# EVALUATING ASYMMETRIC EFFECT IN SKEWNESS AND KURTOSIS

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# Evaluating asymmetric effect in skewness and kurtosis.

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## Abstrac:

This paper shows evidence that the higher moments parameters (skewness and kurtosis) of the distribution of financial returns are time-varying. This means that the distribution of financial returns is not i.i.d. as many approaches for portfolio risk management assume. Therefore it may be preferable to assume that the stochastic process for returns has time-varying conditional distributions. The possible dependence of these parameters on the sign and the size of unexpected returns have also been analyzed. The Engel and Ng. (1993) test was used to do so. The tests carried out provide some evidence that the skewness and kurtosis parameters respond asymmetrically to shocks of different signs and sizes. This result suggests that if we are interested in modeling the dynamic behavior of these parameters we should take asymmetric GARCH specifications into account. Besides, it has been detected that in volatile periods, when the size of unexpected returns is bigger, the tails of empirical distributions grow fatter. and the distribution is not as peaky. If the empirical distribution is skewed to the left skewness rises, otherwise it goes down.

**Keywords:** Value at Risk, Parametric model, Skewness t-Generalised Distribution, GARCH Model, Risk Management, Loss function.

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## 1. Introduction

Volatility (variance) is the most popular and traditional risk measure. In fact, the traditional financial theory defines risk as the dispersion of returns due to movements in financial variables. Under this premise, risk can be measured using the conditional variance of the returns of a portfolio.

Another way of measuring risk, which is the most commonly used at present, is to evaluate the losses that may occur when the financial variables change. This is what Value at Risk (VaR) does. The VaR of a portfolio indicates the maximum amount that an investor may lose over a given time horizon and with a given probability<sup>1</sup>. In this case, the concept of risk is associated with the danger of losses. Since the Basel Committee on Bank Supervision at the Bank for International Settlements requires a financial institution to meet capital requirements on the basis of VaR estimates, allowing them to use internal models for VaR calculations, this measurement has become a basic market risk management tool for financial institutions.

Despite VaR's conceptual simplicity, its calculation is complex. Many approaches have been developed to forecast VaR. These include nonparametric approaches, e.g. Historical Simulation; semi-parametrics approaches, e.g. Extreme Value Theory and the Dynamic quantile regression CaViar model (Engle and Manganelli (2004)) as well as parametric approaches e.g. Riskmetrics (JP. Morgan (1996)).

Many of these assume that the distribution of returns standardised by conditional means and conditional standard deviation is independent and identically distributed (i.i.d.). However, there is some empirical evidence that the distribution of financial returns standardised by conditional means and volatility is not i.i.d. (see Mandelbrot 81963), Hansen (1994), Harvey and Siddique (1999), Jondeau and Rockinger (2003), Bali and Weinbaum (2007) and Brooks et al. (2005)). Others studies also suggested that the process of negative extreme returns at different quantiles may differ from one another (Engle and Manganelli (2004), Bali and Theodossiou (2007)).

<sup>&</sup>lt;sup>1</sup> According to Jorion (2001), VaR measure is defined as the worst expected loss over a given horizon under normal market conditions at a given level of confidence.

Focusing on this particular aspect, recent papers such as Ergun and Jun (2010) and Bali et al. (2008) stress that in the context of the parametric method, the techniques that relax the conventional assumption that the distribution of standardized returns is i.i.d. and model the dynamic performance of the high-order conditional moments (asymmetry and kurtosis) provide more reliable results than those considering functions that imply constant high-order moments.

In the line of the studies aforementioned, this paper contributes to the existing literature by showing that the high-order moment parameters of the distribution financial return are time-varying. Therefore, it may be preferable to assume the stochastic process for returns has time-varying conditional distributions. Moreover, it has been observed that the high-order moments parameters depend on the sign and size of unexpected returns so that a leverage effect is detected when taking these parameters into consideration.

The results stress that the probability distribution of the financial return are time-varying and also that the shape of theses distributions depends on the sign and size of unexpected returns. This is what some extensions of the CaViar method implicitely assume, in particular those that introduce asymmetric specifications for quantiles (see Gerlach et al. (2011) and Yu et al. (2010)).

In the following section the Skewness Generalized t distribution (SGT) proposed by Theodosiou (1998) is presented while in section 3 empirical results are discussed. Finally, section 4 includes the main conclusions.

## 2. Methodology

The empirical distribution of the financial return has been documented to be asymmetric and exhibits a significant excess of kurtosis (fat tail and peakness). Therefore, assuming a normal distribution for risk management and particularly to estimate the VaR of a portfolio does not produce satisfying results. Under this assumption, the size of the losses will be much higher than those predicted in the case of a normal distribution.

As the *t*-Student distribution has fatter tails than the normal distribution, this distribution that has been commonly used in finance and risk management, particularly to model conditional asset returns (Bollerslev (1987)). However, although the t-Student distribution can often account satisfactorily for the excess kurtosis found in common asset returns, this distribution does not capture the skewness of the return.

Taking this into account, one direction for research in risk management involves searching for other distribution functions that capture this characteristic. The skewness t-Student distribution (SSD) of Hansen (1994), the exponential generalized beta of the second kind (EGB2) of McDonald and Xu (1995), the generalized error distribution (GED) of Nelson (1991), the skewness generalized-t distribution (SGT) of Theodossiou (1998), the skewness error generalized distribution (SGED) of Theodossiou (2001) and the inverse hyperbolic sign (IHS) of Johnson (1949) are the most commonly used in VaR literature (see Abad at al. (2013), Chen et al. (2012), Polanski and Stoja (2010), Xu and Wirjanto (2010), Bali and Theodossiou (2008), Bali et al. (2008), Haas (2009) and Ausín and Galeano (2007)). On the whole, these papers show that skewness distributions are extremely reliable to forecast VaR.

In a comparison of these distributions, Abad et al. (2013), show that SGT distributions are the most appropriate in fitting financial returns outperforming the normal and t-Student distribution not only in fitting data but also in forecasting VaR.

The SGT introduced by Theodossiou (1998) is a skewed extension of the generalised *t* distribution that was originally proposed by McDonald and Newey (1988). The SGT is a distribution that allows for a very diverse level of skewness and kurtosis, and it has been used to model the unconditional distribution of daily returns for a variety of financial assets. The SGT probability density function for the standardised residual is:

$$f(z_t|\lambda,\eta,k) = C \left( 1 + \frac{|z_t + \delta|^k}{\left( (\eta + 1)/k \right) (1 + sign(z_t + \delta)\lambda)^k \theta^k} \right)^{-\frac{\eta + 1}{k}}$$
(42)

where

$$C = 0.5k \left(\frac{\eta+1}{k}\right)^{-\frac{1}{k}} B\left(\frac{\eta}{k}, \frac{1}{k}\right)^{-1} \theta^{-1} \qquad \theta = \frac{1}{\sqrt{g-\rho^2}} \qquad \rho = 2\lambda B\left(\frac{\eta}{k}, \frac{1}{k}\right)^{-1} \left(\frac{\eta+1}{k}\right)^{\frac{1}{k}} B\left(\frac{\eta-1}{k}, \frac{2}{k}\right)$$
$$g = (1+3\lambda^2) B\left(\frac{\eta}{k}, \frac{1}{k}\right)^{-1} \left(\frac{\eta+1}{k}\right)^{\frac{1}{k}} B\left(\frac{\eta-1}{k}, \frac{3}{k}\right) \qquad \delta = \rho\theta$$

 $\lambda$  is the skewness parameter,  $|\lambda| < 1$ ;  $\eta$  is a tail-thickness parameter,  $\eta > 2$ ; k is a peakness parameter, k>0; sign is the sign function; B(.) is the Beta function;  $\delta$  is the Pearson's skewness; and the mode of f(z<sub>t</sub>);  $z_t = (r_t - \mu_t)/\sigma_t$  is the standardised residual.

The skewness parameter  $\lambda$  controls the rate of descent of the density around the mode of  $z_t$ . In the case of positive skewness ( $\lambda > 0$ ), the density function is skewed to the right. In contrast, the density function is skewed to the left with negative skewness ( $\lambda < 0$ ).

In the following section we fit an SGT distribution to our data set. The results suggest that high-order moment parameters are time-varying, providing evidence that the standardized financial returns are not i.i.d., as assumed in traditional financial literature.

#### 3. Empirical results

#### 3.1 Data and descriptive statistics

The data consist of closing daily returns on six composite indexes from 3/01/2000 to 31/10/2013 (around 3,500 observations). The indexes are: the Japanese Nikkei, Hong Kong Hang Seng, US S&P 500 and Dow Jones, UK FTSE100 and the French CAC40. The data were extracted from the Yahoo Finance web page (.http://es.finance.yahoo.com/).

The computation of the returns of the indexes  $(r_t)$  is based on the formula,  $r_t=\ln(I_t)-\ln(I_{t-1})$  where  $I_t$  is the value of the stock market index for period *t*.

Figure 1 shows the daily returns of the data and Table 1 provides basic descriptive statistics of the data. The unconditional mean of daily return is very close to zero for all indexes. The unconditional standard deviation varies between 1.234 Dow Jones and 1.586 Hang Seng. In August 2007 the financial market tensions started then followed by a global financial and economic crisis leading in turn to significantly rising volatility of returns. This increase was especially important after August 2008 coinciding with the fall of Lehman Brothers. From mid 2008 to the end of 2009, the volatility of the S&P500, Nikkei and Hang Seng indexes, measured using the standard deviation of returns reached 2.585, 2.671 and 2.899 respectively, more than 1 point higher than the standard deviation observed during the whole of the period 2000-2013. A similar increase is observed in the case of all indexes. A reduced volatility has also been observed in the last four years.

Skewness statistics are negative and significant for all indexes considered except in the case of the CAC40, which means that the distribution of those returns is skewed to the left. For all the indexes, the excess kurtosis statistic is very large and significant at 1% level implying that the distributions of those returns have much thicker tails than the normal distribution. Those results are in line with those that were obtained by Bollerslev (1987), Bali and Theodossiou (2007), and Bali et al. (2008) amongst others. All of them find evidence that the empirical distribution of the financial return is asymmetric and exhibits a significantly excessive kurtosis (fat tails and peakness).

In order to capture the non-normal characteristics observed in the data set, an SGT distribution of Theodossiou (1998) has been fitted. The parameters of this distribution are presented in Table 2. The first and second columns of the table provide the estimates for the unconditional mean and unconditional standard deviation of log-returns. Standard errors are presented in brackets. As expected, these estimates are quite similar across distributions and do not differ much from the simple arithmetic means and standard deviations of log-returns presented in Table 1. The unconditional mean is close to zero for all indexes and the unconditional standard deviation varies around 1.5. The third column presents the skewness parameter  $\lambda$ . For all indexes considered, including the CAC40, this parameter is negative and significant at 1% level, which means that the distributions of these returns are skewed to the left.

The fourth and fifth columns provide estimates for the kurtosis parameters  $\eta$  (fat-tail) and  $\kappa$  (peakness). The value of  $\kappa$  varies around 1.5,

except in the case of the Nikkei index, which reaches 1.887. The value of  $\eta$  reaches approximately 4.5, except in the case of the Hang Seng index, which reaches 8.084. These estimates are quite different from those predicted when using a normal distribution ( $\kappa = 2$  and  $\eta = \infty$ ), thus indicating that the set of returns we considered are characterized by excess kurtosis. As the SGT distribution nets the normal distribution it is possible to use a log-likelihood ratio test for testing the null hypothesis of normality against that of SGT. For all indexes considered, the  $LR_{NORMAL}$  statistics are statistically significant at the 1% level, providing evidence against the normality hypothesis (table 2).

#### 3.2 Time-varying scaling parameters

In the above section, the whole sample (2000-2013) was used to fit SGT distributions. To investigate if higher-order moment parameters are time-varying, an SGT distribution was fitted in a recursive manner to analyze the January 2002 - October 2013 period. Initially the sample used was the period dating 03/01/2000 to 31/12/2001. Then the sample was amplified adding a daily data and re-estimating model parameters each time. This allowed for a sample of 3.000 data of the skewness and kurtosis (peakness and fat-tail) parameters.

The estimates of the skewness ( $\lambda$ ), peakness ( $\kappa$ ) and fat-tail ( $\eta$ ) parameters are presented in figures 2, 3 and 4. In table 3 descriptive statistics of that parameter are presented. In these pictures, two vertical lines are drawn. The first line indicates values reached on August,1 2007 and the second line indicates values obtained in September 2008. The former date indicates the beginning of the global financial and economic crisis<sup>2</sup> that led to a significant rise in the volatility of returns. This increase was particularly

<sup>&</sup>lt;sup>2</sup> In August of 2007 there was a liquidity crisis that prompted a substantial injection of capital into the financial markets by the FED, European Central bank and the Bank of England.

important after August 2008 (second line), coinciding with the fall of Lehman Brothers.

In figure 2, skewness parameters show important differences along the sample. In 2002 the empirical distribution of the Ftse, DJA and CAC40 returns was skewed to the left while the empirical distribution of the S&P500, Nikkei and Hans Seng indexes was skewed to the right. However, at the beginning of 2004 all empirical distributions were skewed to the right. Since then, the value of  $\lambda$  has showed a decreasing trend indicating that the empirical distribution of these returns is skewed to the left. In August 2008, just before the fall of Lehman Brothers,  $\lambda$  reaches a minimum value of -0.09 in the case of the CAC40 index, -0.08 in the case of the S&P500 index, -0.07 in the case of the DJA and Ftse indexes, -0.06 in the case of the Nikkei index and -0.05 in the case of the Hang Seng index. Since then the value of  $\lambda$  has increased slightly becoming a little less asymmetric.

The two kurtosis parameters (*k* and  $\eta$ ) perform differently. The peakness parameter (k) is smooth while the fat-tail parameter ( $\eta$ ) is more volatile.

For all indexes considered, the peakness parameter (*k*) shows a decreasing trend. At the beginning of the sample the value of this parameter was close to 2 in the case of most indexes, indicating no peakness of the empirical distribution in the first years of the analysis. Nevertheless, the value of this parameter decreases along the sample. In August of 2008, before the fall of Lehman Brothers, k reaches a minimum figure close to 1.0 in the case of the Hang Seng and S&P500 indexes and reaches an approximate figure of 1.5 in the case of the other indexes observed. Throughout August and September 2008, depending on the index, the value of k jumps up indicating that an increased volatility leads to a reduced peakness of the empirical distribution of the returns.

Differences were observed across the indexes used. The average value of k is 1.8 in the case of the Nikkei, DJ and CAC40 while in the case of the FTSE the average value of k is 1.6 and 1.4 in the case of the S&P500 and Hang Seng indexes.

The fat-tail parameter ( $\eta$ ) fluctuates much more than the skewness and peakness parameter. At the beginning of the sample this parameter was

close to 5 although its value changes significantly along the sample. In the case of the Hang Seng, Nikkei and S&P500 indexes this parameter reaches approximately 30 in August of 2008, just before the fall of Lehman Brothers. The value of Ftse and CAC40 also show an increasing trend but their increase is moderate. It fell suddenly in August and/or September of 2008 indicating that the empirical distribution tail of the returns became fatter due to the global financial crisis. In the case of all indexes it is also observed that August and/or September of 2008, depending on the index, this parameter go down suddenly indicating that with the global financial crisis the empirical distribution tail of the returns became fatter due to the global financial became fatter of 2008, depending on the index, the parameter go down suddenly indicating that with the global financial crisis the empirical distribution tail of the returns became fatter.

From this analysis, it seems evident that the financial returns are not independent over time and identically distributed (i.i.d.) as we find evidence that the high-order moment parameters significantly change over time. In the next section we evaluate if the dynamic behavior of the high-order moment parameter depends on the sign and size of unexpected returns.

#### 3.3 Evaluating asymmetric effect in high-order moment parameter

Having found evidence that the high-order moment parameters are time-varying, it seems reasonable to analyze the dependence of skewness, fat-tail and peakness parameters on the sign and size of the past standardized returns (see Bali et al. (2008)).

To test this hypothesis it has been used the Sing-Bias test and Size-Bias test proposed by Engle and Ng. (1993. The Sign-Bias test focuses on the different impacts that positive and negative innovations have on these parameters. This test is defined as the *t*-ratio of the coefficient  $b_1$  in the OLS regression:  $z_t = a + b_1 S_{t-1}^- + e_t$ . The Size-Bias test focuses on the different impacts that may have over the skewness, peakness and fat tail parameters large and small innovations. The Negative-Size-Bias test is defined as the *t*ratio of the coefficient  $b_2$  in the OLS regression:  $z_t = a + b_2 S_{t-1}^- e_{t-1} + e_t$ . The positive-Size-Bias Test statistic is defined as the *t*-ratio of the coefficient  $b_2$  in the same regression equation with  $S_{t-1}^+=1$ - $S_{t-1}^-$ . It is important to distinguish between positive and negative innovations while examining the size effect of a piece of news of a negative nature compared to that of a positive nature on high-order moment parameters.

The joint test is the F statistics from the OLS regression:  $z_t = a + b_1 S_{t-1}^- + b_2 S_{t-1}^- e_{t-1} + b_3 S_{t-1}^+ e_{t-1} + e_t$ . In all of these regressions  $e_t$ . represents the innovations returns and  $z_t$  represents the high-order moment parameters ( $\lambda_t$ ,  $\eta_t$  and  $k_t$ ).  $S_{t-1}^-$  is a dummy variable that takes a value of 1 if  $e_{t-1}$  is negative and zero otherwise.

For  $\lambda$  parameter, the results of these tests are presented panel (a) of table 4. The coefficient  $b_1$  in the regression  $\lambda_t = a + b_1 S_{t-1}^- + e_t$  is not statistically significant in any cases. This result seems to indicate that the skewness parameter does not depend on the sign of the unexpected returns. However, the coefficient  $b_2$  in the following regression  $\lambda_t = a + b_2 S_{t-1}^- e_{t-1} + e_t$  is statistically significant at 99% confidence level in five of the six indexes considered. The sign of  $b_2$  is positive in all cases, which implies that the higher the impact of a negative piece of news, the lower the value of  $\lambda$ . This means that, for example, if the empirical distribution of the financial return is skewed to the left, skewness will be increased in the case of a piece of news of a negative nature.

In the case of the positive size-bias test the coefficient  $b_2$  is statistically significant at 99% confidence level only in the case of the Hang Seng and DJA indexes. The sign of this coefficient is negative meaning that the stronger the impact, the lower the value of  $\lambda$ . In addition, the value of  $b_2$  in the negative size-bias test is always higher than the value of this coefficient in the positive size-bias test which means that a piece of news of a negative nature has a bigger impact on the skewness parameter than in the opposite case.

Panel (b) of table 4 indicates the results of the test for the fat-tail parameter. The coefficient  $b_1$  in the regression  $\eta_t = a + b_1 S_{t-1}^- + e_t$  is not statistically significant in any cases. This result indicates that the fat-tail parameter does not depend on the sign of the unexpected returns. However, results show that the fat-tail parameter may depend on the size of the unexpected returns. In 50% of the indexes considered (DJA, Ftse, and CAC40) the coefficient  $b_2$  in the regression  $\eta_t = a + b_2 S_{t-1}^+ e_{t-1} + e_t$  is

negative and statistically significant at 99% confidence level. This means that when the volatility increases the fat-tail parameter decreases implying a fatter-tail distribution. Similar results are obtained for the negative size-bias test. The sign of  $b_2$  in the regressions  $\eta_t = a + b_2 S_{t-1}^- e_{t-1} + e_t$  is positive in the case of the DJA and CAC40 indexes, indicating that a piece of news of a negative nature may reduce the value of  $\eta$  increasing the tail-thickness-distribution.

Finally, the results of the test to measure the peakness parameter are presented in panel (c) of table 4. In the case of the S&P500, DJA and CAC40 indexes, the coefficient  $b_1$  in the regression  $k_t = a + b_1 S_{t-1} + e_t$  is statistically significant at 95% confidence level providing some evidence that the peakness parameter depends on the sign of the unexpected returns. In the case of the majority of indexes, there is evidence that this parameter depends on the size of innovations. The size-bias test failed only in the case of the Nikkei and S&P500 indexes. The sign of the coefficients seems to indicate that regardless of the sign of innovations, bigger unexpected returns imply bigger values for the peakness parameter, which in turn means less peakness in the case of an empirical distribution. This result is coherent with those observed in September 2008 coinciding with the fall of Lehman Brothers when the peakness parameter increased significantly. Only in the case of the Hang Seng index did the sign of these coefficients indicate the opposite.

On the whole, the joint test provides significant evidence that the innovations of different sign and size provoke a different impact on high order moment parameters.

These results stress that the shape of the probability distribution of the financial return depends on the sign and size of unexpected returns. This is what some extensions of the CaViar method assume to estimate VaR, in particular those that introduce asymmetric specifications for the quantiles (see Gerlach et al. (2012) and Yu et al. (2010)). These authors show that when we use an asymmetric version of the CaViaR model the VaR estimates improve significantly. The paper published by Sener et al. (2012) supports this hypothesis. By comparing various CaViaR models (asymmetric and

symmetric) they found that the asymmetric CaViaR model outperforms the result obtained when using the standard CaViaR model.

In this line, but in the context of parametric methods the results obtained in this paper suggest that if we want to model the dynamic behavior of the high moment parameters as Ergun and Jun (2010) and Bali et al. (2008) propose we should consider asymmetric GARCH specifications for these parameters. Analyzing if the use of such specifications actually improve VaR estimates it's left for future research.

#### 3.4 Analyzing the behavior of skewness and kurtosis in high volatility periods

The previous analysis showed significant evidence that skewness, peakness and fat-tail parameters respond asymmetrically to shocks of different sizes and different sign. In particular, it has been found that in a volatile period, when the size of the unexpected returns is bigger, the tails of empirical distributions becomes fatter, and the distribution becomes less peaky. With regards to skewness, if the empirical distribution is skewed to the left, skewness increases and if not, it goes down. This explains the fact that while in a stable period a parametric approach under a normal distributions seems to be appropriate to estimate VaR, in a volatile period this approach clearly underestimates risk (Abad and Benito (2013) and Bao et al. (2006)).

In order to assess the robustness of these results, correlations between the conditional standard deviation of the returns and the high-order moment parameters have been calculated (see table 5). The correlation between volatility and skewness parameters is negative in all cases and it is especially important in the case of the Hang Seng (-0.36), the DJA (-0.26) and the S&P500 (-0.12) indexes, for which scatter plots are presented in figure 4. These figures clearly illustrate the inverse link between the volatility and the asymmetry of a distribution.

The correlation between the conditional standard deviation of the returns and the fat-tail parameters are also negative. It is especially significant in the case of the Nikkei (-0.11), the DJA (-0.25) and the CAC40 (-0.20) indexes. It confirms that there is an inverse link between the fat-tail parameter and volatility, which can be checked in figure 5. Finally, the

correlation between the peakness parameter and the conditional standard deviation of the returns is positive for in the case of the DJA (0.11), Ftse (0.25) and CAC40 (0.19) indexes meaning that in this particular case, when volatility increases the distribution of the returns becomes less peaky. For these indexes the positive relationship between volatility and the peakness parameter can be checked in figure 6. In case of the Nikkei, Hang Seng and S\$P500 indexes, the correlation is negative but very close to zero. Only in the case of the Hang Seng index is this correlation relatively high (-0.19). The sign of this correlation is in the line of the results of the size-bias test that shows that in the case of this index, a piece of news that causes a greater impact will imply a greater peakness of the distribution.

#### 4. Conclusion

This paper contributes to the existing literature showing that the distribution of standardised financial returns is not independent and identically distributed (i.i.d.). This, in turn, may have significant implications in the field of risk management.

Empirical distributions of the portfolio returns have been documented in the literature to be asymmetric and to exhibit important kurtosis. In this paper, an SGT distribution was fitted to a set of six stock markets indexes, Nikkei, Hang Seng, S&P500, Dow Jones, Ftse, and CAC40 to capture these results. The sample used ranged from January 2000 to the end of October 2013.

For all indexes considered, estimates of the skewness, peakness and fat-tail parameters are statistically significant, indicating that the empirical return distributions are skewed, peaky and have fatter tails than normal distributions.

In addition, estimating the SGT distribution recursively produce a sample of about 3.000 estimates of skewness, peakness and fat-tail parameters. The analysis of these time series shows that higher-order moment parameters are time-varying. The fat-tail is very volatile while the skewness and peakness parameters are smoother. These results indicate

that the shape of the density function changes over time. This has significant implications in risk management, in particular in forecasting VaR.

In view of these results it does not seem appropriate to assume that the future will be the same as the past, as for instance, the Historical Simulation method implies when forecasting VaR. However, we can use past information in order to learn how and in which direction the density function will change when the volatility of the return increases again.

In this line, having obtained evidence that high-order moment parameters are time- varying, their possible dependence on the sign and size of unexpected returns was questioned. The Engel and Ng. (1993) test was used to test this hypothesis. These tests provide some evidence that the value of these parameters depend on the sign and size of unexpected returns. Consequently, in order to model the dynamic behavior of the high moment parameters we should consider asymmetric GARCH specifications. Moreover, it has also been detected that in the case of a volatile period, when the size of unexpected returns is bigger, the tails of empirical distributions becomes fatter, and the peakness of distribution is reduced. With regards to skewness if the empirical distribution is skewed to the left, skewness will increase, otherwise it will go down. This explains the fact that in the case of a stable period, the parametric approach under normal distributions proves rather efficient when estimating VaR. However, in volatile periods this approach clearly underestimates risk.

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	Mean	Median	Std. Dev	Skewness	Kurtosis
Nikkei	-0.008	0.015	1.581	-0.419 <sup>**</sup> (0.042)	9.245 <sup>**</sup> (0.084)
Hang Seng	0.008	0.019	1.587	-0.068 (0.042)	10.752 <sup>**</sup> (0.083)
S&P 500	0.005	0.057	1.321	-0.173 <sup>**</sup> (0.042)	10.621 <sup>**</sup> (0.083)
Dow Jones	0.015	0.052	1.235	-0.217 <sup>**</sup> (0.041)	9.723 <sup>**</sup> (0.082)
Ftse100	0.000	0.039	1.268	-0.124 <sup>**</sup> (0.041)	8.891 <sup>**</sup> (0.082)
CAC40	-0.009	0.022	1.541	0.028 (0.041)	7.632 <sup>**</sup> (0.082)

## **Table 1. Descriptive Statistics**

Note: This table presents the descriptive statistics of the daily percentage returns of the Nikkei, Hang Seng, S&P 500, Dow Jones, Ftse 100, and CAC-40 indexes. The sample period is from January 3, 2000 to October 31, 2013. Standard errors of skewness and excess kurtosis are calculated as  $\sqrt{6/n}$  and  $\sqrt{24/n}$  respectively. An  $\cdot$  (--) denotes significance at the 5% (1%) level.

	μ	σ	λ.	η	k	Log-L	
Nikkei	0,000 <sup>**</sup> (0.000)	0.015 <sup>**</sup> (0.000)	-0.044 <sup>**</sup> (0.020)	4.893 <sup>**</sup> (0.291)	1.887 <sup>**</sup> (0.075)	9502.6	463.2**
Hang Seng	0.000 <sup>**</sup> (0.000)	0.015 <sup>**</sup> (0.000)	-0.021 (0.015)	8.048 <sup>**</sup> (0.880)	1.189 <sup>**</sup> (0.036)	9792.1	469.6**
S&P 500	0.000 <sup>**</sup> (0.000)	0.013 <sup>**</sup> (0.000)	-0.067 <sup>**</sup> (0.013)	5.097 <sup>**</sup> (0.314)	1.296 <sup>**</sup> (0.038)	10567.3	756.8**
Dow Jones	0.000 <sup>**</sup> (0.000)	0.012 <sup>**</sup> (0.000)	-0.058 <sup>**</sup> (0.015)	4.341 <sup>**</sup> (0.211)	1.539 <sup>**</sup> (0.049)	10753.8	902.6**
Ftse100	0.000 <sup>**</sup> (0.000)	0.012 <sup>**</sup> (0.000)	-0.058 <sup>**</sup> (0.016)	4.240 <sup>**</sup> (0.200)	1.619 (0.053)	10665.4	763.8**
CAC40	0.000 <sup>**</sup> (0.000)	0.015 <sup>**</sup> (0.006)	-0.058 <sup>**</sup> (0.017)	4.694 <sup>**</sup> (0.026)	1.625 <sup>**</sup> (0.054)	10049.0	576.4**

Note:  $\mu$ ,  $\sigma$ ,  $\lambda$  and  $\eta$  are the estimated mean, standard deviation, skewness parameter, and tail-tickness parameter;  $\kappa$  represents the peakness parameter. Log-L is the maximum likelihood value. *LR*<sub>Normal</sub> is the LR statistic from testing the null hypothesis that the daily returns are distributed as Normal against they are distributed as SGT. An  $\cdot$  ( $\cdot\cdot$ ) denotes significance at the 5% (1%) level.

	Mean	Median	Max	Min	Std Dev.	
Nikkei						
Skewness Fat-tail Peakness	-0.020 8.642 1.807	-0.035 7.303 1.799	0.090 24.194 2.264	-0.059 4.509 1.456	0.038 4.572 0.189	
Hang Seng						
Skewness Fat-tail Peakness	-0.011 11.061 1.335	-0.022 8.057 1.211	0.046 47.788 2.226	-0.043 5.663 1.096	0.021 5.826 0.262	
S&P 500						
Skewness Fat-tail Peakness	-0.039 9.088 1.422	-0.056 8.672 1.333	0.057 38.174 2.030	-0.083 4.110 1.089	0.038 5.112 0.202	
DJA						
Skewness Fat-tail Peakness	-0.042 4.876 7.792	-0.056 5.077 1.774	0.011 6.901 2.753	-0.077 3.374 1.502	0.024 0.788 0.251	
Ftse						
Skewness Fat-tail Peakness	-0.048 6.375 1.547	-0.056 5.447 1.577	0.011 28.182 1.695	-0.072 0.000 1.340	0.019 3.540 0.090	
CAC40						
Skewness Fat-tail Peakness	-0.057 5.030 1.791	-0.064 4.997 1.683	0.019 7.334 3.141	-0.089 3.623 1.577	0.020 0.776 0.289	

## Table 3. Descriptive Statistics for the skewness and fat-tail parameter

Note: This table presents the descriptive statistics of the skewness, fat-tail and peakness parameters in the case of the Nikkei, Hang Seng, S&P 500, Dow Jones, Ftse 100, and CAC-40 indexes. These estimates have been obtained by re-estimating the SGT distribution based on the sample that goes from 02/01/2002 to 31/10/2013 on a daily basis. This has allowed for a sample of proxy 3.000 data of the skewness and kurtosis (peakness and fat-tail) parameters. The exact size of these samples depends on the index.

	Sign Bias Test	Negative Size Bias Test	Positive Size Bias Test	Joint Test		
	Panel (a	a) Skewness parame	eter (λ)			
NIKKEI	0.0005	0.066	-0.056	0.76		
	(0.0014)	0.070	0.079	p-value: 0.52		
HANG SENG	0.0003	0.276 <sup>**</sup>	-0.236**	40.45		
	(0.0008)	0.039	0.040	p-value: <b>0.00</b>		
S&P500	0.0019	0.171 <sup>*</sup>	-0.150 <sup>*</sup>	4.15		
	(0.0014)	0.083	0.089	p-value: 0.01		
DJI	0.0002	0.288	-0.234	21.63		
	(0.0009)	0.054	0.058	p-value: <b>0.00</b>		
FTSE	0.0000	0.141	-0.069	6.00		
	(0.0007)	0.043	0.045	p-value: <b>0.00</b>		
CAC 40	-0.0005	0.124	-0.011	4.32		
	(0.0007)	0.038	0.039	p-value: <b>0.00</b>		
	Panel	(b) Fat-tail parameter	er (η)			
NIKKEI	0.2225	-3.297	-25.364	3.27		
	0.1696	8.367	9.411	p-value: <b>0.02</b>		
HANG SENG	-0.2799	-15.718	-0.084	1.37		
	0.2139	11.039	11.344	p-value: 0.25		
S&P500	0.1412	-17.296	-3.303	1.01		
	0.1877	11.020	11.844	p-value: 0.39		
DJI	-0.0012	5.518	-8.655	13.96		
	0.0288	1.810	1.927	p-value: <b>0.00</b>		
FTSE	0.0656	-13.819	3.866	1.38		
	0.1279	7.954	8.347	p-value: 0.25		
CAC 40	-0.0073	3.592	-4.381	7.06		
	0.0282	1.465	1.502	p-value: <b>0.00</b>		
Panel (c) Peaknesss parameter (ĸ)						
NIKKEI	-0.0058	0.467	0.512	1.13		
	0.0070	0.347	0.391	p-value: 0.33		
HANG SENG	0.0134	1.580	-1.817	11.75		
	0.0096	0.497	0.510	p-value: <b>0.00</b>		
S&P500	0.0123	-0.209	-0.011	0.99		
5&P500	0.0074	0.436	0.468	p-value: 0.40		
ווס	0.0155 <sup>*</sup>	-2.020**	1.191 <sup>*</sup>	8.00		
Dui	0.0092	0.578	0.617	p-value: <b>0.00</b>		
FTGE	0.0024	-0.766**	0.865	14.57		
110	0.0033	0.204	0.216	p-value: <b>0.00</b>		
	0.0176	-2.735**	1.575**	16.03		
CAC 40	0.0105	0.545	0.560	p-value: <b>0.00</b>		

#### Table 4. Sign and Size-Bias Tests

The **Sign-Bias Test** is defined as the *t*-ratio for coefficient  $b_1$  in the regression equation:  $z_t = a + b_1 * S_{t-1}^- + e_t$ . The **Negative-Size-Bias Test** is defined as the *t*-ratio of the coefficient  $b_2$  in the OLS regression:  $z = a + b_2 * S_{t-1}^- + e_t$ . **Positive-Size-Bias Test** statistic is defined as the *t*-ratio of the coefficient  $b_3$  in the same regression equation with  $S_{t-1}^+ = 1 - S_{t-1}^-$ . In all these regressions  $e_t$  represents the innovations returns; z represents  $\lambda$  (panel a),  $\eta$  (panel b) and k (panel c);  $S_{t-1}^-$  is a dummy variable that takes a value of 1 if  $e_{t-1}$  is negative and zero otherwise. In the first, second and third column the coefficient for  $b_1$ ,  $b_2$  and  $b_3$  are presented. The *t*-ratio for these coefficients are in parenthesis. \*, \*\* denote the case in which the coefficient b is statistically significant at 5% and 1% confidence level respectively. The **joint test** is the F statistics from the OLS regression:  $z^2 = a + b_1 * S_{t-1}^- + (b_2 * S_{t-1}^- * e_{t-1}) + (b_3 * S_{t-1}^+ * e_{t-1}) + e_t$  which is presented along with its p-value.

	Nikkei	Hang Seng	S&P500	DJA	Ftse	CAC40
Skewness	-0.06 <sup>(*)</sup>	-0.36	-0.12	-0.26	-0.11	-0.09
Fat-tail	-0.11	-0.02	-0.01	-0.25	0.06	-0.20
Peakness	-0.01	-0.19	-0.01	0.11	0.25	0.19

(\*) Coefficient of correlation between the skewness parameter estimates and conditional standard deviation. Both calculated in the case of the Nikkei index.

Note: To estimate the conditional standard deviation a GARCH model has been used under a student-t distribution. Re-estimating the SGT distribution along the sample that goes from 02/01/2002 to 31/10/2013 on a daily basis enabled to produce estimates of the high-order moment parameters. This allowed for a sample of proxy 3.000 data of the skewness and kurtosis (peakness and fat-tail) parameters. The exact size of these samples depends on the index.

# Figure 1. Stock index returns

This figure illustrates the daily evolution of returns of 6 indexes (the French CAC40, UK FTSE100, US Dow Jones Composite Average, S&P 500, the Japanese Nikkei and Hong Kong Hang Seng) from January 3, 2000 to October 31, 2013.



## Figure 2. Skewness Parameters

This figure illustrates the skewness parameters ( $\lambda$ ) estimates obtained to fit the SGT distribution to the returns of 6 indexes (the French CAC40, UK FTSE100, US Dow Jones Composite Average, S&P 500, the Japanese Nikkei and Hong Kong Hang Seng). These estimates were obtained to fit an SGT distribution in a recursive manner rolling the sample forward one day at a time so as to re-estimate model parameters.



# Figure 3. Peakness Parameter

This figure illustrates the peakness parameters ( $\kappa$ ) estimates obtained to fit the SGT distribution to the returns of 6 indexes (the French CAC40, UK FTSE100, US Dow Jones Composite Average, S&P 500, the Japanese Nikkei and Hong Kong Hang Seng). These estimates were obtained to fit an SGT distribution in a recursive manner rolling the sample forward one day at a time so as to re-estimate model parameters.



#### Figure 4. Fat-tails Parameters

This figure illustrates the fat-tail parameters ( $\eta$ ) estimates obtained to fit the SGT distribution to the returns of 6 indexes (the French CAC40, UK FTSE100, US Dow Jones Composite Average, S&P 500, the Japanese Nikkei and Hong Kong Hang Seng). These estimations were obtained to fit a SGT distribution in a recursive way roller the sample forward one day at a time, each time re-estimating the model parameters.





