PUBLIC DEBT AND ECONOMIC GROWTH IN SPAIN, 1851-2013

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Public debt and economic growth in Spain, 1851-2013

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Abstract

In this paper we investigate the long-run relationship between public debt and economic growth in the Spanish economy for the period 1851-2013. We develop a cliometric analysis of the debt-growth nexus adopting linear and non-linear specifications and using novel methods from the time-series literature. We find some support for a negative relationship between public debt and long-run growth, but no clear evidence of a debt threshold. The estimate of long-run elasticity between both variables in a one-break model shows a tendency to decrease over time from a non-significant 0.011 to a -0.070, indicating that a 10 percentage increase in the public debt-to-GDP ratio is associated with 0,70 percentage points lower real economic growth. Indeed, we find for the first subsample (1851-1939) either "decoupling" or "saturation", while in the second subsample (1940-2013) the long-run elasticity coefficient becomes negative and significant.

Keywords: Public debt; Economic Growth; Fiscal policy; Cointegration; Multiple Structural Breaks; Nonlinear Relationship; Threshold Cointegration.

JEL classification: H63; O49; E62; C22

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1 Introduction

The 2008 global financial crisis led to an economic recession in many countries giving rise to an increased activism in both fiscal and monetary policies. While these expansionary policies may have helped smooth the cycle, fiscal stimulus packages and banking sector bail-outs generated an unprecedented increase in public debt across the world. This fact has raised serious concerns about fiscal sustainability and set the basis for the current European sovereign debt problems[1]. Therefore, the interest on the relationship between public debt and long-term economic growth has recently revived giving rise to a hot debate on whether governments should run fiscal stimulus in order to restore growth rates or run austerity programs to reduce deficits as percentage of GDP. Starting from the seminal contribution of Reinhart and Rogoff (2010, 2009b) who pointed out the existence of strong negative effects of high public debt on economic growth, a large strand of literature has investigated this relationship (*R&R hypothesis* hereafter), attempting to identify possible non-linearities. This paper asks whether, in the case of the Spanish economy, the relationship between public debt and economic growth is significantly negative and how this relationship may evolve overtime giving rise to thresholds beyond which it changes in magnitude.

The relationship between debt and growth has been the focus of much work historically. In the analysis of debt-growth nexus, there exist a set of theoretical models which derives inverted "U" shaped curves by having growth increasing with debt until some threshold is passed, after which growth is reduced. The basic idea underlying all these models is that when some threshold debt level is passed, then the economy moves to another regime, with the debt-growth nexus being different between the old and the new regime. In the inverted "U" models, the low-debt regime corresponds to an increasing debt-growth relationship, while in the regime after the threshold the debt-growth relationship is decreasing.

The theoretical literature has distinguished between the positive short-run effects of accumulating public debt in order to enact counter-cyclical policies and the negative long-run growth effects from high levels of debt. The idea is that debt levels that are above a particular threshold value may have different implications for growth compared to more moderate levels of debt. There exists theoretical work suggesting that the effect of public debt on growth may, in fact, be non linear so that there may

[1] Highly indebted Eurozone countries have been required to implement fiscal austerity measures, provoking an increase in the volatility of the bond yields. See Paniagua et al. (2015) for a recent analysis of the spread determinants

exist an optimal level of public debt (Checherita-Westphal and Rother (2012)).

Empirically, evidence for the *R&R hypothesis* is mixed[2]. Two main caveats have appeared regarding the fulfillment of the hypothesis. The first and most obvious is that not all types of countries obey this empirical regularity. Empirical evidence for the R&R hypothesis has been found in a number of studies, mainly for developing or emerging countries, whereas for advanced economies either debt levels rise consistently growth or turning points are extremely high, or very uncertain (Égert (2015a)). Finally, some of the countries that appear to follow the *R&R hypothesis* may in fact exhibit re-linking at higher debt levels with a subsequent upswing again in growth[3], and hence show an "N-curve" rather than an inverted "U-curve".

A second caveat is the diverse evidence found, mainly due to three reasons. First, the fact that other factors can also be driving growth, like for example, the degree of trade protection, the degree of political freedom (index of civil liberties) or the productivity, independently of the public debt level reached[4]. Second, the possibility that for some countries the turning point has not been met yet. Third, a broader problem is that many of these studies have been carried out on a cross-section basis rather than on a more careful panel or time-series framework within specific countries.

In order to overcome the above criticisms authors have been refining the empirical strategies. Estimated relationships, mainly in reduced form specifications, have taken cubic or quadratic forms. Estimation methods have also varied from OLS estimation, panel data estimations with fixed or random effects, Tobit estimations, or semi parametric estimation. In addition, explanatory variables have also been augmented including lagged values, population density, locational variables, micro or macro variables, distributional variables, trade variables, as well as non-economic variables such as literacy rates or political rights. All in all, these efforts have only found wide variations across countries regarding the debt-growth nexus, and therefore this relationship seems to be less robust than previously thought (Égert (2015a,b)).

As the theory predicts a long-run relationship linking debt and growth, most of the empirical research has focused on the long-run relationship employing either linear or non-linear regression techniques. However, the use of time series cointegration techniques has been quite scarce up to now. This point is relevant since the empirical evidence suggest that public debt and growth may be jointly determined, so that

[2]See, for example, Panizza and Presbitero (2013) for a recent review of the literature

[3]Beyond some threshold some countries may be able to arrive to a reschedule of its debt or even some way of debt redemption program that allows new growth impetus

[4]See Kourtellos et al. (2013)

any constraint put on fiscal policy to help reduce debt will have effects (positive or negative) on economic growth and endogeneity problems can easily be present in this framework [5]. The applied research shows mixed results with no clear consensus emerging in cross country studies. Moreover, most of these studies have do not account for possible discontinuities in the analysis of the linear relationships[6]. The lack of control for structural breaks in the series may be reflected in the parameters of the estimated models that, when used for inference or forecasting, can induce to misleading results. This problem is especially true in the case of very long time series that cover different historical stages that can be subject to discontinuities. In general, structural breaks are a problem for the analysis of economic series, since they are usually affected by either exogenous shocks or changes in policy regimes. As a consequence, the assumption of stability in the long-run relationship between debt and growth would be too restrictive, so that not allowing for structural breaks would be an important potential shortcoming.The debt-growth nexus has probably changed overtime due to variations in macroeconomic and market forces, changes in the structure of the economy, supply or demand shocks, and regulation reforms. Thus, the information content of this nexus is subject to change overtime and all the empirical modeling work that does not take into account the possible variations and instabilities will fail to explain the evolution of the debt-growth relationship.

In the case of Spain, the descriptive analysis performed by Comín (2012) for the period 1850-2012 shows several cycles in the debt service payments of the Spanish economy. A relevant issue in this analysis is that nonlinearity and instability generally are difficult to distinguish and both are compatible. Particularly, the instability in this relationship could lead to nonlinearity, and vice versa. Therefore, we will analyze both possibilities in this research. Comín (2012) signals the existence of five different periods or *default cycles* in the debt that mainly coincide with those reported in Reinhart and Rogoff (2009b) for the world economy excepting the debt crisis cycle of 1930-1950. Therefore, according to the analysis developed by Comín (2012), the first cycle covers the period 1850-1889 from Finance Minister Bravo Murillo's debt restructuring in 1855 to that of Camacho in 1882. The second cycle spans from 1890 to 1920 covering the austerity and rescheduling program implemented by Minister Fernandez-Villaverde in 1899 up to the First World War. The third cycle covers the period of Franco's regime. Both the absence of bonds issues during the Franco's mandate and the steady economic growth of the late 1950s and 1960s reduced the

[5] Only recent studies, like Di Sanzo and Bella (2015) or Gómez-Puig and Sosvilla-Rivero (2015) cover the causality analysis and possible endogeneity in the estimation of the debt-growth nexus

[6] Gómez-Puig and Sosvilla-Rivero (2015) being a remarkable exception.

public debt/GDP ratio, so that during the Franco's era there was no formal public debt crisis. The fourth historical public debt cycle took place under democracy with increasing but sustainable percentages up to 1992-1993 fiscal crises when the general government's debt-to-GDP ratio increased to 52% to reach a peak of 67.4% that diminished until a 36.1% in 2007. After 2008, the stock started an increasing trend up to the present reaching around a 100%. It is worth to say that the level of public debt was almost meeting the criteria of the Growth and Stability Pact of the Eurozone in 2008, and consequently, no sustainability problems were expected whatsoever. However, the contagion of the debt crisis from other peripheral countries provoked a sharp increased in the Spanish risk premium and a self-fulfilling prophecy-type debt crisis where changing expectations were able to lead to multiple equilibria. In order to unveil the relationship of the debt-to-GDP ratio and the real GDP growth rate all over the historical sample period analyzed, a first insight can be made through the visual inspection of Fig. 1a, 1b and 1c. that shows a scatter plot of the debt and growth variables between 1851 and 2013. Due to a lack of homogenous debt data for all the sample range we consider two different periods. In Fig. 1a are depicted the data from Carreras and Tafunell (2005)[7] for the period 1851-2000, while in Fig 2a and 2b we can see the same relationship for two different definitions of debt provided in Banco de España (2014) for the period 1965-2013. In the first one we use as debt the total outstanding liabilities of the general government while in the second, this debt variable is compiled according to the Excessive Deficit Procedure (EDP). A rapid inspection of these figures show that while it is not easy to find out any evidence of a trending relation between both variables for the longer period, the analysis of the shorter sample seems to show a negative relationship using both definitions giving some evidence in favor of the *R&R hypothesis*[8]. Moreover, the inspection of Figures 2, 3, 4 and 5, where the different variables are depicted seem to show the existence of multiple structural breaks in the series. Therefore, we need a more refined empirical cliometric analysis.

In order to answer the previous question, in this paper we extend the existing empirical analysis for the Spanish economy in three ways. First, the originality of our analysis arises from the adoption of recently developed methods from the time series literature which help to improve the way we can empirically model the debt-growth nexus. We use a reduced-form linear model that allows for multiple endogenously determined breaks, and secondly, extent the analysis testing for threshold cointegration

[7]See Section 3 for further details

[8]However, this simple analysis does not find any support of an inverted U-shaped figure (*Debt Laffer curve*).

among the variables studied. More specifically, we use a new approach developed by Kejriwal and Perron (2008), Kejriwal2010 to test for multiple structural changes in cointegrated regression models. They propose a sequential procedure that not only enables detection of parameter instability in cointegration regression models but also allows a consistent estimation of the number of breaks present. Furthermore, we test the cointegrating relationship when multiple regime shifts are identified endogenously. In particular, the nature of the long run relationship between public debt and growth is analyzed using the residual based test of the null hypothesis of cointegration with a single or multiple breaks proposed in Arai and Kurozumi (2007) and Kejriwal (2008), respectively. Second, by focusing on the time series of a single country, we address the crucial question of a specific country's evolution of its debt-growth nexus avoiding problems of cross-sectional dependence and heterogeneity among countries. Moreover, a common criticism to most tests of the *R&R hypothesis* is that the econometric procedures used require a large number of observations. Accordingly, in this paper we use a long span of the data (1851-2013). It will allow us to obtain more robust results.

The rest of the paper is organized as follows. A brief review of the theoretical and empirical literature is provided in Section 2, the description of the data appears in Section 3. The empirical framework for the linear model is presented in Section 4 while Section 5 is devoted to the empirical results. Section 6, describes the econometric methodology for the non linear approach and Section 7 is devoted to report the empirical evidence. Finally, Section 8 concludes with a summary and the main policy implications.

2 Review of the theoretical and empirical literature

2.1 Theoretical approach

The relationship between public debt and economic growth is considered an issue of fundamental importance to empirical and theoretical economics, since strong growth can greatly smooth fiscal adjustment in the short run but, at the same time, a high debt-to-GDP ratio penalizes potential growth. Although it can be imperative to lower public debt over time, however, in the short-run, front-loaded fiscal adjustment is likely to hurt growth prospect, delaying improvements in fiscal indicators, like deficits, debt or service costs. At this point, other policies like monetary, financial, and structural policies (reforms in goods, service, and labor markets that improve economic efficiency) can help to support growth when fiscal policy is tightened. It is clear that this relationship covers both macroeconomic and microeconomic issues.[9] We will focus here on the former ones. All in all, this relationship between economic growth and debt is complex due to the array of feedback loops between both variables in the short and the long run. Thus, disentangling this nexus and assessing its importance and sense is a crucial issue.

From a theoretical point of view, and according to Di Sanzo and Bella (2015), the literature has proposed three main theories of debt. The first one is the *neoclassical school* that asserts that government spending or tax cuts has no impact (are neutral) on GDP due to the *Ricardian equivalence* that postulates that an exogenous increase in the budget deficit (a decline in public savings) will lead to an instantaneous equal increase in private savings (Barro (1974)). In this case, the national saving will stay the same, as will all other macroeconomic variables and there won't be effects of the way the public deficit is financed (via public debt, taxes or money) on growth. Agents fully anticipate the debt burden of the fiscal stimulus, expecting higher taxes in the future (wealth effect). Thus, in order to smooth out their level of consumption they save more as their disposable income increases, leaving private consumption unchanged. There is a crowding out effect of the private sector that fully offsets the increase of the demand from the public sector which renders the fiscal multiplier to zero. According to Pragidis et al. (2015) this is more apparent in periods of growth, since the probability of a more efficient usage of resources from the government is lower than it is during a recession. On the other hand, there is room for a low positive multiplier during recessions, since resources are underused.

The second theory is the so-called *hysteresis theory* proposed by Delong and Summers (2012). The authors argued that, under certain conditions, deep and prolonged recessions may reduce the future potential output (hysteresis effect). For example, a protracted recession could drive workers permanently out of the labor force, with the associated loss of skills, and negative effects on business investment[10]. In presence of hysteresis effects in a depressed economy, this approach pointed out that the policies

[9]The microeconomic effects of fiscal policy on growth are those related to the way the presence of the public sector is felt in the economy above and beyond its macroeconomic balances (deficit and debt). The overall tax pressure, how tax revenues are levied, and the manner in which spending takes place all affect economic incentives and hence investment, employment, economic efficiency and growth.

[10]There is also the waste of talent from educated unemployed young people

of austerity may be counterproductive and can erode the long-run fiscal balance while stimulus can improve it. Thus, in this framework, government spending may have a positive effect on output in both the short and long-run.

Finally, the third theoretical approach is the *conventional theory* (see Elmendorf and Mankiw (1999), for a survey on this topic) that predicts that an increase in government debt (due to fiscal deficit) produces a positive effect on aggregate demand and output in the short-run but crowds out capital and will have a negative effect on output in the long-run leading to an inverted U-shaped relationship between debt and growth. As pointed out by Panizza and Presbitero (2014) there is no a clear theoretical argument for the presence of non-linearities in advanced economies but there is a well-established strand of the literature on the asymmetric effects of fiscal policy that can serve to backup a non-linear effect of public debt on output growth, and more importantly, non-linearities may appear if there is a tipping point of fiscal sustainability. We devote the next paragraphs to derive these arguments.

The positive growth effects of public investment have long been recognized in the theoretical literature. Agénor (2012) outlines a series of ways through which increases in public capital may affect growth. The first is the positive productivity and cost-saving effects in the private sector associated to more public capital; the second is a complementary effect on private capital as more public capital increases the rate of return on private capital; the third is a crowding-out effect, when increases in public capital displaces private investment, and the fourth is the so-called *Dutch vigor* effect, consisting in a raise in the total factor productivity through positive learning-by-doing externalities induced by higher public capital (see Berg et al. (2010)). However, increasing public investment may not always enhance growth: low public investment efficiency and absorptive capacity constraints can significantly discount the growth benefits of public investment (Berg et al. (2013) and Van der Ploeg (2012)). Following Égert (2015b) there are a number of channels through which public debt is likely to hamper long-term growth. First, tax hikes needed to service a higher public debt crowd out private investment by reducing disposable income and saving, raise the distortionary costs of taxation, and are likely to result in non-neutral tax treatment within and across asset classes, thus amplifying distortions. Second, soaring public debt will push up long-term sovereign yields in a nonlinear fashion, as the likelihood of default increases, also does uncertainty, creating expectations of future financial repression, and increasing sovereign risk (Paniagua et al. (2015)). High long-term rates crowd out productive public investment, and more importantly, reduce private investment by increasing the cost of capital. Reduced investment in R&D will have long-lasting negative impacts on growth (Tanzi and Chalk (2000);

Laubach (2009), Elmeskov and Sutherland (2012)). All in all, public debt could have a larger negative effect on economic outcomes as the payment of interest on the debt of the older generations generates an allocation exchange system across generations reducing young people's savings (Teles and Mussolini (2014)), Third, public authorities, especially in countries with weak institutions, may decide to reduce the debt burden via inflation, which has detrimental effect on long-term growth (Woo and Kumar (2015)).

Also, high debt is likely to constrain the scope for countercyclical fiscal policies, which may result in higher volatility and further lower growth (Aghion et al. (2014); Woo (2009)). Moreover, new research finds that the multiplier of fiscal contraction can be positive and vice-versa which means that contractionary fiscal expansions or expansionary fiscal contractions are possible due mostly to a wealth effect[11]. According to Alesina and Perotti (1995) and Giavazzi and Pagano (1990)) fiscal contraction based on expenditure cuts may be expansionary if it is accompanied by currency devaluation[12] or by structural reforms, i. ex. agreements with the unions on labor conditions. In the same vein, Wiese et al. (2015) have found evidence on the key role played by political consensus in order to accomplish a successful fiscal adjustment. The greater this adjustment is the more is being anticipated by the agents leading to more powerful results. In more extreme cases of a debt crisis, by triggering a banking or currency crisis, the adverse effects of hight debt levels can be magnified (Burnside et al. (2012); Hemming et al. (2003). Therefore, fiscal consolidation may reduce uncertainty for the future, leading to an increase in household's wealth today through the decrease of interest rates as a result of the reduction of the risk premium of government bonds (Alesina and Ardagna (2010).

Bertola and Drazen (1993), postulate that the sign of the fiscal multiplier depends on the debt-to-GDP ratio. In a hypothetical economy, where all agents are rational, and debt-to-GDP ratio is low, an increase of the government spending will be neutral to the real economy, featuring a Ricardian or even a negative effect. If the debt to GDP ratio is relatively large a fiscal consolidation signals a trial of the government to stabilize the economy and thus lifting future uncertainty leading to a positive multiplier or to an anti-Keynesian effect, which would create again an inverted U-

[11]Consumers put more weight to future consumption than to current one, so that large fiscal consolidations lead to a revision in expectations about the future tax burden and may also induce a supply-side response if taxes are distortionary. Indeed, several papers conclude that successful fiscal policy adjustments rely on expenditure cuts rather than increased revenues (McDermott and Wescott (1996); Alesina and Ardagna (1998) and Hernández de Cos and Moral-Benito (2013)).

[12]Which is not possible in a monetary union.

shaped relationship between the debt to GDP ratio and growth. As pointed out by Checherita-Westphal et al. (2014), such a relationship imply an optimal, *growth-maximazing*, level of debt. A feasible strategy is to focus on how to create a safe zone or *fiscal space* based on past behavior by examining the fiscal limit or point at which fiscal policies become unsustainable and public finances collapse (Ghosh et al. (2013)). Although economic analysis agrees on the need to maintain the sustainability of debt/GDP levels, the policy debate has devoted most of the attention to deficit/GDP ratios until recently[13].

Fiscal sustainability requires a government to be solvent, which means that it has to be able to repay its debt at some point in the future. The primary budget balance (budget balance net of interests payments) is a key determinant of government debt dynamics, but there are other factors that have been neglected or, at least, underestimated so far by mainstream economic analysis. In fact, gross debt accumulation is driven by three main factors: first, the above-mentioned government primary balance; second, the so-called "snowball effect", which captures the joint impact of interest payments on the outstanding stock of debt and of real GDP growth and inflation rates over the debt-to-GDP ratio; and third, the deficit-debt relationship, also called "the stock-flow adjustment", which relates to all other factors that affect the outstanding stock of debt but are not recorded as part of the primary balance (see European Central Bank (2011)).[14]

Trehan and Walsh (1991) derived that the sufficient and necessary conditions for the Intertemporal Budget Constraint (IBC) to be satisfied are the existence of a cointegration relationship between primary deficit and debt, as well as the stationarity of the quasi-difference of the primary deficit. Moreover, more recently Bohn (2007) showed that the IBC condition is compatible with the variables involved being of any order of integration. Bohn (1998) has suggested that the analysis of the fiscal policy soundness should not be limited to the evaluation of the stationarity of the debt-to-GDP ratio. This author suggests that fiscal policy reaction functions should be used for the assessment of fiscal deficit sustainability. The idea is to determine whether governments are reacting to the evolution of debt by adjusting primary balances in the following periods.

In this section, we derive the algebra for an "ad hoc" version of the IBC and the implied stationarity restrictions. The one-period government budget constraint can

[13] See, for instance, the Stability and Growth Pact in the case of the Euro area.

[14] The above analysis implies that a full assessment of fiscal sustainability requires a comprehensive approach where debt dynamics should capture the feedback effects between fiscal policies, the macroeconomy and the financial sector.

be written as follows:

$$\Delta B_t + \Delta M_t = G_t - T_t = DEF_t, \tag{1}$$

where B_t is the real market value of government debt, M_t is the money base, G_t is real government expenditure inclusive of interest payments, T_t represents real tax revenues and $\Delta = (1 - L)$ is the first difference operator. The deficit (DEF_t) is the one-period difference between outlays and revenues and it also equals the change in public debt and/or money base. In this sense, as claimed by Bohn (2005), while (1) holds in nominal terms, changes in real debt differ from the real value of deficit by an inflation term. Therefore, it is important in this context in order to separate the stock of debt from the outflows of outlays and revenues, to use a scale-invariant definition of debt dynamics.

Denoting i_t as the real interest rate[15] and assuming i_t to be I(0) stationary around a mean i, as in Hakkio and Rush (1991), we can define:

$$G_t = GE_t + i_t B_{t-1}, \tag{2}$$

where GE_t is the real expenditure exclusive of interest payments, and the second term on the right hand side of (2) represents interest payments on the level of debt accumulated at the end of the previous period. Further, we can express the debt as:

$$B_t = (1 + i)B_{t-1} + EXP_t - T_t, \tag{3}$$

where $EXP_t = GE_t + (i_t - i)B_{t-1}$, or, alternatively, $B_t = \left(\frac{1}{1+i}\right)(T_{t+1} - EXP_{t+1}) + \left(\frac{1}{1+i}\right) B_{t+1}$. As the government is subject to the same restriction for periods $t+1,\ t+2, ...$, we can aggregate inter-temporally the different budgetary restrictions for each individual period and obtain:

$$B_t = \sum_{j=0}^{\infty} \left(\frac{1}{1+i}\right)^{j+1} (T_{t+j+1} - EXP_{t+j+1}) + \lim_{j\to\infty} \left(\frac{1}{1+i}\right)^{j+1} B_{t+j+1}. \tag{4}$$

The representation of (4) in terms of the first difference of B_t is the standard specification that is used in the empirical literature to test for fiscal deficit sustainability –

[15] Note that the variables could be expressed in nominal terms, real terms, or as a ratio to GDP as long as i_t is adjusted accordingly (i.e., if the variables are in nominal terms, i_t is the nominal interest rate; if the variables are in real terms, as it is our case, i_t is the real interest rate; if the variables are ratios to GDP, $1 + i_t$ is the growth-adjusted real interest rate that follows from dividing the gross real interest rate by the gross rate of output growth).

see Quintos (1995). If we take first differences on (4) the sustainability of the public finances is associated with the condition:

$$\lim_{j \to \infty} E_t \left(\frac{1}{1+i} \right)^{j+1} \Delta B_{t+j+1} = 0. \tag{5}$$

The link between fiscal sustainability and growth is further highlighted by the basic debt dynamics equation. According to Dreger and Reimers (2013) higher public debt, caused by higher public spending or lower tax revenues can stimulate domestic demand, with expansionary effects on income and output in the short run via increases in investment. However, the positive effect in the short run might be disputed in periods of high debt. Negative effects are more pronounced if debt levels increase uncertainty about default that will call into question fiscal sustainability and trigger higher risk premia and long term real interest rates. Romer (2012) has stressed the relevance of expectations, which may be self-fulfilling. These effects may be stated in the budget constraint of the government in a continuous time setting. In this case, in order to allow for price variation, we define now all the variables in nominal terms and as a ratio to nominal GDP (PY):

$$g - t + ib = \Delta B/PY + \Delta M/PY, \tag{6}$$

where $g = G/PY$, $t = T/PY$ and $b = B/PY$, so that b is the debt-to-GDP ratio. We can approximate $\Delta B/PY$ solving for B in $b = B/PY$, so that $B = bPY$. As the differential of a product with three variables is $dB = \mathrm{d}bPY + \mathrm{d}PbY + \mathrm{d}YbP$ and this expression can be approximate in differences getting: $\Delta B = \Delta bPY + \Delta PbY + \Delta YbP$. Now, we divide the former expression by PY obtaining:

$$\Delta B/PY = \Delta b + b(\pi + y), \tag{7}$$

In the same way, the deficit money financing ratio M/PY can be expanded using M and leading to:

$$\Delta M/PY = \Delta M/MM/PY = \mu m, \tag{8}$$

where $m = M/PY$ and μ is the money growth ratio. Substituting (6) by (7) and (8) and bearing in mind that $r = i - \pi$, we obtain:

$$\Delta b = (g - t) + (r - y)b - \mu m, \tag{9}$$

where b represents the public debt-to-GDP ratio and Δd denotes the change in that ratio per unit of time, $pdef$ is the primary deficit (defined as $(g-t)$, so that $pdef > 0$

implies a deficit), r is the average real rate of interest charged on existing debt, and y is the rate of growth of potential (trend) output in real terms that we present in a compact way in

$$\Delta b = pdef + (r - y)b - \mu m, \tag{10}$$

Following Checherita-Westphal et al. (2014), this is the standard identity for assessing the long-run stability of debt and implies contemporaneous exogeneity even if $(r-y)$ might show endogeneity on the path to its steady-state value. The debt ratio will stabilize ($\Delta b = 0$) when

$$pb + \mu m = (r - y)b, \tag{11}$$

where pb is the bugdet's primary surplus defined as $-pdef$. Thus, the former equation show us that if the real interest rate on debt is higher that the growth rate of the economy ($r > y$), the debt-to-GDP ratio grows out of control and this dynamics only will stop generating primary fiscal surpluses of sufficient size (pb) or using money creation (seignorage) and inflation overtime. On the contrary, if $r < y$, the government could maintain a primary deficit in perpetuity. If the country moves from the steady-state values to a permanently larger deficit, the result would be an increase over time in the debt-to-GDP ratio, but the path is not explosive and would eventually stabilize at a new value of b. All in all, as the ratio of government debt-to-GDP rises, the government's borrowing cost likely rises with it. Once government debt becomes so large as to result in a positive value for $r - y$, sustainability of the fiscal path will one again come back to requiring that (11) be satisfied. One possibility is that the government will be able to maintain quite credibly the future necessary long-run primary surpluses implying a sustainable debt-to-GDP ratio. An alternative that creditors may contemplate is that there could be a partial default or surprise inflation on government debt, bringing the debt-to-GDP ratio back down to a sustainable value. Equation (11) creates a fiscal reaction function where the primary surplus depends, among other factors, on the level of the debt-to-GDP ratio b, characterizing a slippery slope on which governments may find themselves as debt levels rise relative to GDP. Moreover, in case agents are risk averse, three factors have been single out that could lead to a sudden increase in a country's borrowing cost: a) a significant deterioration of the fiscal situation; b) a decrease in the probability that the country will successfully complete the fiscal reforms necessary to return to a sustainable path; and c) an increase in the risk premium on the sovereign debt [16]. However, this level can be at odds with an optimal level in terms of growth

[16] As stated in Greenlaw et al. (2013) a large literature has looked at the determinants of currency and sovereign debt crises, much of it focusing on the experience of developing economies. Reinhart et al. (2003) found that emerging-market economies have a lower tolerance for sovereign debt, with

enhancing in the long-run. Much of fiscal policy is usually committed to long-run targets (public services, social support, education, infrastructure, sustainable finances) and it is not well suited to discretionary stabilization if consistency across time and policy types is not maintained (Checherita-Westphal et al. (2014)). If the primary government surplus that would be necessary to stabilize the debt-to-GDP ratio is far from a country's historical experience and politically plausible level, the government will begin to pay a premium to international lenders as compensation for default or inflation risk, which makes fiscal sustainability even harder to achieve, possibly leading to a fiscal crunch, that is, a tipping point in which sovereign interest rates shoot up and a funding crisis ensues, reducing growth and employment.

2.2 Empirical evidence

The current economic and financial crisis has led to a substantial increase in the general government debt of the OECD countries. This growth in debt, and the difficulty of bringing it to a halt, has placed the sustainability of public finances at the centre of the economic policy debate. This question has triggered an increasing bulk of empirical studies trying to measure the effect of debt on growth from different theoretical and empirical approaches[17].

Recently, Jordà et al. (2015) conduct a historical study of the interaction between public and private debt in advanced economies over the years 1870 to 2012. They find that private debt is responsible for two thirds of the substantial increase in total debt that has taken place in the Western world over the last four decades. An analysis of private and public debt run-ups indicates that historically it has been

defaults at much lower levels of debt to GDP. Reinhart and Rogoff (2010) provided further evidence. Mishkin (1996, 1999) attributed the lower debt tolerance of emerging-market economies to their weaker financial institutions and greater vulnerability to international capital flows. Eichengreen et al. (2005) described the inability of emerging market countries to borrow in their own currencies as *original sin*. The denomination of debt in foreign currencies implies that a currency depreciation increases the debt burden, which can lead to financial crises, a collapse in the economy and further exchange rate depreciation. The possibility of this vicious cycle puts limits on the amount of debt that a country can issue and constrains monetary policy options. Unfortunately, the recent experience has shown that more advanced economies are not immune to potential sovereign-debt problems similar to those widely observed in less developed economies. De Grauwe (2012) argued that the periphery countries of the union are in a similar situation to emerging economies, forced to borrow in a currency (the euro) whose supply they do not control.

[17]See Panizza and Presbitero (2013) and Tomova et al. (2013), for a review of the literature on this topic

the accumulation of private debt that ignited financial crises whereas the build-up of public debt seems to have no predictive power.

However, in general, the evidence available seems to suggest that the existence of high levels of public debt for prolonged periods may have significant macroeconomic repercussions. From the former theoretical revision in the previous subsection and following the Banco de España (2014) we can single out the different mechanisms at stake that explain the debt-growth nexus. First, high levels of debt are usually associated with higher interest rates and, via crowding-out of funding for the private sector, lead to lower medium term GDP growth rates. However, as we will see in more detail, the latest evidence suggests that it is not possible to find a particular threshold of public debt valid for all the countries. Second, high public debt reduces the leeway for a counter-cyclical fiscal policy[18], triggering austerity measures and possible debt crisis episodes. Third, connected to the former mechanism, the sustainability of a high level of public debt, in a context of moderate growth, requires large and sustained primary surpluses, which may affect the composition of public finances and, ultimately, the potential growth of the economy. Finally, a high public debt ratio generates larger borrowing requirements in the short term, which increase the economy's vulnerability to financial market conditions.

Since the work of Aschauer (2000), the empirical literature on the quantitative effects of public capital on output under an aggregate production function approach has grown in volume and sophistication. In a meta-analysis across a large set of empirical studies using industrial country-data (both time series and panel) under the production function approach, Bom et al. (2008) conclude that estimates of the output elasticity of public capital range from 0.175 to 0.917. However, several caveats, such as problems of non-stationarity, endogeneity (potential simultaneity and other forms of reverse causation) and heterogeneity for panel data analysis have dominated much of this literature (see, inter alia, Romp and Haan (2007) for a review). Using the implications of an endogenous growth model Afonso and Alves (2014) document empirically in panel growth regressions that the main impact of fiscal variables on total factor productivity and growth comes through alterations in the pattern of investment in the economy. They identify a crowding-in effect of public investment into private investment that results in an overall positive effect of public investment. Moreover, looking at the budgetary composition and the effects of different categories of government spending on growth, Afonso and Alves (2014) summarize the findings of economic literature on the relation between public finances, more specif-

[18]Indeed, there is evidence associating high levels of public debt with greater volatility of economic growth, which could be a consequence of this lack of fiscal policy leeway.

ically its composition, level and sources of financing, and economic growth. These studies analyze the impact of fiscal variables, including government debt, on long-term interest rates or spreads against a benchmark, as an indirect channel affecting economic growth with somewhat controversial results.

Let us consider first the effect that fiscal policy has on potential growth. These effects operate both through macroeconomic and microeconomic channels. A key macroeconomic channel is the effect of high public debt on potential growth. The discussion of instability and tipping points suggests the possibility of nonlinearities in these relations. For advanced economies, a separate literature has looked at the relation between debt burdens and economic growth rates, and it also finds substantial empirical evidence of nonlinearities or tipping points.

In a seminal study by Reinhart and Rogoff (2010), which analyses (using simple correlation statistics) the developments of public (gross central government) debt and the long-term real GDP growth rate in a sample of 20 developed countries over a period spanning about two centuries (1790-2009), the authors find first, that the relationship between government debt and long-term growth is weak for debt/GDP ratios below a threshold of 90 % of GDP, and second, that above this threshold, the median growth rate falls by one percent point and the average by considerably more (about 4 percentage points). Reinhart et al. (2012) documented that in advanced countries, levels of sovereign debt above 90% of GDP ("debt overhangs") lead to a decline in economic growth. The magnitude of the debt threshold has been partially confirmed by other studies. While Cecchetti et al. (2011) found a threshold of around 85% for the debt-to-GDP ratio at which sovereign debt retards growth, Caner et al. (2010) and Elmeskov and Sutherland (2012) reported even lower turning points of around 70%. Lee and Chang (2009) found a possible nonlinear hump-shaped effect of debt on growth with two regimes around 33% and 67%. With a case study approach, the International Monetary Fund (2012) came to similar conclusions using a 90-100% threshold, but noted that it matters whether a country's debt level is increasing or decreasing. In other recent studies focused on the euro area, Woo and Kumar (2015) and Cecchetti et al. (2011), find a linear inverse relationship between initial debt and subsequent growth in a sample of emerging and advanced economies, with the impact being somewhat smaller in the latter group. Both of them find that beyond a certain threshold about 80-90 percent of GDP higher public debt lowers potential growth. Woo and Kumar (2015) find that higher debt starts affecting growth at a lower threshold (40 percent of GDP), but the effects become statistically significant only at about 90 percent of GDP. According to these results, countries with high debt should address their fiscal problems to avoid a deterioration in their growth perspectives. The creation of fiscal buffers might be an appropriate strategy to compensate for extraordinary shocks. Further literature empirically tests for non-linear interdependencies between debt and growth. Checherita-Westphal and Rother (2012) find a highly significant negative non-linear relationship between the government debt ratio (and other control variables such as investment, population growth or interest rates) and GDP per capita growth rate for a sample of 12 euro area countries during the period from 1970 to 2008. The channels through which public debt is likely to have an impact on economic growth rate are seen to be private saving, public investment, total factor productivity, and sovereign long-term nominal and real interest rates. The estimated debt threshold of 90-100 % of GDP, after which additional debt starts to have a negative impact on growth, is an average for the sample of 12 countries and it may go as low as 70 % of GDP which suggests that for many countries public debt levels may already have detrimental impact on growth as the average public debt ratios are above the lower threshold. In a connected study, Baum et al. (2013) look at short-run effects of debt on growth in EMU countries by applying a dynamic threshold panel method with one-year lagged GDP growth as the dependent variable. The authors find that in the short term, additional debt has a positive impact on GDP growth. This effect, however, decreases to zero and turns non-significant as the debt-to-GDP ratio reaches the level of 67%. In EMU countries with debt levels higher than 95% of GDP, further accumulation of debt has a negative impact on economic growth. These results confirm the previous findings about the negative impact of high debt levels but refine them by indicating that additional debt may be favorable at lower debt levels i.e. when additional stimulus is needed in a low-debt economy that is going through a downturn. However, Panizza and Presbitero (2014) do not find evidence of an effect of public debt on medium-term growth. High debt is expected to result in lower growth because of crowding out effects on private investment, which would thereby lower productivity growth. Finally, Eberhardt and Presbitero (2015) analyze whether the relationship between public debt and economic growth is significantly negative and further investigate the presence of common or country-specific thresholds.The analysis is based on total government debt, measured at face value, as this definition is broadly comparable across countries and makes it possible to use a large and sufficiently long panel dataset. The results find some support for a negative relationship between public debt and long-run growth across countries, but no evidence for a similar or common debt threshold within countries pointing to a diversity across countries. Moreover, there is no evidence of any systematic change in the relationship between debt and growth when countries shift from a ”low” to ”high” debt regime. According to these authors, the commonly found 90% debt threshold is likely to be the outcome of empirical misspecification (using a pooled instead of heterogenous model) and subsequently a misinterpretation of the results.

The approach adopted in this paper addresses several important econometric and modeling issues neglected in previous literature. First, it is highly flexible and can approximate complicated nonlinear relationships without assuming a priori any particular relationship; second, it avoids nonlinear transformations of potentially non-stationary income. Preceding empirical literature has generated substantial criticism. Indeed, many studies risked spurious findings by ignoring that variables like debt and GDP per capita are likely non stationary; moreover, studies considering the non-stationary nature of the variables still risked spurious findings by performing nonlinear (quadratic) transformations of a prospective non-stationary variable (GDP per capita growth); other more recent studies to date that have employed panel unit root and panel cointegration techniques have relied on methods that incorrectly assume that the cross-sections are independent. We use recent advances in time series cointegration techniques that deal with the problem of endogeneity; as debt is likely to be endogenous, the existing literature tries to address endogeneity by using lagged values of the debt-to-GDP ratio (Cecchetti et al. (2011)), GMM estimations with internal instruments (Woo and Kumar (2015)), and by instrumenting the debt-to-GDP ratio with the average debt of the other countries in the sample (Checherita-Westphal and Rother (2012)). While these are useful first steps, we think that they are inadequate to fully address endogeneity. The use of lagged variables is problematic because debt and growth tend to be persistent. The high persistence of debt ratios also limits their validity as internal instruments in the standard GMM estimators developed by Arellano and Bond (1991), Arellano and Bover (1995), and Blundell and Bond (1998). Another drawback found in previous literature is that the polynomial of GDP per capita growth model (either quadratic or cubic) used to study the possible nonlinear nexus of the variables has been criticized for being highly inflexible and for rendering unimportant feasible debt-growth relationships for which it cannot test. For example, the typical polynomial model does allow for the possibility that GDP growth elasticities vary significantly overtime as debt increases but are always positive, i.e. they do not consider a saturation effect or S curve or a possible decoupling in the relationship. Another salient feature of our analysis is that focusing our study on a single country we avoid several problems usually present in this literature: first, we avoid the effects derived from correlations across countries in the estimations; second, we are not concerned about the homogeneity of the equilibrium relationship between debt and growth as well as the existence of a similar debt threshold. Finally, our paper is also closely related to the historical work of Reinhart and Rogoff (2009b,a, 2010) and Balassone et al. (2011) for the case of Italy. Our scope, however, is narrower than that of these papers. In particular, we concentrate on government debt and, unlike Cecchetti et al. (2011) and Reinhart et al. (2012), we do not explore the complex interactions between private and public debt.

3 Data

We use time-series data on the Spanish economy spanning from 1851 to 2000, and 1964 to 2013. The length of these database makes it especially suitable for the econometric approach adopted in this paper. The data and sources are:

1851 – 2000: *a*) public debt, total outstanding liabilities, B_{1t}, from Carreras and Tafunell (2005), Table 12.34, serie 2895; *b*) real GDP, y_{1t}, from Carreras and Tafunell (2005), Table 17.6, serie 4741; *c*) nominal GDP, Y_{1t}, from Carreras and Tafunell (2005), Table 17.7, serie 4744; *d*) the real GDP growth, g_{1t}; *e*) the public debt-to-GDP ratio, $b_{1t} = B_{1t}/Y_{1t}$.

1964 – 2013: *a*) public debt from general government (total outstanding liabilities), B_{21t}, from Banco de España (2014), Table 2.15*a*; *b*) public debt from the general government: debt compiled according to Excessive Deficit Procedure (EDP), B_{22t}, from Banco de España (2014), Table, 2.15.*a*; *c*) real GDP, y_{2t}, from Banco de España (2014), Table 1.3; *d*) the real GDP growth, g_{2t}; e) nominal GDP, Y_{2t}, from Banco de España (2014), Table 1.1; *f*) the public debt-to-GDP ratio (total outstanding liabilities), $b_{21t} = B_{21t}/Y_{2t}$; *g*) the public debt-to-GDP ratio (EDP), $b_{22t} = B_{22t}/Y_{2t}$.

Most of the literature on debt-growth cliometric analysis has used gross government debt, in part because this was the only series available. In our case, the data are also available for net government debt for the more recent sample range (1964-2013) but, as there are arguments for using either measure, we have decided to use gross government all over the study for the sake of homogeneity and comparability of results. However, as it has been recently stated by Eberhardt and Presbitero (2015), the use of total public debt data measured at face value although it facilitates to find a sufficiently long dataset, has also some drawbacks that it is important to bear in mind. First, the exclusion of private debt may be problematic as private debt is a potential source of financial instability; second, our measure of public debt does not consider the proportion of foreign currency-denominated debt and its implicit implications for the fragility of the policy stance[19]. Third, as we have already explained, we are considering gross public debt even if net debt could be a better measure of government indebtedness Panizza and Presbitero (2013). Finally, considering the face value of debt can be misleading as countries can borrow at different maturities and contractual forms Dias et al. (2014).

Bearing all these points in mind, we can start the visual inspection of the series. The evolution of the real GDP growth and the public debt-to-GDP ratios series appear in Figures 2 to 5. The plots suggest that the association between real GDP growth and the public debt-to-GDP ratio may have altered over time. Macroeconomic series appear to often be characterized by broken trend functions, and therefore the long-run relationship could be affected by structural breaks and nonlinear relationships. We proceed to account for this possibility using state-of-the-art time series cointegration analysis.

4 A linear cointegrated model of the public debt-growth nexus with multiple structural changes

In this section, we examine the issue of the link between public debt and growth to account for potential breaks in the long-run relationship between g_t and b_t as well as the cointegration tests with multiple breaks. First, we test for the order of integration of the variables. Second, we test the stability of the public debt-economic growth relationship (and select the number of breaks) using the test proposed in Kejriwal and Perron (2008, 2010). Third, we verify that the variables are cointegrated with tests allowing for a single or multiple structural changes in the coefficients as proposed by Arai and Kurozumi (2007) and Kejriwal (2008), respectively. Finally, we estimate the model incorporating the breaks in order to study if the relationship between public debt and economic growth (the slope parameter β) have altered over time.

[19]See, for instance, the literature on the "original sin" due to Eichengreen et al. (2005) or the more recent literature on the "fragility" of EMU due to De Grauwe (2012)

4.1 Methodology

4.1.1 A linear cointegrated regression model with multiple structural changes

Issues related to structural changes have received a considerable amount of attention in the econometric literature. Bai and Perron (1998) and Perron (2006, 2008) provide a comprehensive treatment of the problem of testing for multiple structural changes in linear regression models. Accounting for parameter shifts is crucial in cointegration analysis since it normally involves long spans of data which are more likely to be affected by structural breaks. In particular, Kejriwal and Perron (2008, 2010) provide a comprehensive treatment of the problem of testing for multiple structural changes in cointegrated systems. More specifically, they consider a linear model with m multiple structural changes (i.e., $m+1$ regimes) such as:

$$y_t = c_j + z'_{ft}\delta_f + z'_{bt}\delta_{bj} + x'_{ft}\beta_f + x'_{bt}\beta_{bj} + u_t \qquad (t = T_{j-1}+1, ..., T_j) \tag{12}$$

for $j = 1, ..., m+1$, where $T_0 = 0$, $T_{m+1} = T$ and T is the sample size. In this model, y_t is a scalar dependent $I(1)$ variable, $x_{ft}(p_f \times 1)$ and $x_{bt}(p_b \times 1)$ are vectors of I(0) variables while $z_{ft}(q_f \times 1)$ and $z_{bt}(q_b \times 1)$ are vectors of I(1) variables.[20]The break points $(T_1, ..., T_m)$ are treated as unknown.

The general model (12) is a partial structural change model in which the coefficients of only a subset of the regressors are subject to change. In our case, we suppose that $p_f = p_b = q_f = 0$, and the estimated model is a pure structural change model with all coefficients of the I(1) regressors and constant (slope and the intercept in (15)) are allowed to change across regimes:

$$y_t = c_j + z'_{bt}\delta_{bj} + u_t \qquad (t = T_{j-1}+1, ..., T_j) \tag{13}$$

Generally, the assumption of strict exogeneity is too restrictive and the test statistics for testing multiple breaks are not robust to the problem of endogenous regressors. To deal with the possibility of endogenous I(1) regressors, Kejriwal and Perron (2008), Kejriwal and Perron (2010) propose to use the so-called dynamic OLS regression (DOLS) where leads and lags of the first-differences of the I(1) variables are added as regressors, as suggested Saikkonen (1993) and Stock and Watson (1993):

[20]The subscript b stands for "break"and the subscript f stands for "fixed"(across regimes).

$$y_t = c_i + z'_{bt}\delta_{bj} + \sum_{j=-l_T}^{l_T} \Delta z'_{bt-j}\Pi_{bj} + u_t^*, \qquad ifT_{i-1} < t \leq T_i \tag{14}$$

for $i = 1, ..., k+1$, where k is the number of breaks, $T_0 = 0$ and $T_{k+1} = T$.

In order to test the relationship between public debt and economic growth, the empirical studies commonly used a linear regression model such as:

$$g_t = \alpha + \beta b_t + \varepsilon_t \tag{15}$$

where g_t is the annual real GDP growth and b_t is the public debt-to-GDP ratio.

4.1.2 Structural Break Tests

We test the parameter instability in cointegration regression using the tests proposed in Kejriwal and Perron (2008, 2010). They present issues related to structural changes in cointegrated models which allows both I(1) and I(0) regressors as well as multiple breaks. They also propose a sequential procedure which permits consistent estimation of the number of breaks, as in Bai and Perron (1998).

Kejriwal and Perron (2010) consider three types of test statistics for testing multiple breaks. First, they propose a $\sup Wald$ test of the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there are a fixed (arbitrary) number of breaks ($m = k$):

$$\sup F_T^*(k) = \sup_{\lambda \in \Lambda\varepsilon} \frac{SSR_0 - SSR_k}{\hat{\sigma}^2} \tag{16}$$

where SSR_0 denote the sum of squared residuals under the null hypothesis of no breaks, SSR_k denote the sum of squared residuals under the alternative hypothesis of k breaks, $\lambda = \{\lambda_1, ..., \lambda_m\}$ as the vector of breaks fractions defined by $\lambda_i = T_i/T$ for $i = 1, ..., m, T_i$, and T_i are the break dates.

Second, they consider a test of the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there is an unknown number of breaks given some upper bound $M(1 \leq m \leq M)$:

$$UD\max F_T^*(M) = \max_{1\leq k\leq m} F_T^*(k) \tag{17}$$

In addition to the tests above, Kejriwal and Perron (2010) consider a sequential test of the null hypothesis of k breaks versus the alternative hypothesis of $k+1$ breaks:

$$SEQ_T(k+1|k) = \max_{1\leq j\leq k+1} \sup_{\tau\in\Lambda_{j,\varepsilon}} T\left\{SSR_T(\hat{T}_1,...,\hat{T}_k\right\} \tag{18}$$

$$-\left\{SSR_T(\hat{T}_1,...\hat{T}_{j-1},\tau,\hat{T}_j,...,\hat{T}_k\right\}/SSR_{k+1} \tag{19}$$

where $\Lambda_{j,\varepsilon} = \left\{\tau : \hat{T}_{j-1} + (\hat{T}_j - \hat{T}_{j-1})\varepsilon \leq \tau \leq \hat{T}_j - (\hat{T}_j - \hat{T}_{j-1})\varepsilon\right\}$. The model with k breaks is obtained by a global minimization of the sum of squared residuals, as in Bai and Perron (1998).

4.1.3 Cointegration tests with structural changes

Kejriwal and Perron (2008, 2010) show that the structural change tests can suffer from important lack of power against spurious regression (i.e., no cointegration). This means that these tests can reject the null of stability when the regression is really a spurious one. In this sense, tests for breaks in the long run relationship are used in conjuction with tests for the presence or absence of cointegration allowing for structural changes in the coefficients.

First, we use the residual-based test of the null of cointegration with an unknown single break against the alternative of no cointegration proposed in Arai and Kurozumi (2007). They propose a LM test based on partial sums of residuals where the break point is obtained by minimizing the sum of squared residuals and consider three models: i) Model 1, level shift; ii) Model 2, level shift with trend; iii) and Model 3, regime shift.

The LM test statistic (for one break), $\tilde{V}_1(\hat{\lambda})$, is given by:

$$\tilde{V}_1(\hat{\lambda}) = (T^{-2}\sum_{t=1}^{T} S_t(\hat{\lambda})^2)/\hat{\Omega}_{11} \tag{20}$$

where $\hat{\Omega}_{11}$ is a consistent estimate of the long run variance of u_t^* in (14), the date of break $\hat{\lambda} = (\hat{T}_1/T, ..., \hat{T}_k/T)$ and $(\hat{T}_1, ...\hat{T}_k)$ are obtained using the dynamic algorithm proposed in Bai and Perron (2003).

The Arai and Kurozumi (2007) test is restrictive in the sense that only a single structural break is considered under the null hypothesis. Hence, the test may tend to reject the null of cointegration when the true data generating process exhibits cointegration with multiple breaks. To avoid this problem, Kejriwal (2008) has recently extended their test by incorporating multiple breaks under the null hypothesis of cointegration. The Kejriwal (2008) test of the null of cointegration with multiple structural changes is denoted with k breaks as $\tilde{V}_k(\hat{\lambda})$.

4.2 Empirical results

4.2.1 Stationarity of time series

The first step in our analysis is to examine the time series properties of the series by testing for a unit root over the full sample. Trend breaks appear to be prevalent in macroeconomic time series, and unit root tests therefore need to make allowance for these if they are to avoid the serious effects that unmodelled trend breaks have on power.[21] In a seminal paper, Perron (1989) shows that failure to account for trend breaks present in the data results in unit root tests with zero power, even asymptotically. Consequently, when testing for a unit root it has become a matter of regular practice to allow for this kind of deterministic structural change. In order to avoid this pitfall, we run tests to assess whether structural breaks are present or not in g_t and b_t series.

Firstly, we have used the Perron and Yabu (2009) test for structural changes in the deterministic components of a univariate time series when it is a priori unknown whether the series is trend-stationary ($I(0)$ case) or contains an autoregressive unit root ($I(1)$ case). The Perron-Yabu test statistic, called $Exp - W_{FS}$, is based on a quasi-Feasible Generalized Least Squares (FGLS) approach using an autoregression for the noise component, with a truncation to 1 when the sum of the autoregressive coefficients is in some neighborhood of 1, along with a bias correction. For given break dates, Perron and Yabu (2009) propose an F-test for the null hypothesis of no structural change in the deterministic components using the Exp function developed in Andrews and Ploberger (1994).

[21] See, *inter alia*, Stock and Watson (1996, 1999, 2005) and Perron and Zhu (2005).

The specification that is chosen to test for the presence of one structural break in all variables is given by Model II in Perron and Yabu (2009), which considers that the structural break may affect the slope of the time trend. The results of the $Exp-W_{FS}$ test are presented in Table 1. Results show that the null hypothesis of absence of structural breaks is rejected for all variables.

Secondly, we have used the GLS-based unit root tests with multiple structural breaks both under the null and the alternative hypotheses proposed in Carrion-i Silvestre et al. (2009). The commonly used tests of unit root with a structural change in the case of an unknown break date (Zivot and Andrews (1992), Perron (1997), Vogelsang and Perron (1998), Perron and Vogelsang (1992a), Perron and Vogelsang (1992b)), assumed that if a break occurs it does so only under the alternative hypothesis of stationarity. The methodology developed by Carrion-i Silvestre et al. (2009) solves many of the topical problems of standard unit root tests with a structural change in the case of an unknown break date.[22]

Carrion-i Silvestre et al. (2009) propose a class of modified tests, called M^{GLS}, originally developed in Stock and Watson (1999) as M tests and analyzed in [23]. These tests use GLS detrending of the data as proposed in Elliott et al. (1996), and using the Modified Akaike Information Criteria ($MAIC$) to select the order of the autoregression k[24].

We use the Model II proposed by Carrion-i Silvestre et al. (2009), which considers that the structural break may affect the slope of the time trend. We use the procedure that allows for up to three breaks. As can be seen in Table 2, the null hypothesis of a unit root with multiple structural breaks cannot be rejected at the 5% level of significance in any of the tests applied for all five series.

Thirdly, Harvey et al. (2013) show that the fixed magnitude trend break asymptotic theory of Carrion-i Silvestre et al. (2009) does not predict well the finite sample power functions of M tests, and power can be very low for the magnitudes of trend breaks typically observed in practice. In response to this problem Harvey et al. (2013) propose a unit root test that allows for multiple breaks in trend both under the null and the alternative hypotheses, obtained by taking the infimum of the sequence (across

[22]See Carrion-i Silvestre et al. (2009) for more details.

[23]These tests are the MZ_{α}^{GLS}, MSB^{GLS}, $MZ_t^{GLS}$and MP_T^{GLS}. For the MZ_{α}^{GLS} and MZ_t^{GLS} tests the null hypothesis is a unit root while for the MSB^{GLS} and MP_T^{GLS} tests the null hypothesis is non stationarity. See Ng and Perron (2001) and Perron and Rodríguez (2003) for more details.

[24]Modified information criteria suggested by Ng and Perron (2001) with the modification proposed by Perron and Qu (2007).

all candidate break points in a trimmed range) of local GLS detrended augmented Dickey–Fuller-type statistics, MDF_m. They show that this procedure has power that is robust to the magnitude of any trend breaks, thereby retaining good finite sample power in the presence of plausibly-sized breaks. They also demonstrate that, unlike the OLS detrended infimum tests of Zivot and Andrews (1992), these tests display no tendency to spuriously reject in the limit when fixed magnitude trend breaks occur under the unit root null.

Table 3 presents results for MDF_1 and MDF_2 Harvey et al. (2013) tests applied at the nominal asymptotic 5% significance level. We find that the test which permits only a single break (MDF_1) fails to reject the unit root for all series. However, when allowance is made for two breaks (MDF_2), the test rejects in favor of stationarity for $g1_t$ and $g2_t$. Consequently, we can conclude that the public debt-to-GDP ratio series $[b1_t, b21_t, b22_t]$ are I(1) with trend breaks, while the real GDP growth series $[g1_t, g2_t]$ could be I(1) with trend breaks or stationary around a broken trend path.

4.2.2 Long-run relationship

Once the order of integration of the series has been analyzed, we will estimate the long-run or cointegration relationship between g_t and b_t. Initially, we will estimate and test the coefficients of the cointegration equation by means of the Dynamic Ordinary Least Squares (DOLS) method from Saikkonen (1993) and Stock and Watson (1993) and following the methodology proposed by Shin (1994). This estimation method provides a robust correction to the possible presence of endogeneity in the explanatory variables, as well as serial correlation in the error terms of the OLS estimation. Also, in order to overcome the problem of the low power of the classical cointegration tests in the presence of persistent roots in the residuals of the cointegration regression, Shin (1994) suggests a new test where the null hypothesis is that of cointegration. First, we estimate a long-run dynamic equation including the leads and lags of all the explanatory variables, the so-called DOLS regression; in our case (variables in logarithms):

$$g_t = c + \Phi t + \gamma b_t + \sum_{j=-q}^{q} \gamma_j \Delta b_{t-j} + \upsilon_t \tag{21}$$

Secondly, the Shin test is based on the calculation of two LM statistics from the DOLS residuals, C_μ and C_τ, to test for stochastic and deterministic cointegration, respectively. If there is cointegration in the demeaned specification given in (21), that occurs when $\Phi = 0$, this corresponds to deterministic cointegration, which implies that the same cointegrating vector eliminates deterministic trends as well as stochastic trends. But if the linear stationary combinations of $I(1)$ variables have nonzero linear trends (that occurs when $\Phi \neq 0$) as given in (21), this corresponds to stochastic cointegration.[25] The parameter γ is the long-run cointegrating coefficient estimated between the public debt-to-GDP ratio and the annual real GDP growth rate.

The results in Tables 4a to 4b show that the null of deterministic cointegration between g_t and b_t is not rejected at the 1% level of significance in three cases. The estimated value for γ is always negative and significantly different from zero. These results imply that a 10 percentage increase in the public debt-to-GDP ratio is associated with 0,17 to 0,38 percentage point lower real economic growth. Therefore, public debt has a significantly negative effect on GDP growth. For example, in the period 2007-2013 the public debt-to-GDP ratio (measured as total outstanding liabilities) increased 84 percentage points, which according to our estimates could be associated with 2.18 percentage point lower economic growth.

Accounting for parameter shifts is crucial in cointegration analysis, which normally involves long spans of data, which are more likely to be affected by structural breaks. Consequently, the link between the public debt and growth has probably changed due to variations in macroeconomic and market forces, such as changes in the structure of the economy and supply and demand shocks. Therefore, it is important to account for structural breaks in our cointegration relationship.

We now consider the tests for structural change that have been proposed in Kejriwal and Perron (2010). We use 15% trimming so that the maximum number of breaks allowed under the alternative hypothesis is 3. Both the intercept and the slope of equation (21) are allowed to change. Tables 5a to 5c present results of stability tests as well as the number of breaks selected by the sequential procedure (SP) and the information criteria BIC and LWZ proposed by Bai and Perron (2003).

For three cases the test results do suggest instability and the sequential test of the null hypothesis of k breaks versus the alternative hypothesis of $k + 1$ breaks (SP) selects one break and provide evidence against the stability of the long run relationships and suggest a model with one break estimated at 1939, 1971 and 1971, respectively. Since the above reported stability tests also reject the null of coefficient stability when the regression is a spurious one, we still need to confirm the presence of cointegration

[25]See Ogaki and Park (1997) and Campbell and Perron (1991) for an extensive treatment of deterministic and stochastic cointegration.

among the variables. We use the residual based test of the null of cointegration against the alternative of cointegration with unknown multiple breaks proposed in Kejriwal (2008), $\tilde{V}_k(\hat{\lambda})$.

Arai and Kurozumi (2007) show that the limit distribution of the test statistic, $\tilde{V}_k(\hat{\lambda})$, depends only on the timing of the estimated break fraction $\hat{\lambda}$ and the number of I(1) regressors m. In our case (one break model), critical values are obtained for $m = 1$ by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.* $N(0, 1)$ random variables. Since we are interested in the stability of per-capita income coefficient, γ, we consider only model 3 that permits the slope shift as well as a level shift. Table 6a to 6c shows the results of the Arai-Kurozumi-Kejriwal cointegration tests allowing one break. Again, the level of trimming used is 15%. We find that the test $\tilde{V}_1(\hat{\lambda})$ cannot reject the null of cointegration with one structural break at the 1% level of significance in three cases.

Overall, the results of the Kejriwal-Perron tests and Arai-Kurozumi-Kejriwal cointegration tests suggest for $[g1_t, b11_t]$ relationship a cointegrated model with one break estimated at 1939 and two regimes, 1851-1939 and 1940-2000. Similarly, for $[g2_t, b21_t]$ and $[g2_t, b22_t]$ relationships the test results suggest a cointegrated model with one break estimated at 1971 and two regimes, 1965-1971 and 1972-2013.

In order to compare coefficients estimated from a break model with those reported from a model without any structural break, we proceed to estimate the cointegration equation (21) for the two sub-samples, and the results for $[g1_t, b11_t]$ relationship are shown in the last two columns of Table 4a. When we split the sample after performing the structural break tests for $[g2_t, b21_t]$ and $[g2_t, b22_t]$ relationships, the reduced span of the first subsample does not advise to perform estimation using the DOLS procedure as results would be scarcely reliable.

For $[g1_t, b11_t]$ case, the results of C_μ statistics show that the null of deterministic cointegration between two variables is not rejected at the 1% level of significance in the two regimes.The coefficient estimated between the public debt-to-GDP ratio and the annual real GDP growth rate in a one-break model change over time. Thus the coefficient in the first regime (1851-1939) is positive but neither large nor significant. For the second regime (1940-2000) the coefficient is negative and significant, indicating that a 10 percentage increase in the public debt-to-GDP ratio is associated with 0,70 percentage point lower real economic growth. This value is twice as much the estimated on the full sample (0,38 percentage point).

5 Nonlinear effect of public debt on economic growth

In this section, we examine the issue of the possible nonlinear relationship between public debt and growth. At the empirical level we estimate threshold time series models of the public debt-growth nexus associated with a threshold income level. Two main research issues in our study are to ascertain: first, the possibility of the presence of a threshold in the long-run relationship, and second the asymmetric movements between the public debt-to-GDP ratio and the annual real GDP growth rate. We estimate a threshold cointegrated model, as proposed by Hansen and Seo (2002).

5.1 Methodology: A threshold cointegrated model

The concept of threshold cointegration was introduced by Balke and Fomby (1997) as a feasible way to combine nonlinearity and cointegration. As is well known, systems in which variables are cointegrated can be characterized by an error correction model (ECM), which describes how the variables respond to deviations from the equilibrium. In this way, the ECM can be characterized as the adjustment process through which the long-run equilibrium is maintained. The traditional approach, however, assumes that such a tendency to move towards the long-run equilibrium is present every period.

Balke and Fomby (1997) stressed the possibility that this movement towards the long-run equilibrium might not occur in every time period, due to the presence of some adjustment costs on the side of economic agents. In other words, there could be a discontinuous adjustment to equilibrium so that, only when the deviation from the equilibrium exceeds a critical threshold, the benefits of adjustment are higher than the costs, and economic agents move the system back to equilibrium. Threshold cointegration would characterize this discrete adjustment as follows: the cointegrating relationship does not hold inside a certain range, but holds if the system gets 'too far' from the equilibrium; i.e., cointegration would hold only if the system exceeds a certain threshold.

This type of discrete adjustment could be particularly useful to describe the behavior of fiscal authorities. More specifically, fiscal authorities would intervene by cutting budget deficits only when these are 'too large', in order to meet the IBC. The concept of threshold cointegration would capture the possibility of a nonlinear relationship between government public debt and growth, so that mean-reverting dynamic behavior in the nexus between both variables should be expected only when a certain threshold is reached.

When testing for threshold cointegration, Balke and Fomby (1997) proposed applying several univariate tests previously developed in the literature, to the known cointegrating residual (i.e., the error-correction term). Further contributions include Forbes et al. (1999), who developed a Bayesian estimation procedure; and Lo and Zivot (2001), who extended Balke and Fomby's approach to a multivariate threshold cointegration model with a known cointegrating vector, using Tsay (1998) and multivariate extensions of Hansen (1996) tests. More recently, Hansen and Seo (2002) have contributed further to this literature by examining the case of an unknown cointegration vector. In particular, these authors proposed a vector error-correction model (VECM) with one cointegrating vector and a threshold effect based on the error-correction term, and developed a Lagrange multiplier (LM) test for the presence of a threshold effect. This will be the approach followed in this paper.

Hansen and Seo (2002) considered a two-regime threshold cointegration model, or a nonlinear VECM of order $l+1$, such as:

$$\Delta x_t = \begin{cases} A_1' X_{t-1}(\beta) + u_t if w_{t-1}(\beta) \leq \gamma \\ A_2' X_{t-1}(\beta) + u_t if w_{t-1}(\beta) > \gamma \end{cases} \tag{22}$$

with

$$X_{t-1}(\beta) = \begin{pmatrix} 1 \\ w_{t-1}(\beta) \\ \Delta x_{t-1} \\ \Delta x_{t-2} \\ \vdots \\ \Delta x_{t-l} \end{pmatrix}$$

where x_t is a p-dimensional $I(1)$ time series which is cointegrated with one $p \times 1$ cointegrating vector β, $w_t(\beta) = \beta' x_t$ is the $I(0)$ error-correction term, u_t is an error term, A_1 and A_2 are coefficient matrices, and γ is the threshold parameter.

As can be seen, the threshold model (22) has two regimes, depending on whether deviations from the equilibrium (defined by the value of the error-correction term) are below or above the threshold, where A_1 and A_2 describe the dynamics in each of the regimes. In one of the regimes there would be no tendency for the variables x_t to revert to an equilibrium (i.e., the variables would not be cointegrated); on the contrary, in the other regime there would be a tendency for the variables x_t to move towards some equilibrium (i.e., the variables would be cointegrated).

Next, Hansen and Seo (2002) proposed two heteroskedastic-consistent LM test statistics for the null hypothesis of linear cointegration (i.e., there is no threshold effect), against the alternative of threshold cointegration (i.e., model (22)). The first test would be used when the true cointegrating vector is known *a priori*, and is denoted as:

$$\sup LM^0 = \sup_{\gamma_L \leq \gamma \leq \gamma_U} LM(\beta_0, \gamma) \tag{23}$$

where β_0 is the known value of β (in the case analyzed below, $\beta_0 = 1$); whereas the second test would be used when the true cointegrating vector is unknown, and is denoted as:

$$\sup LM = \sup_{\gamma_L \leq \gamma \leq \gamma_U} LM(\tilde{\beta}, \gamma) \tag{24}$$

where $\tilde{\beta}$ is the null estimate of β. In both tests, $[\gamma_L, \gamma_U]$ is the search region set so that γ_L is the π_0 percentile of $\widetilde{w}_{t-1}$, and γ_U is the $(1-\pi_0)$ percentile; Andrews (1993) suggested setting π_0 between 0.05 and 0.15. Finally, Hansen and Seo (2002) developed two bootstrap methods to calculate asymptotic critical values and p-values.

5.2 Empirical results

We have applied the tests of threshold cointegration proposed by Hansen and Seo (2002), namely $\sup LM$ (for an estimated β). For the two tests, the p-values are calculated using a parametric bootstrap method (with 5,000 simulation replications), as proposed by Hansen and Seo (2002). To select the lag length of the VAR, we have used the Akaike and Bayesian information criteria, both of them leading to $l = 1$. The results of the tests are reported in Table 7. Beginning for $[g1_t, b11_t]$ case, that corresponds to the period 1851-2000, the null hypothesis of linear cointegration is not strongly rejected. Therefore, we focus on the other sample analyzed in our study, 1965-2013, for the two different definitions of debt adopted by the Bank of Spain to look for the existence of threshold cointegration.

We first analyze the the case of general government debt, $b21_t$, and second, the public debt from the general government according to the Excessive Deficit Procedure definition, $b22_t$. Starting by the $[g2_t, b21_t]$ case, threshold cointegration would now appear at the 2% significance level for the $\sup LM$ test, with β estimated at -0.03. This value is the same to the estimated parameter at linear cointegration model (21). The estimated threshold would be now $\hat{\gamma} = 1.93$. The first or usual regime would include 75% of the observations (with $w_{t-1} \geqslant 1.93$), and the second or unusual regime the remaining 25% ($w_{t-1} < 1.93$). The estimated two-regime threshold VAR (heteroskedasticity-consistent standard errors in parentheses) is shown in Table 8a where significant error-correction effects appear only in the second or unusual regime in $\Delta g2_t$ (economic growth acceleration). On the contrary, in the first or usual regime the error-correction effects and dynamics are minimal, both in terms of significance and size of the coefficients. Figure 6 allows further visual interpretation of the results. In this figure it can be seen the strong error-correction effect for both the annual real GDP growth rate and the public debt-to-GDP ratio on the left-hand side of the estimated threshold (when $w_{t-1} < 1.93$). In contrast, we may observe the minimal error-correction effect on the right-hand side of the estimated threshold (when $w_{t-1} \geqslant 1.93$). For both variables, asymmetry is implied in the sense that there is astronger error-correction effect in the unusual regime compared with the typical one.

Finally, for $[g2_t, b22_t]$ case, threshold cointegration would nowappear at the 6% significance level for the $\sup LM$ test, with β estimated at -0.01. This value is near to the estimated parameter at linear cointegration model (21). The estimated threshold would be now $\hat{\gamma} = 0.84$. The first or usual regime would include 83% of the observations (with $w_{t-1} \geqslant 0.84$) , and the second or unusual regime the remaining 17% ($w_{t-1} < 0.84$).

On the other hand, the estimated two-regime threshold VAR (heteroskedasticity-consistent standard errors in parentheses) is shown in Table 8b, where significant error-correction effects appear both in the first or usual regime in $\Delta b22_t$ equation and in the second or unusual regime in $\Delta g2_t$ equation. On the contrary, in the other two equations displayed in Table 8b the error-correction effects and dynamics are minimal, both in terms of significance and size of the coefficients. Figure 7 allows further visual interpretation of the results. In this figure it can be seen the strong error-correction effect for both the annual real GDP growth rate and the public debt-to-GDP ratio on the left-hand side of the estimated threshold (when $w_{t-1} < 0.84$). In contrast, we may observe the minimal error-correction effect on the right-hand side of the estimated threshold (when $w_{t-1} \geqslant 0.84$). For both variables, asymmetry is again implied in the sense that there is a stronger error-correction effect depending on the regime considered. However, while the adjustment performed by $\Delta g2_t$ equation is stronger in the unusual regime compared with the typical one, in the case of $\Delta b22_t$

equation the adjustment in faster in the usual regime.

6 Concluding remarks

This study contributes to the empirical literature on the analysis of the debt-growth nexus. We investigate the link between government debt-to-GDP ratio and real per capita income growth in Spain over an exceptionally long period: 1851-2013. Unlike most of previous literature relying on panel data analysis over short time spans we focus on deep time series for a single country and use state-of-the-art time series econometrics to identify tipping points beyond which growth and public debt are negatively associated. We emphasize the quest of break points and parameter instability through algorithms that do not impose a functional form a priori and that estimates elasticities for different regimes that are robust to non-stationarity and cointegration.

Accounting for parameter shifts is crucial in cointegration analysis, which normally involves long spans of data, and therefore is more likely to be affected by structural breaks. In this paper we extend the existing empirical analysis of the linear model of the *Debt Laffer curve* in two ways. First, to avoid the econometric problems mentioned in previous empirical literature, we make use of recent developments in cointegrated regression models with multiple structural changes. Specifically, we use a new approach to test for multiple structural changes in cointegrated regression models. They propose a sequential procedure that not only enables detection of parameter instability in cointegration regression models but also allows consistent estimations of the number of breaks present. Furthermore, we test the cointegrating relationship when multiple regime shifts are identified endogenously. In particular, the nature of the long run relationship between debt-to-GDP and GDP growth is analyzed using the residual based test of the null hypothesis of cointegration with a single or multiple breaks. Second, a common criticism to most tests of the *Debt Laffer curve* is that the econometric procedures used require a large number of observations. Accordingly, due to homogeneous data set availability constraints, in this paper we use a long span of data distinguishing two different periods: 1851-2000 and 1965-2013, respectively. The results are consistent with the existence of linear cointegration between debt-to-GDP and GDP growth, with a vector (1, -0.038) for the full historical period analyzed. As for the most recent period, we have used two different definitions of gross public debt, either complying or not with the Excessive Deficit Procedure (EDP). For the first definition, the cointegrating vector is (1,

-0.026) while for the second one is (1, -0.017). These results imply that a 10 percentage increase in the public debt-to-GDP ratio is associated with lower real economic growth within the range 0,17 to 0.38 percentage points. Therefore, public debt has a significantly negative effect on GDP growth. For example, in the period 2007-2013 the public debt-to-GDP ratio (measured as total outstanding liabilities) increased 84 percentage points, which according to our estimates could be associated with a 2.18 percentage points lower economic growth.

Second, our empirical results show also that the cointegrating relationship has changed over time. In particular, Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models would suggest a model of two regimes. The estimate of long-run elasticity between debt-to-GDP and GDP growth in a one-break model shows a tendency to decrease over time (from a non-significant 0.011 to a -0.070), indicating that a 10 percentage increase in the public debt-to-GDP ratio is associated with 0,70 percentage point lower real economic growth. This value is twice as much the estimated on the full sample (-0,038 percentage points) pointing to a process of fiscal "fatigue" or "saturation". Indeed, we find for the first subsample either "decoupling" (where debt no longer affected growth in a statistically significant way but with a positive coefficient) or "saturation" (where the growth elasticity of debt is declining, less than proportional, but still positive), while in the second subsample the long-run elasticity coefficient becomes negative and significant. The Spanish Civil War may explain the placement of such structural change in 1939. After the war, the Bank of Spain started a process of monetization of public deficit and debt that gave rise to a inflationary tax process, reducing the real value of the public debt. This monetization of debt went in parallel with the repudiation of the liabilities issued by the republican government during the Civil War. After this process starts a prolonged period of low debt-to-GDP level and sustained economic growth that lasted until 1975.

Similarly, the results of the Kejriwal-Perron tests and Arai-Kurozumi-Kejriwal cointegration tests suggest for the second dataset (1965-2013) a cointegrated model with one break estimated at 1971 and two regimes, 1965-1971 and 1972-2013. The breakpoint can be placed around the time of the first oil crisis, just before the end of the Franco's regime.

Although the existence of a linear cointegration relation cannot be rejected according to our results for the whole sample studied, we have examined the possibility of non-linear cointegration for the more recent period analyzed (1965-2013). Our analysis is especially relevant for a country as Spain that has faced problems of fiscal sustainability in last years, and which has recently accomplished an important fiscal consolidation program. According to our results, the null hypothesis of linear cointegration between government debt and growth is rejected in favor of a two-regime threshold cointegration model, although the trigger points cannot be easily dated. These results would suggest the presence of a significant nonlinear behavior in Spanish fiscal policy as put forward by Bertola and Drazen (1993) and other authors, who concluded that fiscal authorities would cut deficits only if they (and the cumulated debt-to-GDP ratio) were large enough, ensuring their sustainability in the long run.

All in all, our results support the most recent empirical literature (Eberhardt and Presbitero (2015), Égert (2015b) in the sense of finding some support for a negative relationship between public debt and long-run growth but no clear evidence for a debt threshold.

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Table 1

Perron-Yabu tests for structural changes in the deterministic components

Variable	$Exp - W_{FS}$ test
$g1_t$	1.36^{**}
$g2_t$	2.82^{*}
$b1_t$	4.68^{*}
$b21_t$	31.65^{*}
$b22_t$	6.69^{*}

Notes:

[a] * and ** denote significance at the 5% and 10% levels, respectively.

[b] For the applications, we use a trimming parameter $\epsilon = 0.15$.

[c] The critical values are taken from Perron and Yabu (2009), Table 2.b.

Table 2

GLS-based unit root tests with multiple structural breaks of Carrion-i Silvestre et al. (2009)

Variable	m	MZ_{α}^{GLS}	MZ_{t}^{GLS}	MSB^{GLS}	MP_{T}^{GLS}
$g1_t$	1	-8.02	-1.84	0.229	19.53
	2	-10.93	-2.31	0.212	21.17
	3	-20.01	-3.12	0.156	14.83
$g2_t$	1	-17.37	-2.91	0.167	9.11
	2	-10.64	-2.06	0.194	22.16
	3	-7.31	-1.90	0.260	39.98
$b1_t$	1	-19.18	-2.95	0.153	9.19
	2	-16.90	-2.89	0.171	12.46
	3	-10.94	-2.32	0.212	26.85
$b21_t$	1	-6.34	-1.77	0.280	26.00
	2	-16.44	-2.86	0.174	13.99
	3	-5.12	-1.60	0.312	58.69
$b22_t$	1	-9.57	-2.07	0.216	15.22
	2	-20.38	-3.18	0.156	10.30
	3	-8.75	-2.00	0.229	31.80

Notes:

[a] * denotes rejection the null at the 5% level.

[b] m = number of breaks.

[c] Model II: structural break may affect the slope of the time trend.

[d] The critical values were obtained by simulations using 1,000 steps to approximate the Wiener process and 10,000 replications.

Table 3

Unit root tests with multiple structural breaks using minimum DF-statistics of Harvey et al. (2013)

Variable	MDF_1	MDF_2
$g1_t$	-2.54	-10.79*
$g2_t$	-3.53	-4.62*
$b1_t$	-2.70	-3.40
$b21_t$	-2.95	-3.23
$b22_t$	-2.91	-3.15

Notes:

[a] * denotes rejection the null at the 5% level.

[b] Structural break (m = 1, 2) may affect the slope of the time trend.

[c] Using the Modified Akaike Information Criteria ($MAIC$) to select the order of the autoregression k.

[d] The critical values for MDF_1 and MDF_2 are taken from Harvey et al. (2013), Table 1

Table 4a

Estimation of long-run relationships: Stock-Watson-Shin cointegration tests

1851-2000, $[g1_t, b1_t]$

Parameter estimates	Full sample 1851-2000	First regime 1851-1939	Second regime 1940-2000
c	5.51	0.35	6.74
	(6.0)	(0.1)	(5.3)
γ	-0.038	0.011	-0.070
	(-3.2)	(0.5)	(-2.1)
R^2	0.38	0.19	0.72
C_μ	0.072	0.060	0.050

Notes:

[a] t-statistics are in brackets. Standard Errors are adjusted for long-run variance. The long-run variance of the cointegrating regression residual is estimated using the Barlett window which is approximately equal to $INT\left(T^{1/2}\right)$ as proposed in Newey and West (1987).

[b] We choose $q = INT\left(T^{1/3}\right)$ as proposed in Stock and Watson (1993).

[c] C_μ and C_τ are LM statistics for cointegration using the DOLS residuals from deterministic and stochastic cointegration, respectively, as proposed in Shin (1994).

[d] The critical values for C_μ are taken from Shin (1994), table 1.

Table 4b

Estimation of long-run relationships: Stock-Watson-Shin cointegration tests

1965-2013

Parameter estimates	Full sample $[g2_t, b21_t]$	Full sample $[g2_t, b22_t]$
c	4.42	4.35
	(6.8)	(7.3)
γ	-0.026	-0.017
	(-1.7)	(-1.5)
R^2	0.83	0.83
C_μ	0.094	0.094

Notes:

[a] t-statistics are in brackets. Standard Errors are adjusted for long-run variance. The long-run variance of the cointegrating regression residual is estimated using the Barlett window which is approximately equal to $INT\left(T^{1/2}\right)$ as proposed in Newey and West (1987).

[b] We choose $q = INT\left(T^{1/3}\right)$ as proposed in Stock and Watson (1993).

[c] C_μ and C_τ are LM statistics for cointegration using the DOLS residuals from deterministic and stochastic cointegration, respectively, as proposed in Shin (1994).

[d] The critical values for C_μ are taken from Shin (1994), table 1.

Table 5a

Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models: equation (14) and (21)[a,b]

1851-2000, $[g1_t, b1_t]$

	Specifications[a]		
$d_t = \{g1_t\}$	$z_t = \{1, b1_t\}$	$x_t = \{\emptyset\}$	$M = 3$
	$q = 2$	$p = 0$	$h = 29$
	Tests[b,c]		
$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$UD\max$
9.11^{*}	7.72^{**}	5.28^{**}	9.11^{*}
	Number of Breaks Selected		
SP	1		
LWZ	0		
BIC	0		
	Breaks		
$\hat{T}_1$	1939		

Notes:

[a] d_t, z_t, q, p, h, and M denote the dependent variable, the regressors, the number of I(1) variables (and the intercept) allowed to change across regimes, the number of I(0) variables, the minimum number of observations in each segment, and the maximum number of breaks, respectively.

[b] *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. The critical values are taken from Kejriwal and Perron (2010), Table 1.10 (critical values are available on Kejriwal-Perron website), trending case with $q_b = 1$.

Table 5b

Kejriwal-Perron tests for testing multiple structural breaks

in cointegrated regression models: equation (14) and (21)[a,b]

1965-2013, $[g2_t, b21_t]$

	Specifications[a]		
$d_t = \{g2_t\}$	$z_t = \{1, b21_t\}$	$x_t = \{\emptyset\}$	$M = 3$
	$q = 2$	$p = 0$	$h = 8$
	Tests[b,c]		
$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$UD\max$
8.61*	8.38**	8.53***	8.61*
	Number of Breaks Selected		
SP	1		
LWZ	0		
BIC	0		
	Breaks		
$\hat{T}_1$	1971		

Notes:

[a] d_t, z_t, q, p, h, and M denote the dependent variable, the regressors, the number of I(1) variables (and the intercept) allowed to change across regimes, the number of I(0) variables, the minimum number of observations in each segment, and the maximum number of breaks, respectively.

[b] *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. The critical values are taken from Kejriwal and Perron (2010), Table 1.10 (critical values are available on Kejriwal-Perron website), trending case with $q_b = 1$.

Table 5c

Kejriwal-Perron tests for testing multiple structural breaks

in cointegrated regression models: equation (14) and (21)[a,b]

1965-2013, $[g2_t, b22_t]$

	Specifications[a]		
$d_t = \{g2_t\}$	$z_t = \{1, b22_t\}$	$x_t = \{\emptyset\}$	$M = 3$
	$q = 2$	$p = 0$	$h = 8$
	Tests[b,c]		
$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$UD\max$
8.93^{*}	6.23^{*}	5.93^{**}	8.93^{*}
	Number of Breaks Selected		
SP	1		
LWZ	0		
BIC	0		
	Breaks		
$\hat{T}_1$	1971		

Notes:

[a] d_t, z_t, q, p, h, and M denote the dependent variable, the regressors, the number of I(1) variables (and the intercept) allowed to change across regimes, the number of I(0) variables, the minimum number of observations in each segment, and the maximum number of breaks, respectively.

[b] *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. The critical values are taken from Kejriwal and Perron (2010), Table 1.10 (critical values are available on Kejriwal-Perron website), trending case with $q_b = 1$.

Table 6a

Arai-Kurozumi-Kejriwal cointegration tests with one structural break: equation (14) and (21)[a,b]

1851-2000, $[g1_t, b1_t]$

Test $\tilde{V}_1(\hat{\lambda})^a$	$\hat{\lambda}_1$	$\hat{T}_1$
0.071***	0.59	1940

Notes:

[a] *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

[b] Critical values are obtained by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.* $N(0,1)$ random variables.

Critical values:	10%	5%	1%
$\tilde{V}_k(\hat{\lambda})$	0.106	0.131	0.221

Table 6b

Arai-Kurozumi-Kejriwal cointegration tests with one structural break: equation (14) and (21)[a,b]

1965-2013, $[g2_t, b21_t]$

Test $\tilde{V}_1(\hat{\lambda})^a$	$\hat{\lambda}_1$	$\hat{T}_1$
0.127***	0.16	1971

Notes:

[a] *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

[b] Critical values are obtained by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.* $N(0,1)$ random variables.

Critical values:	10%	5%	1%
$\tilde{V}_k(\hat{\lambda})$	0.166	0.228	0.375

Table 6c

Arai-Kurozumi-Kejriwal cointegration tests with one structural break: equation (14) and (21)[a,b]

1965-2013, $[g2_t, b22_t]$

Test $\tilde{V}_1(\hat{\lambda})^a$	$\hat{\lambda}_1$	$\hat{T}_1$
0.099***	0.16	1971

Notes:

[a] *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

[b] Critical values are obtained by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.* $N(0,1)$ random variables.

Critical values:	10%	5%	1%
$\tilde{V}_k(\hat{\lambda})$	0.166	0.228	0.375

Table 7

Hansen-Seo tests of threshold cointegration

Estimates	$[g1_t, b11_t]$	$[g2_t, b21_t]$	$[g2_t, b22_t]$
Cointegrating vector β	—	-0.03	-0.01
Threshold parameter γ	—	1.93	0.84
$\sup LM$ test value	5.51	15.76	14.92
Residual Bootstrap C.V.	16.52	14.52	15.24
(p-value)	0.98	0.02*	0.06*

Notes:

The model estimated is the bivariate specification with the real GDP growth and the public debt-to-GDP ratios. For p-values, the number of bootstrap replications is set to 5000.

Table 8a

Estimation of threshold VECM[a,b]

$[g2_t, b21_t]$

Dependent variable	$\Delta g2_t$		$\Delta b21_t$	
	Regime 1	Regime 2	Regime 1	Regime 2
w_{t-1}	-0.38	-1.15***	0.39	1.57
	(0.25)	(0.37)	(0.27)	(1.43)
intercept	1.49	1.51***	2.30	1.34
	(1.25)	(0.58)	(1.61)	(2.09)
$\Delta g2_{t-1}$	0.31	-0.09	-0.30	0.18
	(0.23)	(0.19)	(0.24)	(0.43)
$\Delta b21_{t-1}$	0.05	0.004	0.70***	0.55**
	(0.10)	(0.06)	(0.20)	(0.23)

Notes:

[a] Eicker-White standard errors in parenthesis.**,*** coefficient is significant at the 5% and 1% significance level, respectively.

[b] Regime 1: $w_{t-1} \geq 1.93$. Regime 2: $w_{t-1} < 1.93$. The model estimated is defined in equation (22).

Table 8b

Estimation of threshold VECM[a,b]

$[g2_t, b22_t]$

Dependent variable	$\Delta g2_t$		$\Delta b22_t$	
	Regime 1	Regime 2	Regime 1	Regime 2
w_{t-1}	-0.24	-0.91***	-0.68***	2.93
	(0.22)	(0.28)	(0.28)	(1.68)
intercept	0.51	1.82***	3.70***	2.78
	(0.87)	(0.38)	(1.41)	(1.49)
$\Delta g2_{t-1}$	0.28	-0.25***	-0.25	0.91***
	(0.22)	(0.07)	(0.28)	(0.24)
$\Delta b22_{t-1}$	-0.03	-0.09***	0.60***	0.86***
	(0.08)	(0.03)	(0.21)	(0.21)

Notes:

[a] Eicker-White standard errors in parenthesis.**,*** coefficient is significant at the 5% and 1% significance level, respectively.

[b] Regime 1: $w_{t-1} \geq 0.84$. Regime 2: $w_{t-1} < 0.84$. The model estimated is defined in equation (22).

Figure 1a. Scatter plot of government debt as % of GDP and real GDP growth in Spanish economy, 1851-2000 [g1, b1]

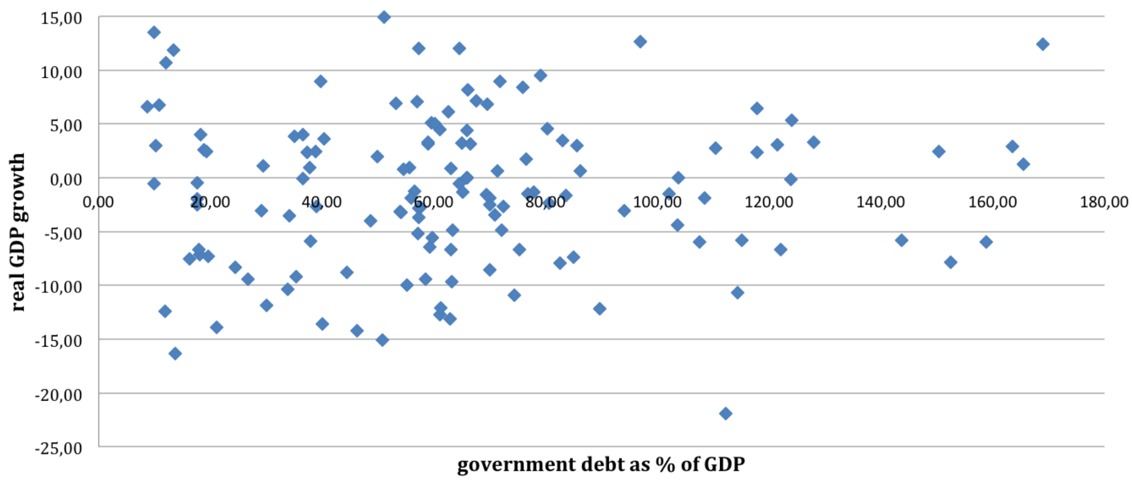
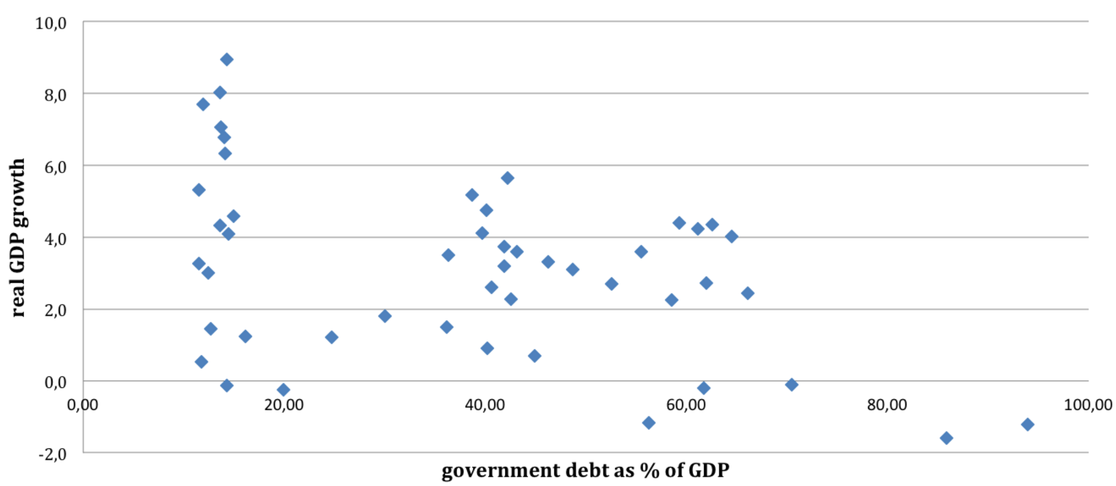


Figure 1b. Scatter plot of government debt as % of GDP and real GDP growth in Spanish economy, 1965-2013 [g2, b21]



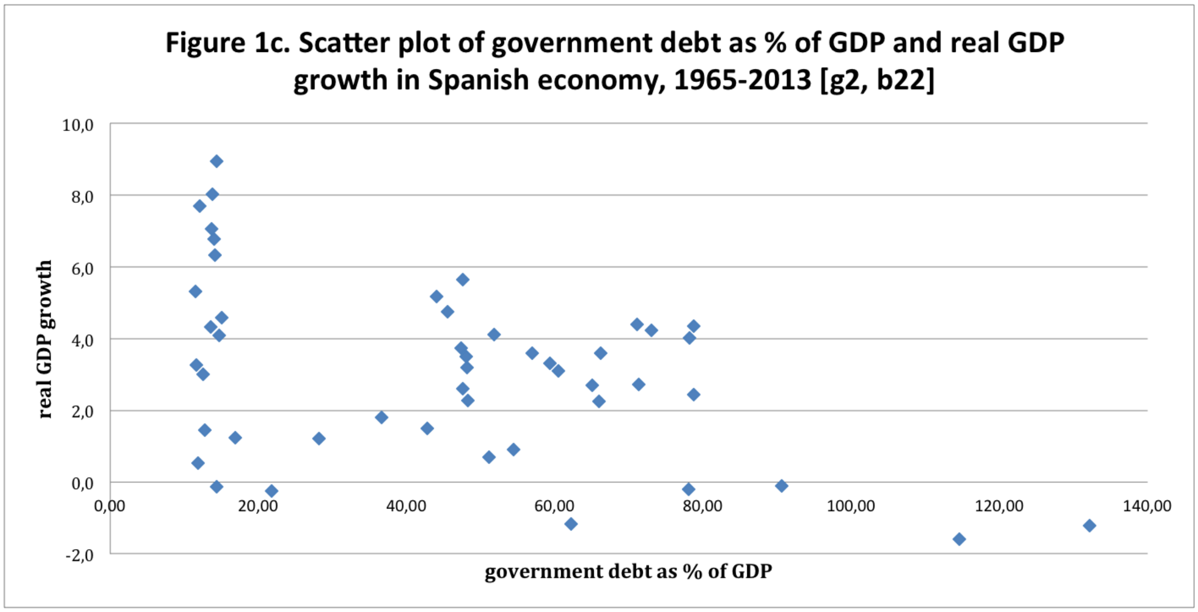


Figure 1c. Scatter plot of government debt as % of GDP and real GDP growth in Spanish economy, 1965-2013 [g2, b22]
real GDP growth
government debt as % of GDP
10,0
8,0
6,0
4,0
2,0
0,0
-2,0
0,00
20,00
40,00
60,00
80,00
100,00
120,00
140,00

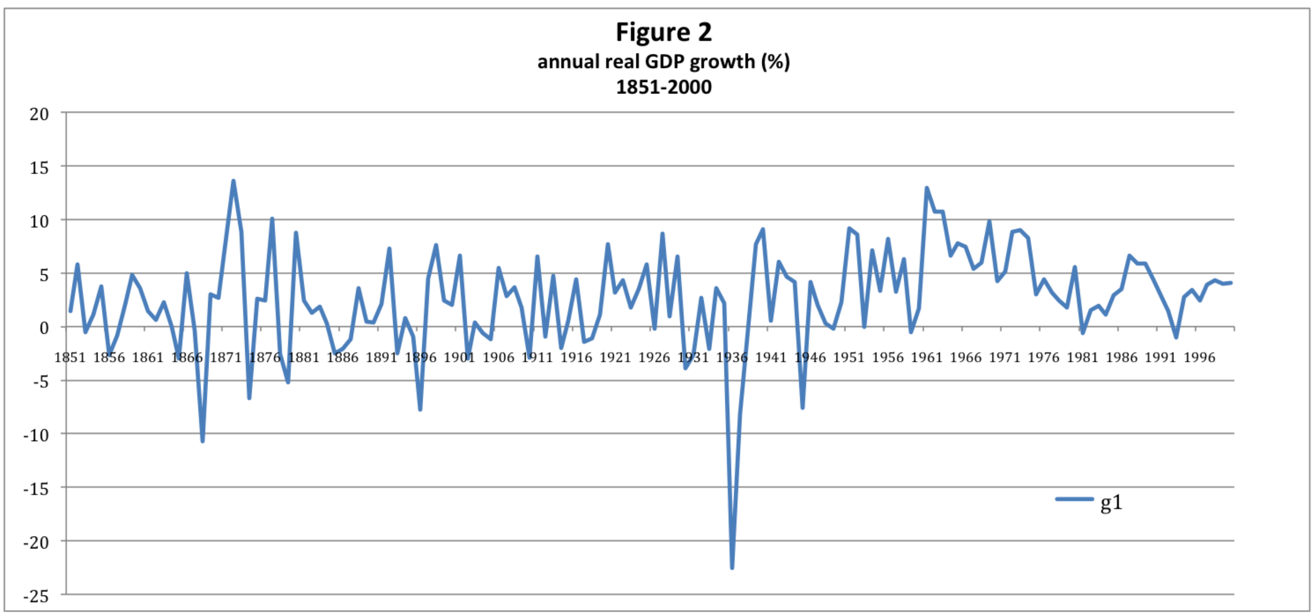


Figure 2
annual real GDP growth (%)
1851-2000
g1
20
15
10
5
0
-5
-10
-15
-20
-25
1851 1856 1861 1866 1871 1876 1881 1886 1891 1896 1901 1906 1911 1916 1921 1926 1931 1936 1941 1946 1951 1956 1961 1966 1971 1976 1981 1986 1991 1996

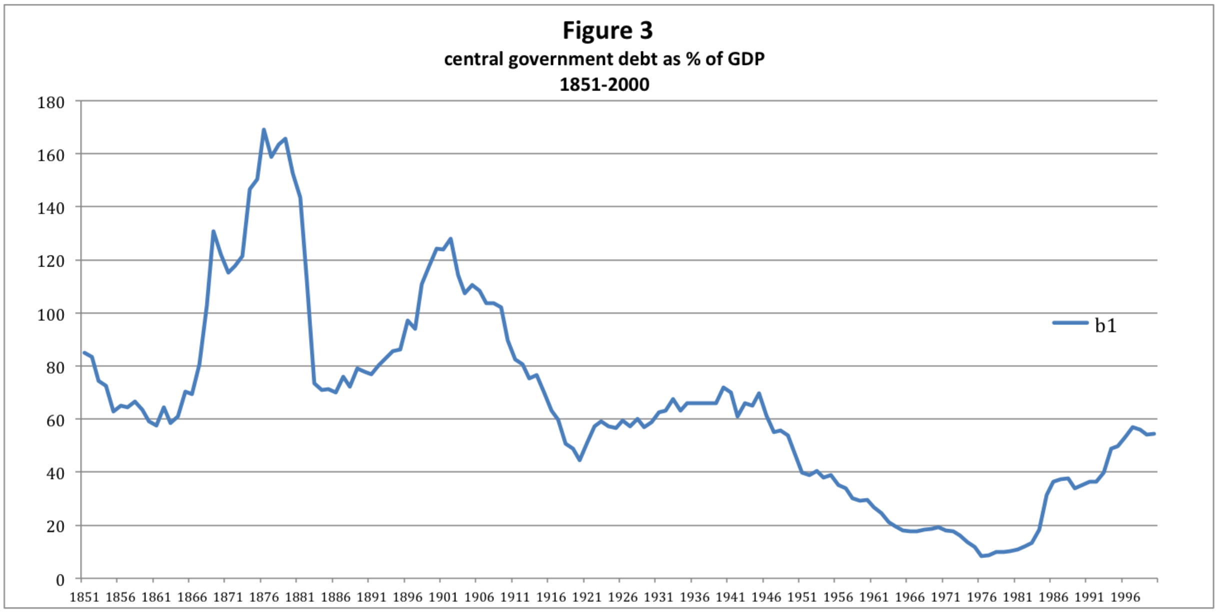


Figure 3
central government debt as % of GDP
1851-2000
180
160
140
120
100
80
60
40
20
0
1851 1856 1861 1866 1871 1876 1881 1886 1891 1896 1901 1906 1911 1916 1921 1926 1931 1936 1941 1946 1951 1956 1961 1966 1971 1976 1981 1986 1991 1996
b1

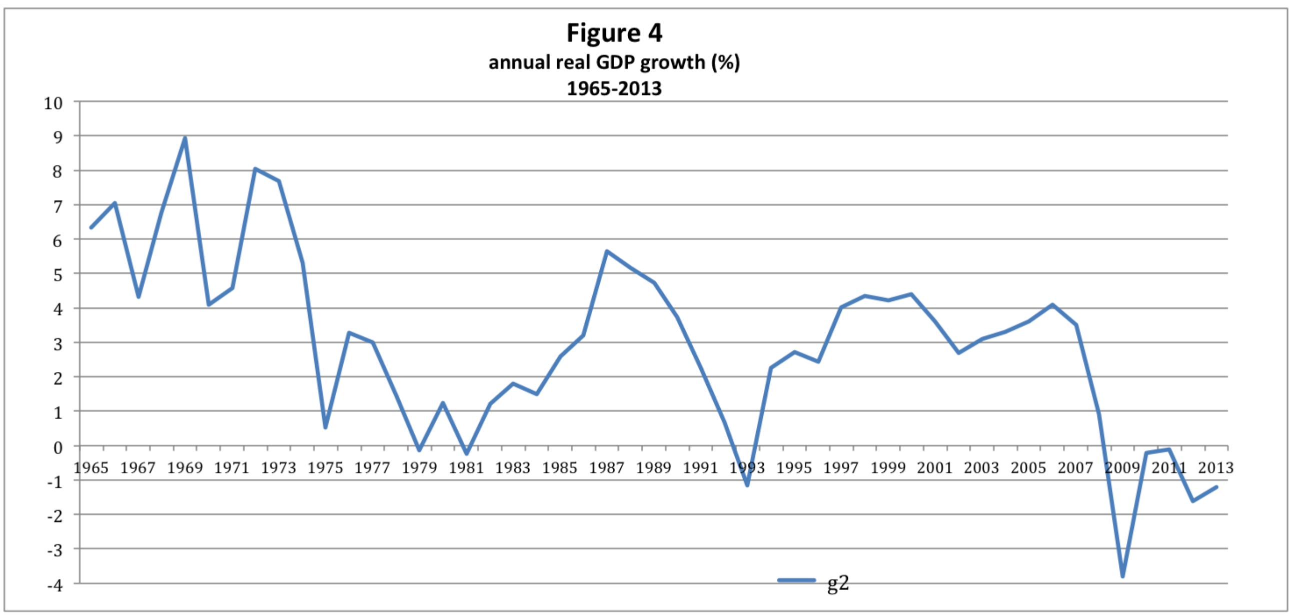


Figure 4
annual real GDP growth (%)
1965-2013
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013
g2

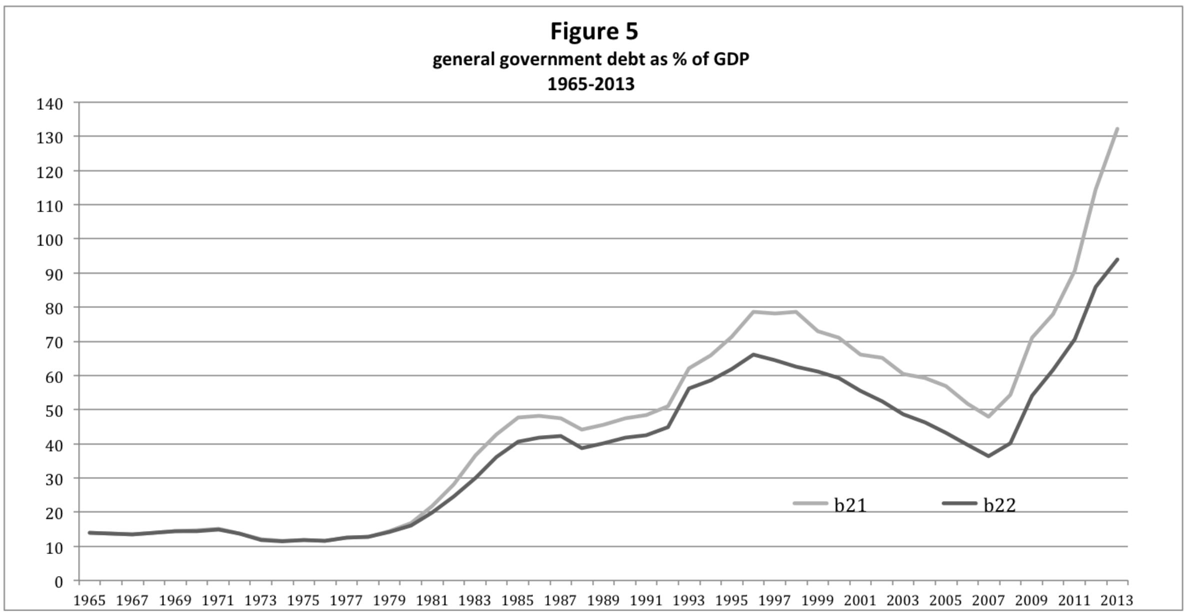


Figure 5
general government debt as % of GDP
1965-2013
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013
b21
b22

Figure 6
Response of growth and public debt to error correction
debt according to the EDP, 1965−2013

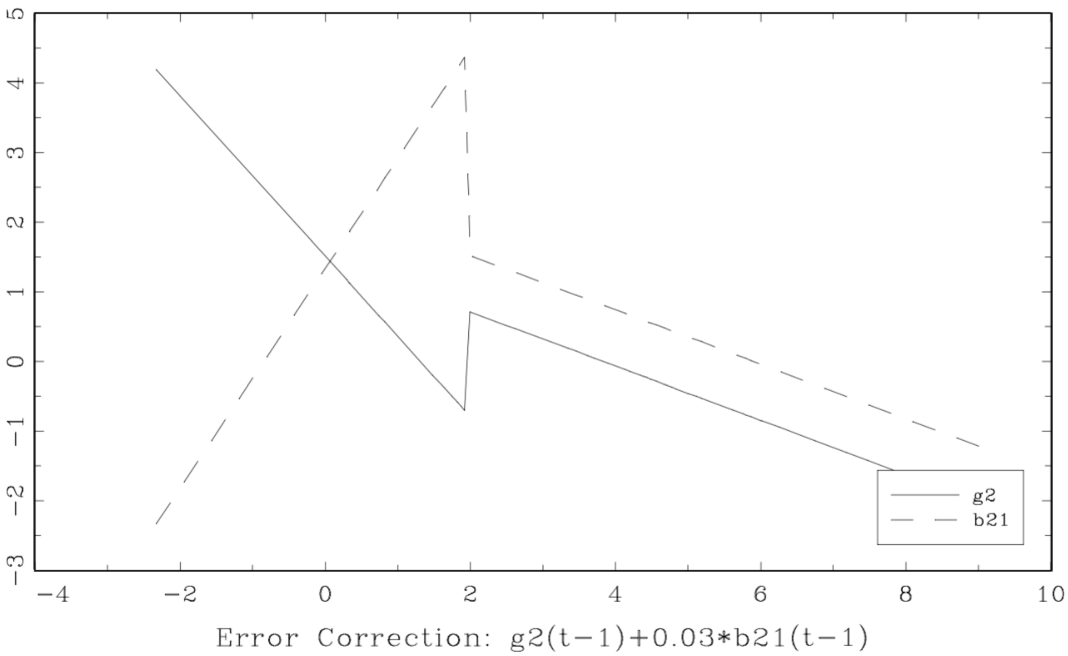


Figure 7
Response of growth and public debt to error correction
total outstanding liabilities, 1965−2013