differently. Finally, we also provide some conditions that make the model empirically falsifiable and testable.

The main strength of the model we propose is its tractability. It provides a simple way of modeling behavioral inertia that can be easily used in standard economic analysis without the need to change the methods usually adopted or depart too much from the standard framework of rational choice. The implications of the model are many and can be potentially important not only for discussions on early life opportunities but also to have a better understanding of the structure of dynamic competition among firms. If consumers are described by our model, then the *pioneering advantage* or *first-mover advantage* emerge with clarity and the model might help to design better policies to avoid dominant market position. Nevertheless, the model is not able to incorporate some ideas that might be sensible in some context, like the idea of satiation. In some circumstances, it is sensible to assume that the DM becomes satiated with an alternative the more he chooses it. That is, the probability of choosing an alternative should decrease the more the DM is exposed to it. This is exactly the opposite model with respect to the one we are proposing here. Nonetheless, it should be possible to accommodate such ideas just by changing the properties we propose in this work. In particular EB would need to be changed and also some more technical assumptions would be needed to correctly specify the model, *i.e.,* to avoid probabilities becoming negative. This a route we leave to future research.

To conclude, we think it is important to try to find the main cognitive process that might cause some of the biases that have been documented in the literature in the last years. We think that this chapter might help to shed some new light on the kind of behavioral inertia that is widely accepted in economics nowadays. In particular, the novelty of the framework we propose is to take into consideration the inherently dynamic relationship between experiences, inertia and choices.

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APPENDIX C

Appendix to Chapter 3

Proof of Remark 2. Suppose the DM, at the moment t has m units of money and he has to decide whether to buy an alternative x that he has chosen n times up to t. Then his WTP_t^x will be equal to $u(x) + f(n) - u(0)$. Now suppose the DM chooses x. Then, in $t + 1$, the WTA_{t+1}^x of the DM will be equal to $u(x) + f(n + 1) - u(0)$. But then, given that $f(n + 1) > f(n)$ due to the exposure effect, we must have that $WTA_{t+1}^x > WTP_t^x$, that is, the DM experiences the endowment effect.

Proof of Remark 3. Suppose we have a sequence of choices made by the DM that has m units of money, in which he chose the alternative x n times up to t. Once the DM chooses x, his WTA_{t+1}^x will be equal to $u(x) + f(n) - u(0)$. This implies that the WTA is increasing with n due to the fact that f is increasing and the result follows.

Proof of Remark 4. This is a straight forward implication of the exposure effect. Suppose we have a sequence of choice of length t in which the DM chose n times alternative x. Define 1.1. as the set containing the alternatives dominating x at the moment in time t, *i.e.*, $\mathfrak{U}_x^t = \{x' \in X | u(x') + f(n_{x'}) > u(x) + f(n)\}$. Then, given f is increasing in its argument, *ceteris paribus,* the set of alternatives that dominates x cannot increase with exposure.

Proof of Remark 5. Suppose the DM chooses an alternative x in t that he has chosen n times up to t. Then the value in $t + 1$ of this alternative for him will be $u(x) + f(n + 1)$. Thus, if he loses it the difference in utility will be $u(0) - (u(x) + f(n + 1))$. On the other hand, if he has to decide in t whether to take the alternative, the perceived value of the alternative is $u(x) + f(n)$ due to his unawareness regarding the exposure effect in $t + 1$. Thus, the perceived gain when taking alternative x is $u(x) + f(n) - u(0)$. Clearly, given f is increasing, $|u(0) - (u(x)+ f(n+ 1))| > |u(x)+f(n) - u(0)|$ and the result follows.

Proof of Remark 6. When at s, the difference between the utility of consumption at time t and the one of consumption at time $t + 1$ for a naïve DM described by a EBLM will be $\Delta_s = |u(c_t) - u(c_{t+1})|$. On the other hand, once at t we have $\Delta_t = |u(c_t) + f(1) - u(c_{t+1})|$. Clearly, due to the exposure bias, $\Delta_s < \Delta_t$, and the remark follows.

Proof of Proposition 7. Notice that a Lin-EBLM defines a random process with reinforcement equivalent to the classical Pólya urn process presented in Eggenberger and Pólya 1923.

Definition 5 (Pólya Urn Process). *A Pólya Urn Process is a process following which at any t a ball is drawn from an urn containing only two colors, white and black. Whenever a ball of a given color is drawn, it is returned to the urn in addition to k* $\in \mathbb{R}_+$ *balls of the same color.*

Notice that the initial number of balls of each color does not have to be an integer, it can be any positive real number.

To see that a Lin-EBLM can be seen as a Pólya urn, let the alternatives be the different colors in the urn and the initial utility values of the alternatives be equivalent to the number of balls of each color in the urn. Then choices are draws and the exposure bias determines how many balls of each color are added in every draw. In their classical result Eggenberger and Pólya show that an urn containing balls of two colors, will converge to a Beta distribution with parameters equal to the initial numbers of the two kind of balls in the urn each divided by k.

Given this result, it is immediate to see that our process should converge to a Dirichlet distribution with parameters equal to the basic utilities of the different alternatives divided by k. In fact, notice that we can take any alternative x and define a fictitious alternative $\overline{x}$ representing all other alternatives. This would still define a Pólya urn with two colors, hence the result in Eggenberger and Pólya 1923 would directly apply. Given the generality of the reasoning, it must be that the final distribution of the process has Beta distributions as marginals, that is, the process must converge to a Dirichlet distribution with parameters equal to the utilities of the different alternatives divided by k.

Proof of Theorem 5. *Necessity:* In the text.

Sufficiency: We proceed following three steps that will allow us to construct the functions on which a EBLM is based. In the first step we are going to define a general utility function for every alternative that only depends on the number of times an alternative has been chosen. Then we are going to show that it is possible to separate such function into two components that will characterize the utility function and the exposure function that is not alternative specific and non decreasing in exposure. Finally, we will show that the exposure function has to be concave.

First, notice that EIIA implies that the relative probabilities of two distinct alternatives depend neither on the other alternatives in the set nor on the particular sequence the alternatives are in. Hence, given $\mathcal{D}$, we can apply the results in Luce 1959 and construct a utility function $v : \mathbb{Z}_{+} \times X \rightarrow \mathrm{R}_{++}$ that assigns a real value to an alternative $x \in X$ that depends on the number of times the alternative has been chosen that is defined as follows. For any $x \in X$ having been chosen n_x times up until t, and for any $\mathrm{A} \subseteq \mathrm{X}$:

$$p_t(x \mid A) = \frac{v(n_x, x)}{\sum_{y \in A} v(n_y, y)}$$

That is, $v(n_x, x)$ will represent the utility of x after having been chosen n_x times.

Now, we want to show that v can be decomposed into a utility function $u : X \rightarrow \mathbb{R}_{++}$ that represents the preferences of the DM not influenced by exposure and another function $f : \mathbb{N} \rightarrow \mathbb{R}_{+}$ that is not alternative specific and that is non-decreasing in exposure. W.l.o.g., take $x, y \in X$ such that haven bee chosen 0 times up until $t \in D$ and n times up until t'

$\in D'$ and a third alternative $z \in X$ that has been chosen n' times up until t and t'. First notice that EIIA implies the following equalities:

$$\frac{p_t(x|\{x,z\})}{p_t(z|\{x,z\})} = \frac{p_t(x|\{x,y,z\})}{p_t(z|\{x,y,z\})}$$

$$\frac{p_{t'}(x|\{x,z\})}{p_{t'}(z|\{x,z\})} = \frac{p_{t'}(x|\{x,y,z\})}{p_{t'}(z|\{x,y,z\})}$$

$$\frac{p_t(y|\{y,z\})}{p_t(z|\{y,z\})} = \frac{p_t(y|\{x,y,z\})}{p_t(z|\{x,y,z\})}$$

$$\frac{p_{t'}(y|\{y,z\})}{p_{t'}(z|\{y,z\})} = \frac{p_{t'}(y|\{x,y,z\})}{p_{t'}(z|\{x,y,z\})}$$

Then AB, by induction, implies the following:

$$\frac{p_{t'}(x|\{x,y,z\})}{p_{t'}(z|\{x,y,z\})} - \frac{p_t(x|\{x,y,z\})}{p_t(z|\{x,y,z\})} = \frac{p_{t'}(y|\{x,y,z\})}{p_{t'}(z|\{x,y,z\})} - \frac{p_t(y|\{x,y,z\})}{p_t(z|\{x,y,z\})}$$

Using the representation we obtained by applying the classic result from Luce 1959, we can rewrite the previous equality as follows:

$$\frac{v(n,x)}{v(n',z)} - \frac{v(0,x)}{v(n',z)} = \frac{v(n,y)}{v(n',z)} - \frac{v(0,y)}{v(n',z)}$$

Given the generality of the argument, this is true for any $x, y \in X$ and for all $n \in \mathbb{Z}_+$. This implies that we can separate v into two components. We can let $u : X \to \mathbb{R}_{++}$ be equal to $v(0,\cdot)$ and define a function $\hat{f} : \mathbb{Z}_+ \to \mathbb{R}_+$ such that for all $n \in \mathbb{Z}_+$, $\hat{f}(n) = v(n, x) - v(0, x)$ for some $x \in X$. It is immediate to see that $\hat{f}(0) = 0$. Moreover, by Luce 1959, u as it is defined rationalizes the choice of the DM in the absence of the exposure effect.

Now let $f\, \mathbb{R}_+ \to \mathbb{R}_+$ be the affine extension of $\hat{f}$. It is immediate to see that EB implies that f is non-decreasing. In fact, EB implies that $v(n + 1, x) - v(n, x) \geq 0$ for any $x \in X$ and $n \in \mathbb{Z}_+$.

Finally, it is immediate to see that MDB implies that f is concave. In fact, by MDB and the definition of affine extension, for any $n \in \mathbb{R}_+$ we have $f(n + 1) - f(n) \leq f(n) - f(n - 1)$, that is, $f(n + 1) + f(n - 1) - 2f(n) \leq 0$ which implies that the function is concave.

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