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Abstract

This paper examines whether the comparison of VaR models depends on the loss function used for such purpose. We show a detailed comparison for several VaR models for two groups of loss functions (designed for regulators and for risk managers). Additionally, we propose a firm's loss function that exactly measures the opportunity cost of the firm when the losses are covered. We find that the VaR model that minimises the total losses is robust within groups of loss function but differs across firm's and supervisor's loss functions.

Key words: Value at Risk, GARCH Model, Risk Management, Loss Function, Backtesting.

JEL classification: G32, G15, C22

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1.- Introduction

The global financial crisis suffered in the last years has taught us the importance of measuring risk accurately. Because the Basel Committee on Bank Supervision (BCBS) at the Bank for International Settlements requires that financial institutions meet capital requirements for the base Value at Risk (VaR), this methodology has become a basic market risk management tool. Consequently, the last decade has witnessed the growth of literature proposing new models to estimate the VaR. To know which is the best of these models has been and still is a primary aim of the empirical literature.

Several studies dedicated to comparing VaR models have used a standard backtesting procedure (see Bhattacharyya and Ritolia (2008), Yu et al. (2010), Nozari et al. (2010), Bao et al. (2006), and Mittnik and Paolella (2000), among others). The standard backtesting is based on calculating the number of times that losses exceed the VaR and comparing this value with the expected number using statistical tests. Jorion (2001) defines backtesting as an ex-post comparison of a risk measure generated by a risk model against actual changes in the portfolio value over a given period. The Basel Committee on Banking Supervision (1996a) and the amendments of the Basel Committee on Banking Supervision (1996b) developed several statistical tests to evaluate the accuracy of the VaR estimates. More recently, in Basel III (2010), the committee pointed outnotes the necessity of verifying the model’s accuracy through frequent backtesting, although no particular backtesting technique is recommended.

A different perspective is given by Lopez (1998, 1999) who indicates that it is also important to know the size of the losses not covered. To calculate the uncovered losses, he proposes using a loss function. The loss function is based not on a hypothesis-testing framework such as the statistical test but on examining the distance between the observed returns and the forecasted VaR(α) when the losses are uncovered. Some papers dedicated to comparing VaR models use both backtesting procedures: statistical tests and loss function (see Abad and Benito (2013), Orhan and Köksal (2012), Marimoutou et al. (2009) and Angelidis and Degiannakis (2007), among others).

There is the trade-off between the regulators and the financial enterprises regarding the aims in the market risk management tool. Supervisors are concerned about how many times losses exceed the VaR and the size of the non-covered losses. However, the risk managers have a conflict between the goal of safety and the goal of profit maximisation. An excessively high VaR forces them to hold too much capital, imposing the opportunity cost of capital upon the firm. Considering this factor, Sarma et al. (2003) propose a firm's loss function.

This paper focuses on loss functions. We examine whether the results of comparing the VaR models depend on the loss function used. In a comparison of a large set of VaR models, we compare these models using several loss functions proposed in the literature from the point of view of the regulator and from the point of view of the firm. Additionally, we propose a new firm's loss function, in line with Sarma et al. (2003). This function has the advantage of precisely computing the opportunity cost of the firm when the losses are covered.

The relevance of this study is twofold. First, it fills a gap in the literature regarding the comparison of VaR models, as this is the first paper to analyse whether the results of the VaR model comparison are robust to the loss function used. Second, we propose a new loss function that better captures the aim of the firm. Our results can help market participants, supervisors and risk managers to select the best VaR models, taking into account the different utility functions facing each.

The rest of the paper is organised as follows: in the next section, we describe the backtesting procedure, focusing mainly on the role of the loss function. In section 3, we present the data we have used in the paper and the results of the empirical application. The last section includes the main conclusions.

2.- Loss Functions

Since the late 1990s, a wide variety of tests have been proposed for evaluating the performance of the VaR models. The backtesting procedures used in the literature can be classified into two groups: backtesting based on any statistical test and backtesting based on the loss function.[1]

The unconditional coverage test (Kupiec (1995)), the conditional coverage test and the independence test of Christoffersen (1998), the Dynamic Quantile test proposed by Engle and Manganelli (2004) and the Backtesting Criterion Statistic are the most usual backtesting procedures based on any statistical test. To implement all these tests, the exception indicator (I_t) must be defined. We have an exception when $r_t<VaR(\alpha)$, and then I_t is equal one (zero otherwise).

[1] There is no general agreement in the literature addressing what backtesting really comprises

The unconditional coverage test assumes that an accurate VaR(α) measure provides an *unconditional coverage*; i.e., the percentage of exceptions observed ($\hat{\alpha}$) should be consistent with the theoretical proportion of failures (α). Thus, the null hypothesis of this test is $\hat{\alpha}=\alpha$. A similar test for the significance of the departure of $\hat{\alpha}$ from α is the *backtesting criterion* statistic.

The *conditional coverage test* proposed by Christoffersen (1998) jointly examines whether the model generates a correct proportion of failures and whether the exceptions are statistically independent from one another. The independence property of exception is an essential property because the measures of risk must reply automatically to any new information; a model that does not consider this factor would provoke exceptions clustering.

The *Dynamic Quantile* test proposed by Engle and Manganelli (2004) suggests another approach based on a linear regression model, examining whether the exception indicator is uncorrelated with any variable that belongs to the information set Ω_{t-1} available when the VaR was calculated. This is a Wald test of the hypothesis that all slopes in the regression model $I_t = \beta_0 + \sum_{i=1}^{p} \beta_i I_{t-1} + \sum_{j=1}^{q} \mu_j X_j + \varepsilon_t$ are zero, where X_j are explanatory variables contained in Ω_{t-1}. VaR(α) is usually an explanatory variable to determine whether the probability of an exception depends on the level of the VaR.

The backtesting procedures based on certain statistical tests present a drawback; they only show whether the VaR estimates are accurate, so this toolbox does not allow us to rank the models.

Backtesting based on the loss function pays attention to the magnitude of the failure when an exception occurs. Lopez (1998, 1999), who is a pioneer in this area, proposes to examine the distance between the observed returns and the forecasted VaR(α). This difference represents the loss that has not been covered. The loss function enables the financial manager to rank the models. The model that minimises the total loss will be preferred to the other models.

Lopez (1999) proposed a general form of the loss function:

$$L_t = \begin{cases} f\left(r_t, \mathrm{VaR}\right) & \text{if} \quad r_t < \mathrm{VaR} \\ g\left(r_t, \mathrm{VaR}\right) & \text{if} \quad r_t \geq \mathrm{VaR} \end{cases} \qquad (1)$$

where $f\left(r_t, \mathrm{VaR}\right)$ and $g\left(r_t, \mathrm{VaR}\right)$ are functions such that $f\left(r_t, \mathrm{VaR}\right) \geq g\left(r_t, \mathrm{VaR}\right)$, thereby penalising to a greater extent those cases where the real returns fall bellow the VaR estimations. He considers three loss functions: (i) the Binomial loss function that assigns the value 1 when the VaR estimate is exceeded by its loss and 0 otherwise, (ii) the Zone loss function based on the adjustments to the multiplication factor used in market risk amendment (see Sajjad et al. (2008), Hass (2001) and Lopez (1998) among others), and (iii) the Magnitude loss function, which assigns a quadratic numerical score when a VaR estimate is exceeded by its loss and 0 otherwise. Subsequently, not only the VaR exception but also the magnitude of the losses is incorporated.

Depending on the form adopted by $f\left(r_t, \mathrm{VaR}\right)$ and $g\left(r_t, \mathrm{VaR}\right)$, we can speak of two types of functions: regulator's loss functions and firm's loss functions.

The regulator's loss functions pay attention to the magnitude of the losses only when they occur. Thus, the Lopez's Magnitude loss function has the following quadratic specification:

$$RQL = \begin{cases} 1+\left(VaR_t - r_t\right)^2 & if \quad r_t < VaR_t \\ 0 & if \quad r_t \geq VaR_t \end{cases} \qquad (2)$$

In this loss function, the quadratic term ensures that large failures are penalised more than small failures. This function was built mainly for regulatory purposes for evaluating the bank internal models. Applications of this loss function are numerous (see Ozun et al. (2010), Campell (2005), Marimoutou et al. (2009), Zatul (2011), Osiewalski and Pajor (2012) and Orhan and Köksal (2012), among others).

Since the reporting of Lopez (1998, 1999), many authors have proposed other alternative functions with the same goal, to measure the distance between returns and VaR estimates when an exception occurs. In column 1 of Table 1, we report some of these functions.

Sarma et al. (2003) defined the regulator's loss function as follows:

$$RQ = \begin{cases} \left(VaR_t - r_t\right)^2 & if \quad r_t < VaR_t \\ 0 & otherwise \end{cases} \qquad (3)$$

Applications of this function can be found in Angelidis et al. (2007) and Abad and Benito (2013), among others. Caporin (2008) notes that there is an open issue with the function aforementioned. At a parity exception, we may reject a correctly specified and identified model only because it provides higher losses. For this author, what is important is not the losses uncovered but their relative size. To solve this point, he divides $f\left(r_t, \mathrm{VaR}\right)$ by VaR. The mathematical expression of these functions can be found in the first column of Table 1.

The aforementioned loss function only takes into account the magnitude of the failure but does not consider the cases in which the returns exceed the VaR estimates. This is an important point because a too high VaR overestimation would lead firms to hold much more capital than necessary, thus imposing an opportunity cost of capital above. Firms must resolve the conflict related to safety, in the same way that a regulator does, but they also have the objective of maximising their owner profits. For this purpose, Sarma et al. (2003) define a firm's loss function (FS), where the non-exception days are penalised according to the opportunity cost of the reserved capital held by the firm for risk management purposes:

$$FS = \begin{cases} \left(\mathrm{VaR}_t - r_t\right)^2 & \text{if} \quad r_t < \mathrm{VaR}_t \\ -\alpha.\mathrm{VaR}_t & \text{if} \quad r_t \geq \mathrm{VaR}_t \end{cases} \qquad (4)$$

where α is the cost of capital for the firm. Thus, a model that may be adequate because it provides few exceptions becomes inadequate if the opportunity capital cost is high. Caporin (2008) suggests applying the same loss function not only to the exceptions but also to the entire sample, (an exception occurs and does not), i.e., he suggests applying a function such as $f\left(r_t, \mathrm{VaR}\right) = g\left(r_t, \mathrm{VaR}\right)$.

In line with these papers, we propose a new loss function to capture the aim of the firm. The expression of the function we propose is as follows:

$$FABL = \begin{cases} \left(\mathrm{VaR} - r_t\right)^2 & \text{if} \quad r_t < \mathrm{VaR} \\ \alpha\left(r_t - \mathrm{VaR}\right) & \text{if} \quad r_t \geq \mathrm{VaR} \end{cases} \qquad (5)$$

As can be determined in this function, the exceptions are penalised as usual in the literature, following the instructions of the regulator. When there are no exceptions, the loss function penalises the difference between the VaR and returns weighted by a factor α that represents an interest rate. This product is the opportunity cost of the capital, i.e., the excess capital held by the firm.

Sarma et al. (2003) penalises the cases in which there are no exceptions for multiplying the VaR estimate by a factor α. From our point of view, this product does not precisely capture the opportunity cost of the capital. Unlike Sarma et al. (2003), we are committed to measuring the real cost of opportunity, rather than the cost of security imposed by Basel. On the other hand, Sarma et al. (2003) do not identify factor α. We propose the price of the capital opportunity cost to be an interest rate. Other firm´s loss functions are presented in the second column of Table 1.

Table 1. Loss functions

Regulator's loss function (RLF)		Firm's loss functions (FLF)	
Lopez's quadratic (RQL)	$\begin{cases} 1+(VaR_t - r_t)^2 & \text{if} \quad r_t < VaR_t \\ 0 & \text{if} \quad r_t \geq VaR_t \end{cases}$	**Sarma (FS)**	$\begin{cases} (VaR_t - r_t)^2 & \text{if} \quad r_t < VaR_t \\ -\alpha.VaR_t & \text{if} \quad r_t \geq VaR_t \end{cases}$
Lineal (RL)	$\begin{cases} (VaR_t - r_t) & \text{if} \quad r_t < VaR_t \\ 0 & \text{if} \quad r_t \geq VaR_t \end{cases}$	**Caporin_1 (FC_1)**	$\left\lvert 1-\left\lvert \frac{r_t}{VaR} \right\rvert \right\rvert \quad \forall r_t$
Quadratic (RQ)	$\begin{cases} (VaR_t - r_t)^2 & \text{if} \quad r_t < VaR_t \\ 0 & \text{if} \quad r_t \geq VaR_t \end{cases}$	**Caporin_2 (FC_2)**	$\frac{(\lvert r_t \rvert - \lvert VaR \rvert)^2}{\lvert VaR \rvert} \quad \forall r_t$
Caporin_1 (RC_1)	$\begin{cases} \left\lvert 1-\left\lvert \frac{r_t}{VaR} \right\rvert \right\rvert & \text{if} \quad r_t < VaR \\ 0 & \text{if} \quad r_t \geq VaR \end{cases}$	**Caporin_3 (FC_3)**	$\lvert VaR-r_t \rvert \quad \forall r_t$
Caporin_2 (RC_2)	$\begin{cases} \frac{(\lvert r_t \rvert - \lvert VaR \rvert)^2}{\lvert VaR \rvert} & \text{if} \quad r_t < VaR \\ 0 & \text{if} \quad r_t \geq VaR \end{cases}$	**Abad_Benito_López (FABL)**	$\begin{cases} (VaR_t - r_t)^2 & \text{if} \quad r_t < VaR_t \\ \alpha(r_t - VaR) & \text{if} \quad r_t \geq VaR_t \end{cases}$
Caporin_3 (RC_3)	$\begin{cases} \lvert VaR-r_t \rvert & \text{if} \quad r_t < VaR \\ 0 & \text{if} \quad r_t \geq VaR \end{cases}$		

This table presents the different loss functions used in this paper. In the first column, we show the regulator's loss functions (Lopez' magnitude loss function (RQL), lineal regulatory function (RL), Sarma et al. (2003) quadratic loss function (RQ) and the three loss function suggested by Caporin (2008) ((RC_1), (RC_2), and (RC_3)). The second column lists the firm's loss functions (Sarma et al. (2003) (FS), the three loss function suggested by Caporin (2008) (FC_1), (FC_2), and (FC_3) from the viewpoint of the firms, and our new loss function (FABL)).

3. Empirical results.

The purpose of this paper is to check whether the comparison of different VaR models is independent of the loss function used to perform the selection of the best model. With this aim, we replicate Marimoutou et al. (2009). In this paper, the authors compare several VaR models using a two-stage selection approach. In the first stage, Kupiec and Christoffersen's tests are applied. In the second stage, and only for the survived models, they calculate the Lopez's quadratic loss function given by the expression (2). The VaR models included in the comparison are as follows: Historical Simulation (HS), Filtered Historical Simulation (FHS), Conditional and Unconditional Extreme Value Theory (GPD) and Parametric approach below a normal and *t*-Student distribution. Next, consider the conditional and unconditional volatility measure. The study has been performed using two spot prices for crude oil: the Brent and the West Texas Intermediate (WTI).

3.1.- Data

In this study, we have only taken one of the spot prices for oil used in Marimoutou et al. (2009). The data consists of daily prices of the Brent crude oil extracted from the Datastream database. The data set covers the same period (from May 20 1987 to January 24 2006). The computation of the daily returns (r_t) is based on the given expression, $r_t = \ln(P_t/P_{t-1})$ where P_t is the price of the oil for period *t*.

Figures 1 and 2 show the daily prices of crude oil Brent measured in US $/Barrel and daily returns, respectively. Table 2 provides basic descriptive statistics of the data.

Figure 1: Daily prices of crude oil Brent (US $/Barrel)

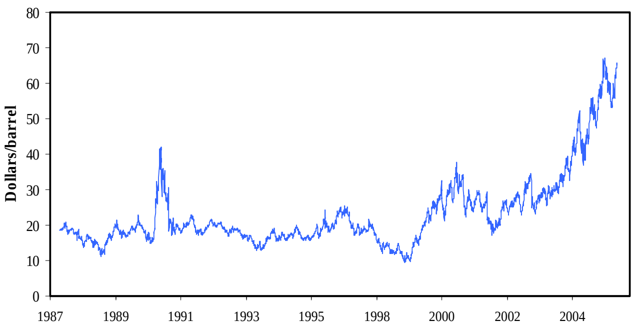

Figure 2: Daily returns of crude oil

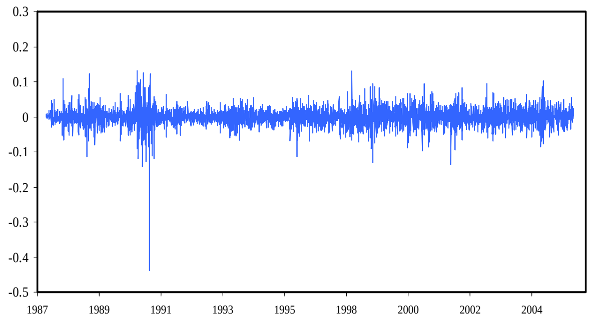

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Table 2. Descriptive statistics of the daily returns of the crude oil

	Mean (%)	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
BRENT	0.026* (0.0004)	0.000	0.131	-0.439	0.024	-1.427* (0.035)	28.204* (0.070)	162849 (0.001)

Note: This table presents the descriptive statistics of the daily returns of the Brent crude oil. The sample covers from May 20th, 1987, to January 24th, 2006. The return is calculated as $r_t = \ln(P_t)/(P_{t-1})$, where P_t is the price level for period *t*. The standard errors of the skewness, the mean and excess kurtosis are calculated as $\sqrt{6/n}$, $\sigma/\sqrt{n}$ and $\sqrt{24/n}$, respectively. The Jarque-Bera statistic is distributed as the Chi-square with two degrees of freedom. (*) denotes significance at the 5% and 1% levels, respectively.

The unconditional mean daily return is very close to zero (0.026%), which is significant, and the unconditional standard deviation is especially high (2.4%). The skewness statistic is negative, and the distribution of the returns is skewed to the left. The kurtosis coefficient (28.20) implies that the distribution has much thicker tails than the normal distribution does. Similarly, the Jarque-Bera statistic is statistically significant, rejecting the assumption of normality. All of this evidence shows that the empirical distribution of the return must not be fit by a normal distribution, as it exhibits a significantly excess of kurtosis and asymmetry (fat tails and peakness).

Going back to Figure 2, we can see that the range fluctuation of the returns is not constant, which means that the variance of these returns changes over time. The volatility of the series was high during the early 1900s, coinciding with the beginning of the Gulf War. In the last years of the sample, we observe a period more stable. According to Marimoutou et al. (2009), we estimate an AR(1)-GARCH(1,1) specifications using a rolling window of 1000 days data. Table 3 shows our estimated parameters for the AR(1)-GARCH(1,1) of the mean and volatility of the returns.

Table 3. AR(1)-GARCH(1,1) estimation result for Brent returns

$$\mu_t = \alpha_0 + \alpha_1 r_{t-1}$$
$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \gamma \sigma_{t-1}^2$$

Parameter	Estimates	*p*-value
α_0	0.001*	0.018
α_1	-0.079	0.992
β_0	0.000*	0.016
β_1	0.890*	0.000
γ	0.048*	0.001

The asterisk denotes parameters significantly different from zero at the 5% confidence level.

The set of models we have included in the comparison are the same models used in Marimoutou et al. (2009). In Table 4, we show the expressions for the different approaches. The VaR is calculated the forthcoming day for two confidence levels: 99.5% and 99.9%[2].

Table 4. Statistical approaches for estimating the Value at Risk

Normal	$VaR_{t+1} = \phi^{-1}(p)$
Historical Simulation	$VaR_{t+1} = \text{Quantile}\left\{\{r_t\}_{t=1}^{n}\right\}$
Filtered Historical Simulation	$VaR_{t+1} = \mu_{t+1} + \sigma_{t+1}\text{Quantile}\left\{\{r_t\}_{t=1}^{n}\right\}$
Unconditional GPD	$VaR_{t+1} = \hat{u} + \frac{\hat{\sigma}}{\hat{\xi}}\left[\left(\frac{n}{N_u}(1-p)\right)^{-\hat{\xi}} - 1\right]$
Conditional Normal	$VaR_{t+1} = \mu_{t+1} + \sigma_{t+1}\phi^{-1}(p)$
Conditional Student	$VaR_{t+1} = \mu_{t+1} + \sigma_{t+1}\sqrt{\frac{\nu-2}{\nu}}F^{-1}(p)$
Conditional GPD	$VaR_{t+1} = \mu_t + \sigma_{t+1}VaR_t(Z)$

Note: $\phi^{-1}(p)$ is the quantile of the standard normal distribution, $\hat{u}$ is the threshold, $\hat{\sigma}$ is the estimated scale parameter, $\hat{\xi}$ is the estimated shape parameter, N_u is the number of exceedances over the threshold, μ_{t+1} and σ_{t+1} are the conditional forecasts of the mean and the standard deviation, $F^{-1}(p)$ and *v* are the quantile the *t*-distribution and the degrees of freedom, respectively, and *Z* is the standardised residual series.

For evaluating the performance of each model in terms of the VaR, we also use a backtesting procedure in two stages. Unlike Marimoutou et al. (2009) who apply only two

[2] In Marimoutou et al. (2009), the VaR is calculated for eight levels of confidence (0.1%, 0.5%, 1% and 5% to left and right tails). We believe that two levels are sufficient.

accuracy tests, in the first stage, we use five accuracy tests (the unconditional coverage test (LR_{UC}), Backtesting criterion (BTC), the conditional coverage test (LR_{cc}), the independence test (LR_{ind}) and the Dynamic Quantile test (DQ). If a model survives in all tests, the model is accurate. In the second stage, we evaluate the magnitude of the loss function for the first stage overcoming models, but unlike Marimoutou et al. (2009) who only use a single loss function, eleven loss functions have been used in our comparative: six loss functions, which reflect the utility function of the regulator and five loss functions from the viewpoint of the firms.

Overall, we design an exhaustive comparison. We estimate the VaR by using seven different models for two levels of confidence and follow a two-stage backtesting procedure to assess the forecasting power of each model: five accuracy tests and eleven loss functions.

3.2.- Results

The results of the accurate tests are presented in Table 5. We show the percentage of exception obtained with each VaR model at the 99.9% and 99.5% confidence levels. The table reports the *p*-value obtained for the different tests. When the null hypothesis that "the VaR estimate is accurate" has not been rejected by any test, we have shaded the percentage of exceptions.

According to Table 5 and for both levels of confidence, we can assert that the VaR estimates obtained by the unconditional and conditional normal distribution are very poor. The Historical Simulation, Filtered Historical Simulation, unconditional GPD and conditional Student yield good VaR estimations at 0.1% and 0.5%. The conditional GPD model performs well only at 0.1%. For the survived models, in comparing our results with these obtained by Marimoutou et al. (2009), we observe some differences. There are more models that pass the first stage than these found by Marimoutou et al. (2009). In table 5, we denote with an asterisk (*) the models that pass the first stage in Marimoutou et al. (2009). The differences in the data source and the tests used may explain the discrepancies.

Table 5. Accuracy tests

	0.1%	0.5%
Normal	**0.59**	**0.90**
LR_{UC}	0.0000	0.0352
BTC	0.0000	0.0007
LR_{IND}	0.7298	0.5180
LRcc	0.0001	0.0883
DQ	0.0000	0.0000
HS	**0.10**	**0.46**
LR_{UC}	0.9662	0.8352
BTC	0.3981	0.3800
LR_{IND}	0.9522	0.7870
LRcc	0.9973	0.9435
DQ	1.0000	0.9997
FHS	**0.15**	**0.44***
LR_{UC}	0.5101	0.7164
BTC	0.2225	0.3449
LR_{IND}	0.9283	0.7986
LRcc	0.8018	0.9062
DQ	0.8567	0.9923
Uncond.GPD	**0.13**	**0.52**
LR_{UC}	0.7182	0.9253
BTC	0.3387	0.3949
LR_{IND}	0.9403	0.7639
LRcc	0.9344	0.9517
DQ	0.9926	1.0000
Cond. Normal	**0.57**	**1.26**
LR_{UC}	0.0000	0.0002
BTC	0.0000	0.0000
LR_{IND}	0.7411	0.7670
LRcc	0.0002	0.0009
DQ	0.0000	0.0000
Cond.Student	**0.08***	**0.59***
LR_{UC}	0.7604	0.5967
BTC	0.3614	0.2834
LR_{IND}	0.9643	0.7298
LRcc	0.9536	0.8190
DQ	0.9971	0.9466
Cond.GPD	**0.13**	**0.10**
LR_{UC}	0.7182	0.0050
BTC	0.3387	0.0009
LR_{IND}	0.9403	0.9522
LRcc	0.9344	0.0193
DQ	0.9926	0.0000

VaR violation ratios of the daily returns (%) are boldfaced. The table reports the *p*-values of the following tests: (i) the unconditional coverage test (LRuc); (ii) the backtesting criterion (BTC); (iii) statistics for serial independence (LRind); (iv) the conditional coverage test (LRcc); (v) the Dynamic Quantile test (DQ). A *p*-value greater than 5% indicates that the forecasting ability of the VaR model is accurate. The shaded cells indicate that the null hypothesis that the VaR estimate is accurate is not rejected by any test. An asterisk indicates that the results are identical to the results obtained in Marimoutou et al. (2009).

For the survived models, we calculate the loss functions. The model that provides the lowest loss function value will be considered the best. Tables 6 and 7 reports the results obtained by the regulator´s loss functions and the firm's loss functions, respectively. For each loss function, we marked in bold the model that provides the lowest losses. To calculate the FABL firm's loss function, we proxy the price of capital with the interest rate of the Eurosystem monetary policy operations of the European Central Bank since the first of 1999 and the Deutsche Bundesbank's interest rate for the previous period.

According to the quadratic loss function of Lopez (1998, 1999) from the viewpoint of the regulator, our results are the same as Marimoutou et al. (2009) (see Table 6). At the 99.9% confidence level, the best model is the conditional Student and for 99.5% level of confidence is the Filtered Historical Simulation. Furthermore, all the regulator´s loss functions provide the same results. Thus, the results of the comparison are robust to the regulator´s loss function, but depend on the level of confidence. However, the firm's loss functions point to other models as being optimal (see Table 7). From the viewpoint of the firm, the best model at the 99.9% confidence level is the conditional GDP, whereas at the 99.5% confidence level, the conditional Student provides the lowest losses. This result is robust to the firm´s loss function.

Table 6. Magnitude of the regulator's loss functions

	level	HS	FHS	Uncond.GPD	Cond.Student	Cond.GPD
RQL	**0.10%**	4.0023	6.0019	5.0043	**3.0015**	5.0024
	0.50%	18.0127	**17.0061**	20.0142	23.0104	
RL	**0.10%**	0.0687	0.0773	0.1183	**0.0623**	0.092
	0.50%	0.3367	**0.2094**	0.3593	0.3228	
RQ	**0.10%**	0.0023	0.0019	0.0043	**0.0015**	0.0024
	0.50%	0.0127	**0.0061**	0.0142	0.0104	
RC_1	**0.10%**	0.8631	0.9345	1.424	**0.6944**	1.1295
	0.50%	5.1171	**3.2809**	5.6399	5.0781	
RC_2	**0.10%**	0.0319	0.0212	0.0526	**0.0155**	0.0278
	0.50%	0.1942	**0.0889**	0.2255	0.154	
RC_3	**0.10%**	0.0687	0.0773	0.1183	**0.0623**	0.092
	0.50%	0.3367	**0.2094**	0.3593	0.3228	

Note: The table reports the values of the sum of the different loss functions of each VaR model at both confidence levels. The boldface figures denote the minimum value of the function.

Table 7. Magnitude of the firm's loss functions

	level	HS	FHS	Uncond.GPD	Cond.Student	Cond.GPD
FS	**0.10%**	38.6488	17.4877	24.6294	13.5053	**13.1329**
	0.50%	13.7859	9.7707	13.4868	**9.1305**	
FC_1	**0.10%**	3437.374	3269.0552	3327.208	3218.6995	**3189.4691**
	0.50%	3121.3939	2993.6661	3093.4115	**2942.9809**	
FC_2	**0.10%**	662.0202	323.4116	420.2661	254.7756	**242.3176**
	0.50%	227.066	171.1829	217.8154	**152.3774**	
FC_3	**0.10%**	769.5652	427.0749	524.5131	357.17	**343.8888**
	0.50%	325.4232	267.5742	315.3856	**247.2837**	
FABL	**0.10%**	38.6826	17.5287	24.6655	13.5359	**13.1708**
	0.50%	13.8533	9.8392	13.5575	**9.2071**	

Note: The table reports the values of the sum of different loss functions for each VaR model at both confidence levels. The boldface figures denote the minimum value of the function.

Overall, our comparison points to some interesting results. First, the optimal VaR model is a function of the confidence level. Second, the optimal model depends on the group of loss functions (regulator's loss function versus firm's loss function) used to evaluate the losses. Third, the best VaR model is robust within both groups of loss functions.

4. *Conclusions*

This paper investigates whether the results of the comparison VaR models depend on the loss function used for such purpose. We use daily returns of the Brent price from May 1987 to January 2006. Following Marimotou et al. (2009), the models that we have included in the comparison are Historical Simulation (HS), Filtered Historical Simulation (FHS), Conditional and Unconditional Normal, Conditional Student and Conditional and Unconditional Extreme Value (GPD).

The best model is selected by a two-stage selection approach. First, we apply a backtesting procedure based on five statistical tests. For the models that survive the first stage, we compute the losses using several loss functions from two groups: firm´s loss functions and regulator/supervisor's loss functions. Our results indicate that in terms of their ability to forecast the VaR, the best model is robust to the regulator's loss function. The same results are obtained with the firm's loss function. However, we find important differences in terms of the main actor, the regulator/supervisor and the risk manager.

In particular, we find that from the viewpoint of the regulator, the best models are conditional Student at the 99.9% confidence level and Filtered Historical Simulation at the 99.5% confidence level. From the viewpoint of the firm, the best model at the 99.9% confidence level is the conditional GPD and at 99.5% confidence level is conditional Student.

Finally, although the model that minimises the losses is robust to the firm's loss functions, we consider that the loss function has to compute the real opportunity cost of the firm and, particularly, if the VaR models are very prudent. We propose a firm's loss function that meets this condition.

Our results can help market participants make effective selections between VaR models. The market participants (supervisors and risk managers) must consider that they have specific loss functions, and the final result depends on who is the end-user of the VaR model. Finally, our results can also help researchers understand the different results presented in the compared literature.

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