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On the Finite-Sample Biases in Nonparametric Testing for Variance Constancy[1]

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Abstract

In this paper we analyse the small-sample size distortions of nonparametric CUSUM tests for variance stability resulting from the long-run variance estimation. The long-run variance estimator is a key factor necessary to ensure asymptotically pivotal test statistics. We discuss the large sample properties of these tests under standard and recently developed fixed- b bandwidth asymptotic theory for kernel heteroskedasticity and autocorrelation consistent (HAC) estimators, and analyse the finite sample performance for different data generation processes of major empirical relevance. Despite the good properties evidenced by the large-sample theory, important distortions may arise when the empirical processes exhibit strongly-persistent volatility and excess kurtosis even in relatively large samples. In this context, consistent (inconsistent) HAC estimators may lead to over-sized (under-sized) tests. Hence, nonparametric tests may lack power to distinguish between a strongly persistent -yet stationary- process and a process with a structural break.

JEL Classification: C12, C15, C52
Keywords: CUSUM tests, HAC estimators, variance constancy, structural breaks.

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1. Introduction

Detecting breaks in variance is a topic which is receiving increasing attention in recent literature. The statistical methods specifically designed to analyse variance constancy and which have been widely used in the applied framework are based on the cumulative sum (CUSUM) of squares test.[4] The widespread use of these tests is not only due to its tractability and simplicity, but mainly to its statistical appeal. In the application of this procedure, no previous knowledge of the timing of the shift is required, and given that the CUSUM principle does not specify a particular pattern of variation, the tests have non-trivial local power against several alternatives, including parameter instability and distribution changes. Furthermore, the class of nonparametric tests ensures asymptotic invariance against a fairly general class of generating processes. These properties make this procedure well-suited for empirical applications on financial variables, which typically involve conditional heteroskedasticity patterns of unknown form and non-normal distributions.

These compelling properties are achieved by means of a consistent estimation of the spectral density at frequency zero of the usual proxies for variance, the so-called long-run variance (LRV) estimation. Whereas the test size and other statistical properties may be asymptotically correct in presence of short-run dynamics or conditional heteroskedasticity, LRV estimation turn out to be particularly inefficient in samples that exhibit (realistic) forms of strong dependence. Hence, significant biases may arise and translate into size distortions. Given that the observed measures of variability are characteristically persistent in financial data, detecting structural breaks accurately through nonparametric CUSUM-type tests may be particularly problematic even in the relatively large samples that are usually available for financial variables. Furthermore, the maintained assumption of fourth-order finiteness, necessary to show weak convergence of the test statistic to the standard distribution, may turn out to be too restrictive in practice and lead to further complications.

In this paper, we conduct theoretical and experimental analyses to characterize the size distortions due to the LRV estimation which may arise in testing unconditional variance constancy through nonparametric CUSUM tests. We discuss the asymptotic properties of these tests given kernel heteroskedasticity and autocorrelation consistent (HAC) estimators under both the conventional and the recently developed fixed-bandwith asymptotic theory (so-called fixed- b asymptotics) proposed in Kiefer, Vogelsang and Bunzel (2000), and Kiefer and Vogelsang (2002, 2005). The latter strategy delivers an inconsistent HAC estimate that, nevertheless, may lead to better testing properties compared to the standard

[4]Into this category fall the tests discussed, among others, in Brown, Durbin and Evans (1975), Ploberger (1989), Ploberger and Krämer (1986), Pagan and Schwertz (1990), Loretan and Phillips (1994), Inclán and Tiao (1994), Kokoszka and Leipus (1998, 2000), Kim, Cho and Lee (2000), Lee and Park (2001), Sansó, Aragó and Carrión (2004), Chen, Choi and Zhou (2005), Deng and Perron (2006), and Rodrigues and Rubia (2006). Alternative procedures to CUSUM testing are discussed in Andreou and Ghysels (2004) and Horváth, Kokoszca and Zhang (2006).

consistent approach. Hence, it merits attention to examine whether these estimates provide an useful alternative in the particular context of CUSUM testing. Finally, we also analyze the finite-sample size properties of these classes of estimators under the realistic case of volatility persistence. In particular, we consider a wide parametric range of GARCH and stochastic volatility models. Our interest in these models is that they are of major relevance for practical volatility modelling and have different probabilistic properties. HAC estimation is addressed in terms of $i)$ consistent deterministic-based HAC estimators, $ii)$ consistent data-dependent (random) HAC estimators, and $iii)$ fixed- b HAC estimators.

This papers shows under general assumptions that the CUSUM test has correct asymptotic properties, namely, invariant asymptotic distribution and consistency to date (multiple) breaks, both for consistent and inconsistent estimators. However, the finite-sample properties of nonparametric HAC-based CUSUM tests do not conform with the expected asymptotic approximation when the volatility process is strongly conditionally heteroskedastic and the data shows excess kurtosis, which is exactly the context of interest for high-frequency financial variables. The important implication is that nonparametric CUSUM tests may lack power to disentangle whether a seemingly strong dependence in volatility is an artifact due to some form of nonstationarity, such as breaks, or the actual consequence of some form of long-range dependence.
The rest of the paper is organized as follows. In Section two we introduce the theoretical framework of the paper and discuss the asymptotic properties of nonparametric CUSUM tests given consistent and inconsistent estimates of the LRV parameter. In Section three we describe the major features of the experimental design. In Section four we report the finite sample performance from a Monte Carlo investigation. Finally, Section five summarizes and concludes.

In what follows, '$\overset{p}{\rightarrow}$' denotes convergence in probability, while '$\overset{d}{\rightarrow}$' denotes weak convergence of the associate probability measures as the sample size diverges. The conventional notation $O_p(1)$ $(o_p(1))$ is used to represent a series of random numbers that are bounded (converge to zero) in probability; the notation $[\cdot]$ and $\mathbb{I}_{(\cdot)}$ is used to denote the greatest integer and the indicator functions, respectively. Finally, the limiting forms of the test statistics appearing in the paper are functionals of standard Wiener processes with time parameter $0 \leq \tau \leq 1$, denoted as $W(\tau),$ and Brownian bridge processes, $W^*(\tau) = W(\tau) - \tau W(1).$

2. Testing for structural breaks in variance: large-sample theory

The CUSUM principle tests parameter constancy against the alternative hypothesis of a single break at an unknown date. This can be inferred endogenously by means of a test statistic $\mathcal{T}_T = \varphi(a_T R_T(k))$, given a suitable scaling factor a_T and an empirical processes $R_T(k)$ that takes values in the space of right-continuous functions on [0,1] with left-hand limits (i.e., the space D [0,1]). The continuous functional, $\varphi : (D[0;1]) \rightarrow R^+$, considered when testing variance stability is typically $\varphi(f) = \sup_{0\leq\tau\leq 1}|f(\tau)|$, being other mappings, such as $\varphi(f) = \int_0^1 f(\tau)^2 d\tau$, possible when addressing mean stability. Under appropiate conditions, $\mathcal{T}_T$ has non-trivial asymptotic power against fixed and contiguous alternatives and converges weakly towards $\varphi(W^*(\tau))$ as $T \rightarrow \infty$.

This section aims to characterize a general set of sufficient conditions to discuss the asymptotic properties of a nonparametric test statistisc, $\mathcal{T}_T$, for variance stability. [5] Parametrically testing for changes in volatility involves imposing *a priori* beliefs on the data, which may lead to wrong inference under misspecification. The different papers that have focused on nonparametric testing consider the same statistic, and differ mainly in the set of basic assumptions. We are interested in preserving the nonparametric nature of the CUSUM testing framework, and discuss a fairly general class of encompassing restrictions. It is worth remarking that in testing for breaks in variance, $\mathcal{T}_T$ is normally defined on the squared- or absolute-valued transformations of centered variables, as these convey unbiased information about the dynamics of the second-order moment. Hence, our assumptions basically aim to ensure the existence of a functional central limit theorem (FCLT) for these series. We introduce the notation that shall be used throughout the paper and characterize the sufficient conditions in Assumption $\mathcal{A}$ below.

[5] Finite-sample distributions are only available under strong distribution assumptions which are not realistic for practical purposes on financial data.

Assumption $\mathcal{A}$

Let $\{r_t\}$ *be a real-valued process and* $\mathcal{F}_t = \sigma(r_s, s \le t)$ *its natural filtration.* *Given a* $\mathcal{F}_t$ *-measurable function* $s(\bullet)$ *such that the random variable* $X_{t,s} \coloneqq s(r_t)$ *is integrable, define the centered series* $X^c_{t,s} = X_{t,s} - \sigma^2_s$, $\lim_{T\to\infty} T^{-1}\sum_{t=1}^{T} X_{t,s} \xrightarrow{p} \sigma^2_s > 0$, *and denote* $\gamma_{j,s} = E\left(X^c_{t,s}X^c_{t-j,s}\right)$. *Then:*

i) $E(r_t|\mathcal{F}_{t-1}) = 0$, *almost surely, and* $\sup_t E(|r_t|^{4+\delta}) < \infty$ *for some* $\delta > 0$.

ii) $\sum_{j=-\infty}^{\infty}|\gamma_{j,s}| < \infty$ *and* $\sum_{j=-\infty}^{\infty}\gamma_{j,s} = \mathcal{M}^2_s$, *for a constant* $\mathcal{M}_s > 0$.

iii) For any $0 \le \tau \le 1$, $\lim_{T\to\infty} T^{-1/2}\sum_{t=1}^{[T\tau]} X^c_{t,s} \xrightarrow{d} \mathcal{M}_s W(\tau)$.

iv) The cumulants $\kappa(h,r,p)$ *of* $X_{t,s}$ *satisfy* $\sup_h \sum_{r,s=-\infty}^{\infty}|\kappa(h,r,p)| < \infty$.

Some comments on these assumptions follow. The martingale difference assumption in $i)$ is slightly stronger than necessary but it is very convenient. It should furthermore be noticed that this is not restrictive, since on the one hand, a demeaning regression may be used (see *e.g.* Deng and Perron, 2006, Chen *et al.*, 2005), and, on the other hand, most series of financial returns verify this condition directly. The random variables $X_{t,s} = s(r_t)$ for $s(x) = |x|^u$ and $s(x) = \log(|x|^u)$, $u = 1,2,$ are empirical proxies for the conditional variance and have been used in CUSUM testing (see *e.g.* Andreou and Ghysels, 2002), with r_t^2 being the `natural' approach in the literature. We shall give special attention to the squared transformation, $X_{t,2} \coloneqq r_t^2$, and its related test statistic, $\mathcal{T}_{T,2}$. Assumptions $ii)$ and $iii)$ bound the spectral density of $X_{t,s}$ at the zero frequency, require short-memory, and give rise to a FCLT applicable to the partial sums of $X^c_{t,s}$. A suitable FCLT holds under a variety of regularity conditions on r_t (*e.g.*, mixing conditions), and is very general in the sense of allowing for heteroskedasticy and serial dependence.[6] Major examples of time-series to bear in mind for the purpose of this paper are GARCH-type and stochastic volatility models under suitable parametric restrictions. Assumption $iv)$ is a technichal restriction to ensure consistent estimation of $\mathcal{M}^2_s$ by means of standard nonparametric techniques. It is implied, for instance, if $iii)$ holds from mixing conditions.
The CUSUM test for variance stabilitiy is computed on an empirical process defined on $X_{t,s} = s(r_t)$ as follows:

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[6]Assumption $iii)$ holds in general settings which model dependence under more primary conditions. The theoretical literature has concentrated on the squared transformation. Suitable conditions include, among others, mixing conditions (*e.g.*, Sansó *et al.*, 2004, Deng and Perron 2006), linear processes (Loretan and Phillips 1994). Also, given assumptions $i)$ and $ii)$, the FCLT for r_t^2 would also hold if r_t is L_2 -NED of size $-(1+1/2\delta)$ on a strong-mixing basis.

$$\mathcal{R}_{T,s}(k) = \frac{G_{T,s}(k)}{\sqrt{T}\widehat{\mathcal{M}}_{T,s}}, \quad k = 1,\ldots,T \tag{1}$$

with

$$G_{T,s}(k) = \left(\sum_{t=1}^{k} X_{t,s} - (k/T)\sum_{t=1}^{T} X_{t,s}\right) \tag{2}$$

and $\widehat{\mathcal{M}}_{T,s}$ being an estimator of the (squared-root) LRV parameter $\mathcal{M}_s$. The test statistic in (KLk) combines the testing and estimation in a single step and infers the break-point location, say $\hat{k}_{T,s}$, automatically whenever the null hypothesis is rejected. In particular, variance constancy is rejected for large values of:

$$\mathcal{T}_{T,s} = \max_{1\le k\le T}|\mathcal{R}_{T,s}(k)| \tag{3}$$

and gives rise to $\hat{k}_{T,s} = \arg\max|\mathcal{R}_{T,s}(k)|$. Although the procedure presented is intended for detecting a single break, it can readily be generalized to gain power against multiple breaks; see *e.g.* Inclán and Tiao (1980), Sansó *et al.,* (2002), and Chen *et al.,* (2005). In this case, an iterative algorithm is used to evaluate the test statistics at different parts of the total sample, dividing consecutively after a possible change point is located. The basic asymptotic results under Assumption $\mathcal{A}$ are provided as lemmas below.

Proposition 2.1. *Given the time-series* $\{r_t\}_{t=1}^{T}$ *, assume that the assumptions in* $\mathcal{A}$ *hold true and define* $\mathcal{R}_{T,s}(k)$ *as in (1). Then, for any* $\widehat{\mathcal{M}}_{T,s}$ *such that* $\widehat{\mathcal{M}}_{T,s} - \mathcal{M}_s = o_p(1)$ *, under the null hypothesis* $H_0 : E(r_t^2) = \sigma^2$ *for all* $t \geq 1$, *as* $T \to \infty$, *it follows that,*

$$\max_{1\le k\le T}|\mathcal{R}_{T,s}(k)| \xrightarrow{d} \sup_{\tau\in[0,1]}|W^*(\tau)| \tag{4}$$

where $W^*(\tau) = W(\tau) - \tau W(1)$.

Proof. See appendix.

The restrictions $ii)$ and $iv)$ in Assumption $\mathcal{A}$ allow for consistent estimation of the unknown LRV parameter, $\mathcal{M}_s^2$, by means of any kernel HAC or other spectral estimator. This issue will be discussed more carefuly in the following subsection. The important result to note from this proposition is that a consistent LRV estimator provides the necessary standardization such that the formally unknown dependence structure in the volatility process (such as GARCH-type, stochastic volatility, or any other short-memory volatility admissible model) is not a concern as the sample size is allowed to diverge, even if there exists a considerable degree of dependence or heterogeneity. The critical values are obtained from the *supremum* of a standard Brownian Bridge (SSBB henceforth), whose relevant percentiles are 1.22 (90%), 1.36(95%) and 1.63 (99%). The empirical suitability of these values depends critically on the assumptions of a) fourth-order finiteness, and b) short memory in the sense $\sum_{j=-\infty}^{\infty}|\gamma_{j,s}|< \infty$. Both conditions are not prerequisites for weak convergence as such, but without them the limit distribution cannot formally be stated in terms of functionals of Wiener processes and, therefore, the SSBB does not have theoretical support. Since such conditions may not be verified in real settings, and because we shall use volatility processes, it is interesting to briefly discuss the effects of relaxing these conditions. We discuss the most relevant case for $X_{t,2}$.

Remark 2.1a). If $E(|r_t|^{\delta}) < \infty$, $\delta > 4$, is weakened by instead requiring $E(|r_t|^{\bar{\alpha}}) < \infty$, $2 < \bar{\alpha} < 4$, then $\mathcal{M}_2$ diverges, and so does $\widehat{\mathcal{M}}_{T,2}$ and $T^{-1/2}G_{T,2}(k)$. Nevertheless, under the additional constraint that r_t lies in the domain of atraction of a normal distribution, then an alternative FCLT holds (see *e.g.* Loretan and Phillips 1994). For a kernel HAC-based estimator $\widehat{\mathcal{M}}_{T,2}$ in which the bandwidth $l_T \to \infty$ in such a way that $l_T/T \to 0$, then $\mathcal{R}_{T,2}(k) \xrightarrow{d} L_{\bar{\alpha}/2}(\tau) := U^{*}_{\bar{\alpha}/2}(\tau)\left(\int_0^1 (dU_{\bar{\alpha}/2}(\tau))^2\right)^{-1/2}$, with $U_{\alpha}(\tau)$ being a Lévy α -stable process on $D[0,1]$, and $U^{*}_{\bar{\alpha}/2}(\tau) = U_{\bar{\alpha}/2}(\tau) - \tau U_{\bar{\alpha}/2}(1)$. Hence, $\mathcal{T}_{T,2} \xrightarrow{d} \sup_{\tau\in[0,1]}|L_{\bar{\alpha}/2}(\tau)|$, an even though $\mathcal{T}_{T,2}$ still converges to a well-defined distribution, the critical values from SSBB will no longer be correct because the asymptotic distribution depends on the maximal moment exponent $\bar{\alpha}$ (as $\bar{\alpha}$ is smaller, so is the tail distribution). Consequently, size distortions may be expected if resorting to the `standard' critical values even if having arbitrarily large series.

Remark 2.1b). The short-memory condition $\sum_{j=-\infty}^{\infty}|\gamma_{j,s}|< \infty$ rules out long-range dependence in volatility such as (fractional) integration. Nevertheless, these patterns have received considerable attention in the literature because $|r_t|^u$ often shows slowly-decaying correlations which may be consistent with some form of long-range dependence; see, among others, Ding, Granger and Engle (1993). If $\gamma_{j,2} \sim cj^{2d-1}$ as $j \to \infty$ for some $0 < d < 1/2$, $c > 0$, then the $X_{t,2}$ series has a slowly decaying autocorrelation fuction, unbounded spectrum at zero frequency and $\sum_{j=-\infty}^{\infty}|\gamma_{j,2}|= \infty$, but it exhibits so-called stationary long-memory and therefore a FCLT may still be applicable provided fourth-order finiteness; see Giraitis, Robinson and Surgailis (2000), and Giraitis *et al.*, (2003).[7] Under these conditions, it could be shown for a standard kernel HAC estimation with bandwidth

[7]Long-memory models that allow for stationarity and finite moments are discussed, among others, in Robinson (1991) and Davidson (2004). On the other hand, under IGARCH or FIGARCH patterns (an extension of standard GARCH models to include long-range persistence in the same spirit of ARIMA and ARFIMA models), then $\sum_{j=-\infty}^{\infty}|\gamma_{j,2}|$ diverges and so does $E(r_t^u)$ for any $u \geq 2$.

l_T $(l_T/T \to 0)$ that $T^{-d}\mathcal{R}_{T,2}(k) = O_p(1)$, and $(Tl_T)^{-d}\mathcal{T}_{T,2} \xrightarrow{d} \sup_{0\leq\tau\leq 1}|W_H^*|$, with $W_H^* = W_H(\tau) - \tau W_H(1)$ and $W_H(\tau)$ being a fractional Wiener process with parameter (Hurst coefficient) $H = 1/2 + d$ on $D[0,1]$. Hence, $\mathcal{T}_{T,2}$ diverges, and using the critical values from the SSBB would lead to size departures. Similar results are shown under integrated GARCH models (IGARCH); see Sansó *et al.*, (2004).

Proposition 2.2. *Let* $\hat{\tau}_T$ *be the estimator of the relative break-point,* i.e., $\hat{\tau}_{T,s} = \left[\hat{k}_{T,s}/T\right]$ *with* $\hat{k}_{T,s} = \arg\max_k|\mathcal{R}_{T,s}(k)|$ *. Assume that a permanent break occurs at time* $k_0 = [\tau_0 T]$, $0 < \tau_0 < 1$, *such that* $E(r_t^2) = \sigma_r^2(1 + \Delta_T \mathbb{I}_{t\geq k})$. *If sup* $_tE(\varepsilon_t^\delta) < \infty$ *for* $\delta > 4$, $\sum_{j=-\infty}^{\infty}|\gamma_{j,s}| < \infty$, *and if* $\Delta_T = \Delta$ *(fixed alternative) or* $\Delta_T \to 0$ *as* $T \to \infty$ *such that* $T^{1/2}\Delta_T \to \infty$ *(local alternative) then:*

$$\hat{\tau}_{T,s} \xrightarrow{p} \tau_0. \qquad (5)$$

Proof. See appendix.

Remark 2.2. Short-run dynamics do not interfere in the (asymptotic) ability of the test to date the break consistently under standard conditions. Similar results arise when considering multiple breaks. Hence, under large-sample theory, the nonparametric CUSUM tests achieve correct properties.

2.1. Long-run variance parameter estimation

The LRV estimator plays a critical role in the CUSUM testing as it is necessary to provide the standardization which guarantees valid inference (*i.e.*, free of nuisance parameters) in a model-free theoretical framework. The spectral density at frequency zero can be estimated by means of nonparametric kernel estimators of the general form

$$\widehat{\mathcal{M}}_{T,s}^2 = \sum_{j=-T+1}^{T-1} \omega(jl_T^{-1})T^{-1} \sum_{t=|j|+1}^{T} \hat{X}_{t,s}^c \hat{X}_{t-|j|,s}^c \qquad (6)$$

with $\hat{X}_{t,s}^c = X_{t,s} - T^{-1}\sum_{t=1}^T X_{t,s}$, and given any suitable kernel weighting function $\omega(\cdot)$, with bandwidth parameter l_T. Leading examples of this include Newey and West (1987) and Andrews (1991). The kernel function ensures nonnegative estimates, while the main aim of l_T is to capture the covariance structure of the underlying series correctly.[8] As the

[8]The standard conditions on the kernel weighting funcion assume that this is a continuous, square-integrable,

temporal dependence increases --similarly, as the curvarture of the spectral density at the zero frequency increases-- it is then necessary to account for a larger number of non-zero covariances through a larger value of l_T. Therefore, the correct choice of the bandwidth parameter becomes critical for the correct properties of HAC estimates. Various rules have been suggested in the literature for setting this parameter in empirical applications.

On the one hand, the standard setting considers that $l_T \to \infty$ as $T \to \infty$ in such way that $l_T/T \to 0$, *i.e.*, $l_T = o(T)$. This approach includes both determinist- and random-based choices of l_T (for which $l_T = o_P(T))$, and provides consistent estimates under fairly general assumptions on the underlying data generating process (DGP). Alternatively, Kiefer and Vogelsang (2002, 2005) have advocated for using HAC estimates with bandwidth determined under the rule $l_T/T \to b$, with $0 < b \leq 1$. Since b is held fixed even asymptotically, this approach has been labelled as fixed- b asymptotics. Under this approach, l_T is of order $O_P(T^2)$ and the resultant estimate is not consistent. The experimental analysis in Kiefer and Vogelsang (2002) shows that inference based on an estimator that uses the entire sample ($b = 1$) may result in better size performance than one based on traditional estimates for which $b \to 0$. However, recent results in Cai and Shintani (2006) in the context of unit-root testing show that size improvements may come at the cost of a loss in power. It merits attention to fully characterize the size properties of CUSUM tests for the general context. The asymptotic distribution of the CUSUM tests for variance stability is provided below.

Proposition 2.3. *Consider that the conditons in Assumption* $\mathcal{A}$ *hold true, the kernel HAC estimator* $\widehat{\mathcal{M}}^2_{T,s} = \sum_{j=-T+1}^{T-1} \omega(jl_T^{-1})\hat{\gamma}_{j,s}$ *with bandwidth* $l_T = [bT^a]$, $0 < b \leq 1$, *and the kernel function* $\omega(\cdot)$ *satisfying usual restrictions* $\cdot$ *Then, as* $T \to \infty$,

i) if $a < 1$, $\widehat{\mathcal{M}}^2_{T,s} - \mathcal{M}^2_s = o_P(1)$ *and* $\mathcal{R}_{T,s}(k) \to W^*(\tau)$.

ii) if $a = 1$, $\widehat{\mathcal{M}}^2_{T,s} - \mathcal{M}^2_s = O_P(1)$ *and* $\mathcal{R}_{T,s}(k) \to H_\omega(\tau;b)$,

where $H_\omega(\tau;b)$ *is a known functional on D* $[0,1]$ *which does not depend on unknown parameters. Hence, for* $a < 1$ *, the asymptotic distribution of* $\mathcal{T}_{T,s}$ *has the structure discussed in Proposition 2.1, whereas* $\mathcal{T}_{T,s} \xrightarrow{d} \sup_{0\leq\tau\leq1}|H_\omega(\tau;b)|$ *if* $a = 1$.

Proof. See Appendix.

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and simmetric function such that $\omega(\cdot) : [0,\infty) \to [-1,1]$. We maintain these assumptions througout the paper. Note that the resultant HAC estimator and asymptotic properties depends on both the kernel function and the bandwidth parameter, say $\widehat{\mathcal{M}}^2_{T,s}(\omega, l_T)$. Throughout this paper, we will refer to $\widehat{\mathcal{M}}^2_{T,s}(\omega, l_T)$, for simplicity of notation, as $\widehat{\mathcal{M}}^2_{T,s}$.

Consequently, the limit distribution of $\mathcal{T}_{T,s}$ under fixed- b bandwidth choices depends on the particular kernel function as well as on the ratio l_T/T, which are arbitrarily set by the researcher. The limit distribution differs from the SSBB because now it depends on the asymptotic bias resulting from the inconsistent HAC estimator. These features characterize the shape of $H_\omega(\tau;b)$. For instance, in the case of the Bartlett kernel $\omega(x) = (1-|x|)\mathbb{I}_{x\leq 1}$, the functional $H_\omega(\tau;b)$ is given by the ratio $W^*(\tau)Q^{-1/2}(b)$, with $Q(b) = (2/b)\int_0^1 W^*(r)^2 dr - (2/b)\int_0^{1-b} W^*(r+b)W^*(r)dr$. If the whole sample is used in the estimation, *i.e.*, $l_T = T$, the limit functional reduces to $W^*(\tau)\left(2\int_0^1 W^*(r)^2 dr\right)^{-1/2}$, and the asymptotic distribution of $\mathcal{R}_{T,s}(k)$ is equivalent to the `bias-corrected' statistic $\mathcal{R}'_{T,s}(k) = T^{1/2}G_{T,s}(k)/\sqrt{2\mathcal{S}_T}$, with $\mathcal{S}_T = \sum_{k=1}^{T}\left(\sum_{t=1}^{k}\hat{X}_{t,s}^c\right)^2$. Note that $\mathcal{S}_T$ is an important statistic in the context of stationarity tests, such the well-known KPSS test of Kwiatkowski *et al*., (1992), and the rescaled range (R/S) test of Lo (1991). Apart from changing the asymptotic distribution, using an inconsistent LRV estimator does not affect the consistency of the break-point identification.[9] After analyzing the relevant theory, we can discuss the finite sample properties of the test.

3. Small-sample analysis. Experiment design

In this section we carry out Monte Carlo simulations that examine the size performance of the nonparametric CUSUM tests in small-samples. As finite sample distortions are ultimately an empirical question, we focus on experimental designs based on DGPs of empirical relevance as well as on popular methods to determine the bandwidth parameter in the HAC estimation. We first describe the methods to determine l_T for the kernel HAC estimation in our simulations, and then review the main characteristics of the DGP used in the simulations.

It is widely accepted that the choice of the kernel function is not critical, so for simplicity we only consider the popular Bartlett kernel [BK henceforth] routinely applied in a number of econometric packages.[10] For setting the bandwidth for this kernel, we consider two popular rules belonging to the class of consistent HAC estimators (namely, a deterministic and a data-dependent method) and consider the full-sample bandwidth $l_T = T$ as for the

[9]Note that $\hat{k}_{T,s} = \arg\max_k|\mathcal{R}_{T,s}(k)|$ is scale-invariant, so the test would identify the break-location provided the standard conditions in Proposition 2.2, although correct inference requires the use of the correct critical values, now given by $\sup_{\tau\in[0,1]}|H_\omega(\tau;b)|$. The formal proof is not presented but it is available upon request.

[10]The results we shall discuss do not hingue upon the choice of the kernel function. For instance, similar results arise when using the Quadratic Spectral kernel. Complementary results are available upon request.

inconsistent class (we shall refer to it as l_T^b -rule in what follows) . Deterministic rules simply set $l_T = [bT^a]$, for $b > 0$ and $0 < a < 1.$ A popular choice is $l_T = \left[4(T/100)^{2/9}\right],$ which will be refer to as l_T^d -rule (deterministic). Data-dependent methods try to determine an `optimal' truncation lag as a function on the correlation structure rather than the available number of observation; *e.g.*, Andrews (1991) and Newey and West (1994). We consider the Newey and West's (1994) method for the Bartlett kernel, for which $l_T = \min\left\{T,\ [\theta T^{1/3}]\right\},\quad \theta = 1.1447\left(S_1^2/S_0^2\right)^{1/3},$ where $S_0 = \hat{\gamma}_0 + 2\sum_{j=1}^{l_T^*} \hat{\gamma}_{j,2}$ and $S_j = 2\sum_{i=1}^{l_T^*} i^j \hat{\gamma}_{j,2}$ and given a deterministic pre-bandwidth parameter l_T^* .[11] We shall refer to this as l_T^r -rule (random) and use $l_T^* = l_T^d$ as pre-bandwidth in our simulations.

As discussed previously, the conventional critical values from CUSUM variance tests are formally justified solely for fourth-order weakly dependent processes. Consequently, all our DGPs are defined on the basis of short-memory volatility models. Otherwise, size distortions may be due to the use of an incorrect asymptotic distribution. Since financial returns and other economic variables observed on a daily or weekly basis are characterized by high kurtosis and volatility dependence, we focus on models able to generate these effects parsimoniously, and consider GARCH(1,1) and stochastic volatility (SV) models under suitable parametric restrictions.

In particular, the simulated series are generated from,

$$r_t = \sigma_t(\theta)\eta_t,\ \ t = 1,\ldots,T \tag{7}$$

with $\eta_t \sim iid\mathcal{N}(0,1),\quad \sigma_t(\theta)$ being the volatility process (indepedent of η_t) given a set of parameters $\theta.$ The returns are simulated by generating 25000 series of pseudo-random numbers, $\eta_t,$ with sample size $T = 1000$, for the GARCH and SV volatility specifications, and for different parameter configurations $\theta.$ This sample length seeks a realistic compromise between the number of observations in daily sets of data, and the much more limited one which is typically available at lower frequencies. The empirical size is computed for $\mathcal{T}_{T,2}$ on the basis of squared-series $\{r_t^2\}.$ We concentrated on this approach since no qualitative differences arose for other proxies of variance, and because this is also the standard approach in applied settings and the theoretical properties of GARCH and SV models are better known for it. We describe the main setting of the GARCH and SV model below.

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[11]Here l_T becomes a stochastic variable in itself, which is determined by the characteristics of the data under some optimality criterion related to asymptotic mean squared error formulas. Although not reported, it must be noted that this approach yields qualitatively similar results to those based on pre-whitening and alternative data-dependent strategies, such as Andrews' (1991), since they essentially attempt to determine a HAC estimator as a function of the characteristics of the data. Results for these rules are available from the authors upon request.

3.1. The GARCH(1,1) model

The conditional variance in GARCH models is defined as follows:

$$\sigma_t^2(\theta) = \omega + \alpha r_{t-1}^2 + \beta\sigma_{t-1}^2(\theta) \quad (8)$$

with $\omega > 0,\ \alpha,\beta \geq 0$ to ensure almost-surely positiveness. The statistical properties of this process are well-known in the econometric literature. The parameter $\phi \coloneqq \alpha + \beta$ measures the persistence of the process, with $\{r_t^2\}$ showing an autocorrelation pattern as an ARMA(1,1) model with autoregressive root ϕ . The condition $\phi < 1$ is necessay and sufficient for second-order stationarity, and ensures stationarity and ergodicity. The more restrictive condition

$$\kappa\alpha^2 + 2\alpha\beta + \beta^2 < 1;\ E(\eta_t^4) = \kappa < \infty \quad (9)$$

is necessary and sufficient for the existence of fourth-order moments. As the latter is clearly more restrictive, it follows that $\{r_t, \sigma_t^2\}$ is strict stationary and ergodic when this restriction is verified. Furthermore, Carrasco and Chen (2003) shows that stationary GARCH processes are β -mixing if $\phi < 1$, which in turn ensures the suitability of Assumption $\mathcal{A}$.

In order to carry out simulations, we consider a parametric space which includes empirical values tipically observed in practice. In particular, we set α and ϕ in the range $\Phi_{\alpha,\phi}$: $[0.01, 0.20] \times [080, 0.99]$ with step-increments of size $i_0 = 0.01$. The unconditional variance of r_t is normalized through $\omega = 1 - \phi$. It is worth remarking that the key restriction $\phi < 1$ is always verified in $\Phi_{\alpha,\phi}$, but condition $\left(\text{ref: 4order}\right)$ may not be verified for some values (α, ϕ), in particular when ϕ is close to unity and α is relatively high (recall $\kappa = 3$ for the Gaussian distribution). As remarked before, the conventional critical values do not apply formally if $E(r_t^4) = \infty$, and hence any finite-sample bias may remain even asymptotically. The region in which $E(r_t^4)$ diverges will be signalled conveniently when presenting the results from simulations.

3.2. The Stochastic Volatility model [SV]

The conditional variability of SV models is given by:

$$\ln\sigma_t^2 = \xi \ln\sigma_{t-1}^2 + v_t \quad (10)$$

with $v_t \sim iid\mathcal{N}(0, \sigma_v^2)$ being independent of η_t. The main statistical properties of SV models are reviewed in Ghysels *et al.*, (1996). In particular, condition $|\xi| < 1$ is required for stationarity and ergodicity $\cdot$ Furthermore, in contrast to the GARCH model, if $\kappa < \infty$, $|\xi| < 1$ suffices by itself to ensure $E(r_t^4) < \infty$ without the need for further restricting the parameter space. Carrasco and Chen (2003) show that a stationary SV process is also β -mixing, which again ensures the suitability of the asymptotics discussed in this paper.

As in the case of GARCH models, there exists a strong link between persistence, correlation and kurtosis in SV models. The parameter ξ has also a major influence on the correlation structure of $\{r_t^2\}$, which, in contrast to GARCH models, can show very different patterns as a function of the noise-variability σ_v. This parameter also controls the degree of mixing independently of the degree of smoothness of the conditional variance evolution (Ghysels *et al.*, 1996). Therefore, it seems reasonably to characterize the SV dynamics experimentally through the joint values of (σ_v, ξ).

In order to carry out simulations, the values of (σ_v, ξ) are taken from a parameter space which includes empirical values tipically observed in practice. In particular, we focus on the range $\Phi_{\sigma_v,\xi} : [0.01, 0.60] \times [080, 0.99]$, with steps $i_0 = 0.01$. Note that all the theoretical restrictions that give support to the critical values from the SSBB distribution hold for the $\Phi_{\sigma_v,\xi}$ considered. This feature allows us to obtain further insight into the context in which strong persistence coexists with large degrees of kurtosis such that $E(r_t^4)$ exists, which is not possible in GARCH models.

5. Experiment results

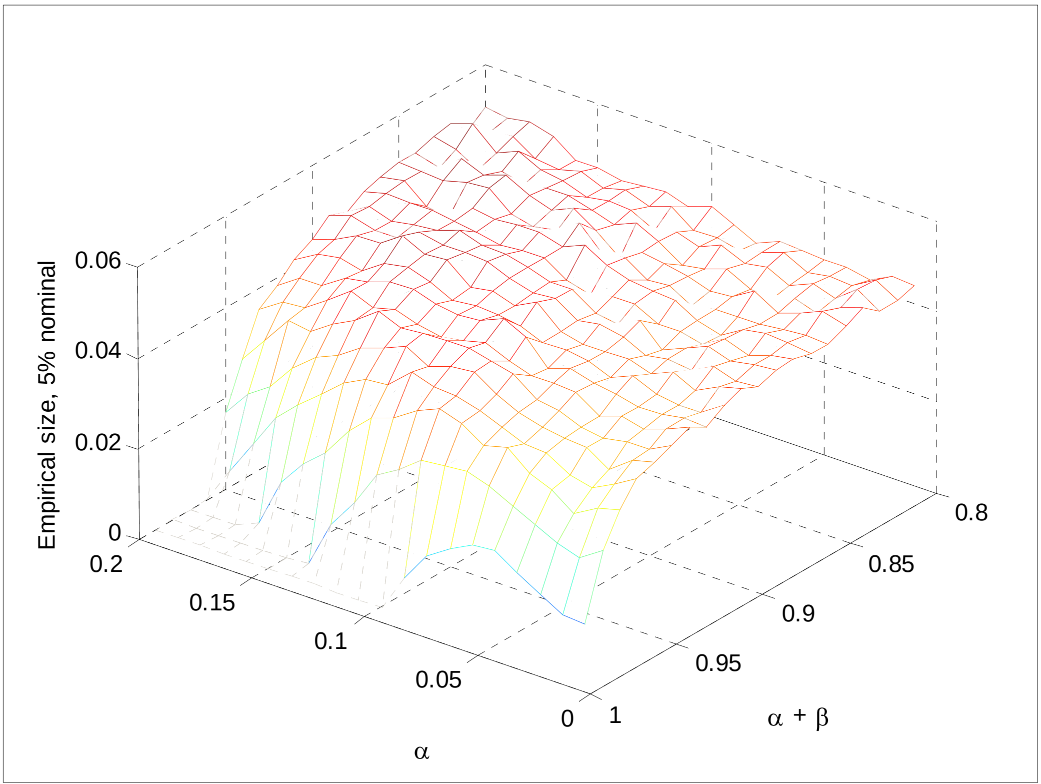
The GARCH(1,1) model

Before presenting the simulation evidence on the performance of the nonparametric CUSUM test against GARCH errors, let us characterize the size distortions that arise when the population $\mathcal{M}_2$ has *not* to be estimated. For fourth-order stationary GARCH models, Kim, Cho and Lee (2000) discussed a closed-form representation of the population LRV of r_t^2 , which is given by

$$\mathcal{M}_2^2 = (\kappa - 1)\omega^2 \frac{(1+\phi)(1+3\alpha^2+\phi^2-4\alpha\phi)}{(1-\phi)^3(1-\phi^2-(\kappa-1)\alpha^2)}. \qquad (11)$$

Therefore, for values (α, ϕ, κ) such that $\mathcal{M}_2$ is well-defined, we can exploit our knowledge of the experimental DGP and use the true population parameter in the CUSUM test. In this experiment, any size distortion comes from the fact that $T^{-1/2}G_{T,s}(k)$ finds its representation as a Brownian bridge in the asymptotic sample. This strategy allows us to define a benchmark free of estimation biases, which is useful to characterize and isolate the effects in the real context in which $\mathcal{M}_2$ must be estimated. The rate of rejection of the null hypothesis at a 5% nominal size is plotted in Figure 1.

Figure 1. Empirical size (asymptotic 5% level) for the nonparametric CUSUM test with GARCH errors and population LRV parameter. The values (α, ϕ) for which the LRV of $\{r_t^2\}$ is not well-defined are plotted in dashed lines.



The simulation shows that $\mathcal{T}_{T,2}$ achieves aproximately correct values for the vast majority of DGPs simulated in our analysis. Under-sized inference is only observed as ϕ approaches unity, and in the (α, ϕ) -region in which $\mathcal{M}_2$ diverges for Gaussian shocks. Note that the testing procedure remains asymptotically valid because $\phi < 1$ (*i.e.*, the drift towards under-sized inference will vanish completely if we allow $T \to \infty$), but it is shown that time-series with extremely persistent conditional heteroskedasticity require larger sample sizes to make precise inference compared to cases in which ϕ is relatively small.[12] Distortions appear as well when, for a fixed persistence ϕ, α is so high that $\mathcal{M}_2$

[12]Intuitively, the persistence is the reason underlying the changes in the rate of convergence which are displayed in Remark 2.1.b).

diverges. Note that increasing $\alpha,$ everything else constant, leads to larger kurtosis and larger autocorrelations, since then α measures the dependence between squared observations and can be interpreted as the parameter leading the volatility dynamics.[13] If volatility follows GARCH models, large combinations of (α, ϕ) represent the worst-case scenario for the critical values of the SSBB to provide a good finite sample approach, and we can expect further statistical complications when inferring $\mathcal{M}_2$. Overall, the picture that emerges from this analysis is that the empirical test size is fairly stable across most values of the DGP, with departures arising only when $E(r_t^4)$ and/or the memory of a process as measured by $\sum|\gamma_{j,2}|$ tends to diverge.

We now turn our attention to the practical case in which $\mathcal{M}_2$ must be inferred. The empirical rate of rejection of the null hypothesis at a 5% nominal level for (α, ϕ) is shown for the Bartlett kernel HAC estimators with bandwidth-rules l_T^d , l_T^r, and l_T^b , see Figures 2, 3, and 4, respectively (the values for which $E(r_t^4) = \infty$ are plotted using dashed lines). At the end of this section, in Table 1, we provide a summary of the results for both GARCH and SV models. We first comment the results for the class of consistent estimators based on the l_T^d derterministic rule.

[13]The autocorrelation function (ACF) of r_t^2 for a GARCH(1,1) model is given by $\rho_1 = \alpha(1 - \phi^2 + \phi\alpha)/(1 - \phi^2 + \alpha^2)$ and $\rho_j = \rho_1\phi^{j-1}, j > 1,$ showing an exponential decay to zero for $\phi < 1$.

Figure 2. Empirical size (asymptotic 5% level) for the nonparametric CUSUM test with GARCH errors and bandwidth parameter set by the l_T^d rule. The values (α, ϕ) for which the LRV of $\{r_t^2\}$ is not well-defined are plotted in dashed lines.

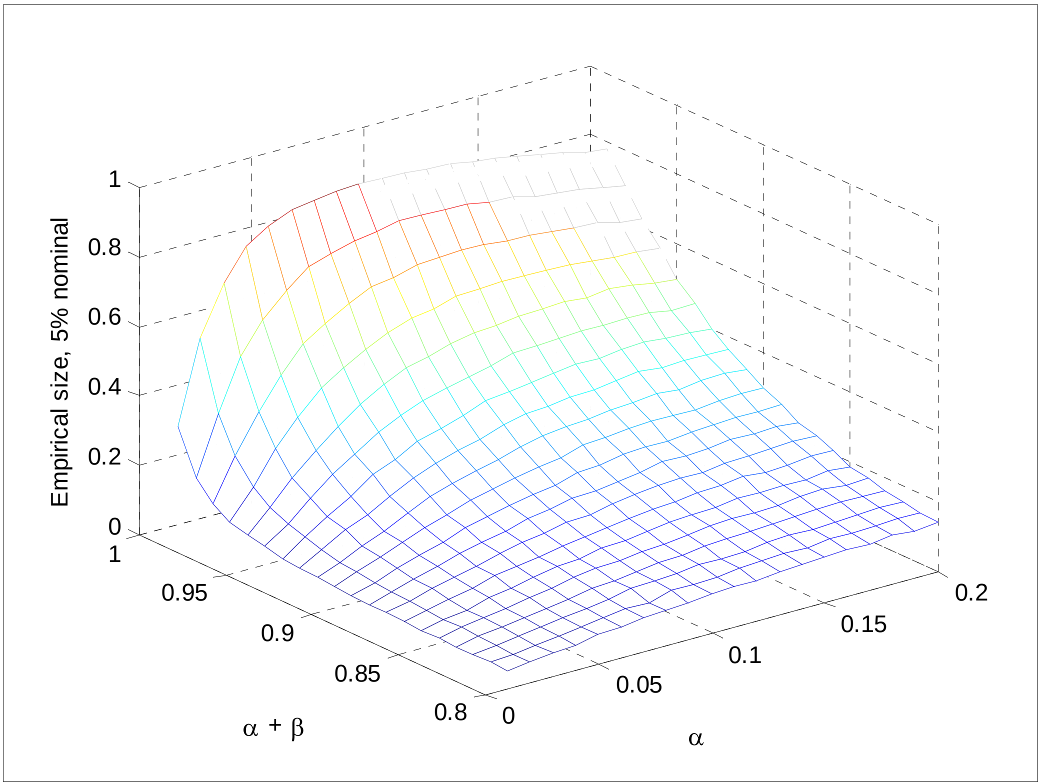
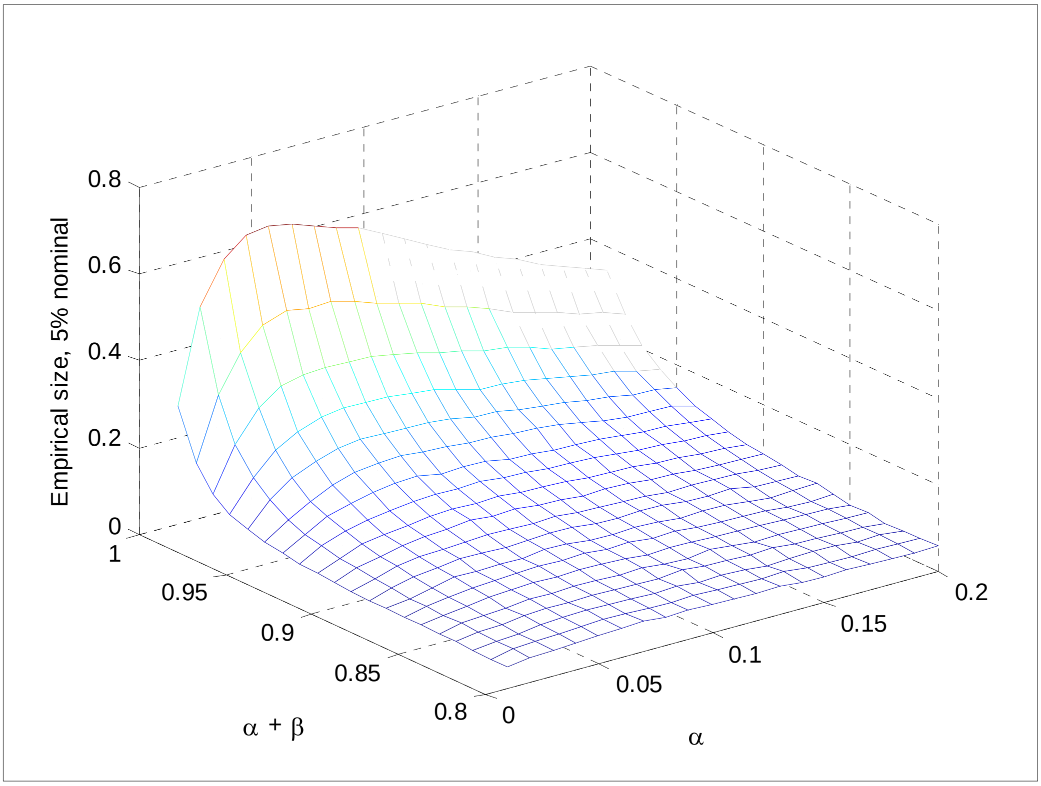


Figure 2 shows large differences between the empirical and the corresponding asymptotic size, given different combinations of values (α, ϕ) which lead to largely over-sized tests. In view of the previous analysis, this is mostly due to severe small-sample bias (underestimation) of the LRV parameter. The largest departures correspond to the case in which ϕ is close to unit (*i.e.*, nearly-integrated variance) and α is relatively high (excess kurtosis and high autocorrelation), such that $E(\varepsilon_t^4)$ is large, yet not necessarily of diverging magnitude. In fact, the shape of the empirical size envelope is convex on ϕ, and slightly concave on α, with the peak of nearly 88% corresponding to a theoretically admisible model $(\alpha = 0.09, \phi = 0.99)$. The empirical size is fairly sensitive and unstable in this area, with small changes in the driving parameters leading to large variations. For instance, for $(\alpha, \phi) = (0.01, 0.98)$ the probability of rejection exceedes 19% , and a sligth variation, $(\alpha', \phi) = (0.02, 0.98)$ or $(\alpha, \phi') = (0.01, 0.99)$ leads to empirical sizes around 32%, nearly doubling the extent of the actual size. Note that it is not unusual at all to find parameter estimates in this range. For instance, Carnero, Peña and Ruiz (2004) report average values of $\hat{\phi}_T \approx 0.98$ and $\hat{\alpha}_T \approx 0.05$ in its recent empirical application with Gaussian GARCH models on exchange rates. The actual size in testing for variance breaks under the l_T^d -rule for these values, and for a sample of $T = 1000,$ would be larger than 66% at a nominal 5% level.

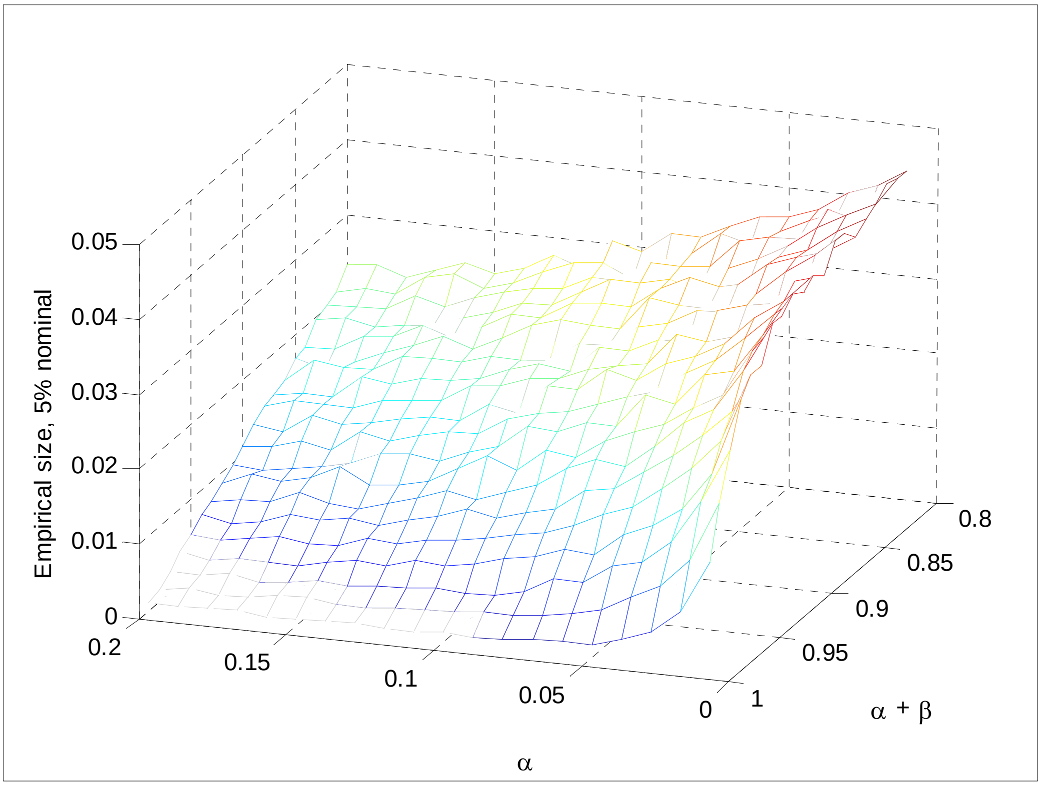
Figure 3. Empirical size (asymptotic 5% level) for the nonparametric CUSUM test with GARCH errors and bandwidth parameter set by the l_T^r rule. The values (α,ϕ) for which the LRV of $\{r_t^2\}$ is not well-defined are plotted in dashed lines.



Using a data-dependent bandwidth parameter has very important effects on improving the finite-sample performance of the CUSUM test, since this rule adds flexibility to the estimation and proves useful in reducing the downwards bias estimation. For small values of α the statistical gain is rather small, and the l_T^d - and l_T^r -rules provide similar results, although l_T^r shows enhacements as ϕ increases. However, as both ϕ and α increase, the empirical test size proves less convex on ϕ and remarkably less sensitive to α when using the l_T^r -rule . Maximal distortions are considerably smaller than before, and increases in persistence and kurtosis lead to overall smaller departures. Nevertheless, the extent of the size distortions for large values of (α,ϕ) is still uncontrolled. Interestingly, the overall degree of concavity on α of the empirical size function is larger than before, with the CUSUM test showing smaller size distortions in the area in which $E(r_t^4)$ diverges. The heuristic reason to explain the difference with respect to the deterministic case is that $\widehat{\mathcal{M}}_{T,2}$ is allowed to diverge faster under the l_T^r -rule when ϕ is very close to unity, and hence the empirical size distortions are attenuated. Note that, whereas the size distortions in the region $E(r_t^4) = \infty$ may or not persist asymptotically, they will vanish completely in $E(r_t^4) < \infty$ as $T \to \infty$.

In summary, the empirical size distortions may considerably be attenuated by using data-dependent procedures in consistent HAC estimation, but esentially large departures will remain in the same region in which the deterministic rule exhibits a really bad performance. For instance, for the empirical values $\hat{\phi}_T \approx 0.98$ and $\hat{\alpha}_T \approx 0.05$ the empirical size for the l_T^r -rule is around 48%, which represents a dramatric reduction of the size distortion with respect to the previous case, but is still far from yielding acceptable size. For GARCH models with large persistence and excess kurtosis, the CUSUM test is expected to be biased towards rejecting too frequently and finding breaks around the middle of the sample.

Figure 4. Empirical size (asymptotic 5% level) for the nonparametric CUSUM test with GARCH errors and bandwidth parameter set by the l_T^b rule. The values (α, ϕ) for which the LRV of $\{r_t^2\}$ is not well-defined are plotted in dashed lines.



The picture changes completely when HAC estimation builds on an inconsistent estimate. In this case, the CUSUM test has less (absolute) distortions as compared to the results based on consistent estimation. Interestingly, the empirical size for small values of α is very similar to that when using the population LRV parameter. However, as α increases, a fairly strong drift towards under-sized inference is observed. Furthermore, the empirical size envelope decreases with both α and $\phi,$ and for large values, the test hardly shows any ability to reject the null. Among the three methods disscused in this section, the inconsistent method is the one that shows the largest sensitivity (in relative terms) to the values of $\alpha.$ The extent of the under-sizing bias is so marked that it allows us to directly predict strong reductions in the power of the test in the same parameter space in which consistent estimates are biased towards rejecting.

Hence, for strongly heteroskedastic GARCH errors, CUSUM tests seem bounded to overreject the null when using consistent estimates, or underreject if resorting to inconsistent estimates even in relatively large samples. Despite the good asymptotic properties of the testing procedure, the characteristics that are usually found in financial data may make the inference obtained under the nonparametric CUSUM principle unreliable because of the statistical difficulties to estimate efficiently the LRV parameter in highly heteroskedastic data. These results agree in essence with previous experimental work; see, for instance, Cai and Shintani (2006), and the theoretical results in Müller (2005), when focusing on persistent short-run dynamics in mean in the context of unit-root testing. The analysis for nearly-integrated volatility dynamics in the spirit of Müller (2005) posses some theoretical difficulties and is left for future research.

The Stochastic volatility model

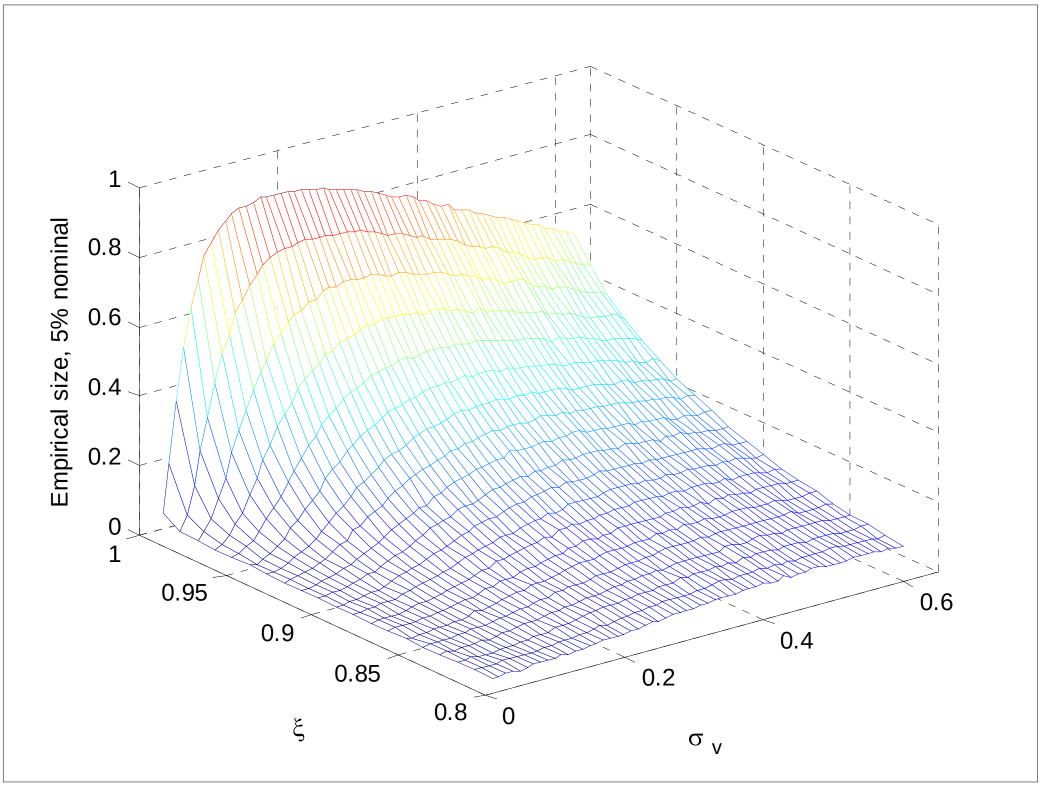
The results for the performance of the CUSUM test when volatility is driven by an SV process are shown in Figures 5, 6, and 7, for the $l_T^d,$ l_T^r and l_T^b rules, respectively. At the end of this section, in Table 1, we summarise the results from all these simulation. As in the case of GARCH errors, the finite distribution of the CUSUM test turns out to be strongly affected by the values of the driving parameters of the volatility process, $(\sigma_v, \xi),$ as well as for the type of bandwidth used in the HAC estimator. Overall, the picture that emerges for SV errors is qualitatively similar to that of the GARCH model.

For moderate degrees of persistence, as measured by $\xi,$ the (consistent) CUSUM test display some degree of robustness against changes in $\sigma_v,$ everything else being equal (see Figures 5 and 6). Furthermore, for small values of $\sigma_v,$ the test suffers moderate oversizing effects even when ξ approaches unity, with l_T^r providing similar results than l_T^d . This is due to the fact that small values of σ_v imply correlations of small magnitude.[14]

[14]The ACF of the squared process for SV models is given by $\rho_j = \frac{\exp\left(\sigma_h^2 \xi^j\right) - 1}{\kappa \exp\left(\sigma_h^2\right) - 1},\ \sigma_h^2 = 1/(1-\xi^2).\ j \geq 1.$

As σ_v increases, the size distortions become more sizeable, with the shape of the empirical size function showing a strong degree of concavity on this parameter: a large kurtosis does not necessarily mean a larger distortion when this is large enough. The reason is that for large σ_v and ξ, there exists a stronger first-order autocorrelation in r_t^2, but higher-order correlations decay fast to zero. The l_T^r -rule provides an overall better performance than l_T^d, but none of them deliver empirical sizes reasonably close to the nominal ones for realistic values of σ_v and ξ.[15] Finally, using a b -fixed bandwidth parameter leads to under-sized tests, as observed for the GARCH errors (see Figure 7).

Figure 5. Empirical size (asymptotic 5% level) for the nonparametric CUSUM test with SV errors and bandwidth parameter set by the l_T^d rule.



[15]For instance, for driving parameters $(\sigma_v, \xi) = (0.18, 0.98)$ --which are highly pausible from an empirical point of view -- the empirical size reaches nearly 79% (46%) if l_T is determined through the l_T^d -rule (l_T^r -rule). For the values reported in Carnero, Peña and Ruiz (2004), the empirical size would not be inferior to 39%.

Figure 6. Empirical size (asymptotic 5% level) for the nonparametric CUSUM test with SV errors and bandwidth parameter set by the l_T^r rule.

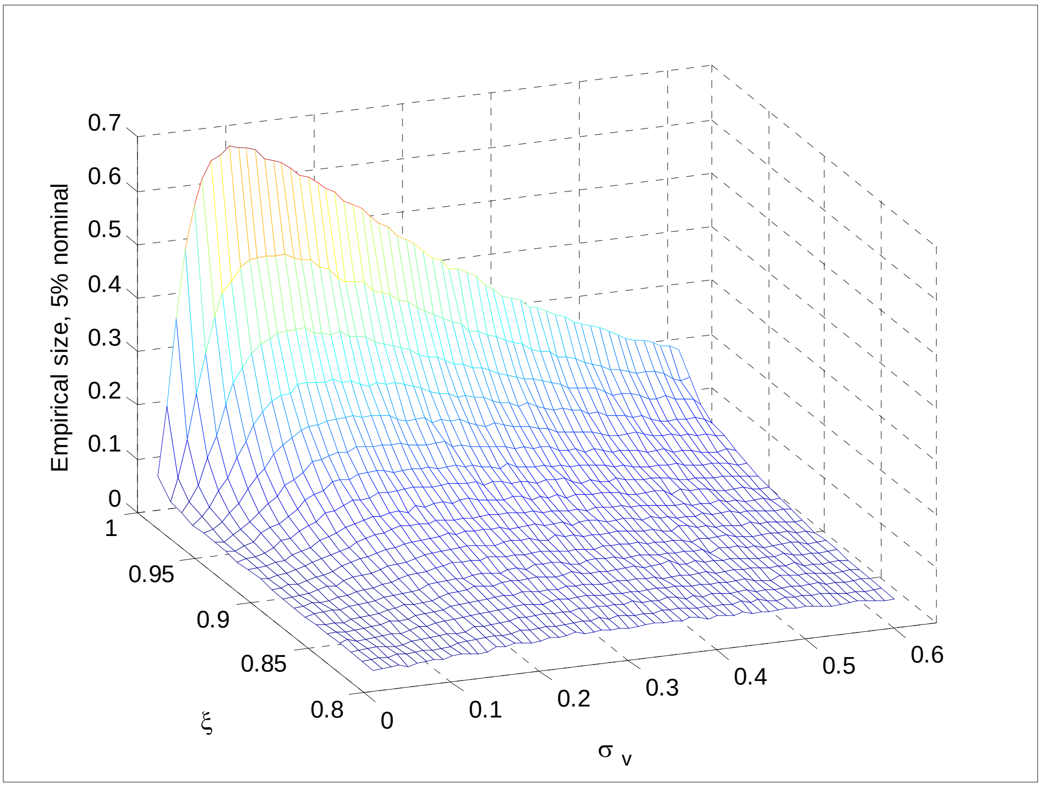


Figure 7. Empirical size (asymptotic 5% level) for the nonparametric CUSUM test with SV errors and bandwidth parameter set by the l_T^b rule.

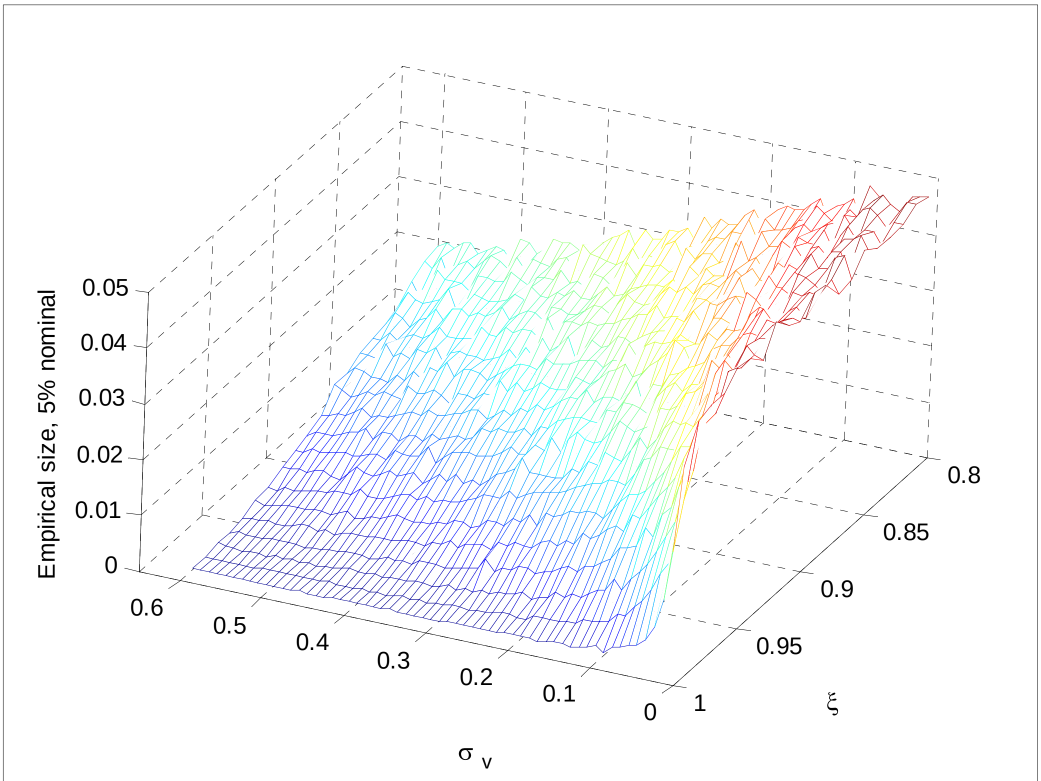


Table 1. Summary of empirical sizes.

DGP	GARCH(1,1)				SV(1,1)		
(θ_1, θ_2)	$\mathcal{M}_2$	l_T^d	l_T^r	l_T^b	l_T^d	l_T^r	l_T^b
$(0.01, 0.80)$	0.0439	0.0496	0.0465	0.0440	0.0428	0.0394	0.0472
$(0.05, 0.80)$	0.0403	0.0820	0.0624	0.0362	0.0422	0.0416	0.0444
$(0.11, 0.80)$	0.0445	0.1175	0.0635	0.0304	0.0486	0.0478	0.0406
$(0.20, 0.80)$	0.0507	0.1408	0.0576	0.0232	0.0704	0.0607	0.0356
$(0.01, 0.90)$	0.0436	0.0618	0.0593	0.0347	0.0422	0.0382	0.0443
$(0.05, 0.90)$	0.0412	0.1608	0.1080	0.02724	0.0498	0.0499	0.0413
$(0.11, 0.90)$	0.0448	0.2594	0.1100	0.0185	0.0928	0.0855	0.0352
$(0.20, 0.90)$	0.0444	0.2994	0.0875	0.0111	0.1786	0.1096	0.0248
$(0.01, 0.95)$	0.0383	0.0952	0.0896	0.0347	0.0438	0.0389	0.0474
$(0.05, 0.95)$	0.0368	0.3358	0.2186	0.0154	0.1000	0.0907	0.0332
$(0.11, 0.95)$	0.0426	0.5038	0.2077	0.0082	0.2776	0.2089	0.0181
$(0.20, 0.95)$	0.0170	0.4988	0.1475	0.0051	0.4452	0.2198	0.0008
$(0.01, 0.97)$	0.0323	0.1415	0.1356	0.0271	0.0456	0.0430	0.0421
$(0.05, 0.97)$	0.0336	0.5332	0.3507	0.0098	0.2109	0.1853	0.0233
$(0.11, 0.97)$	0.0354	0.6729	0.3211	0.0004	0.5218	0.3475	0.0096
$(0.20, 0.97)$	-	0.6285	0.2090	0.0002	0.6725	0.3401	0.0033
$(0.01, 0.99)$	0.0152	0.3149	0.2089	0.0144	0.0852	0.0782	0.0370
$(0.05, 0.99)$	0.0217	0.8246	0.6577	0.0017	0.6526	0.5830	0.0049
$(0.11, 0.99)$	–	0.8702	0.5444	0.0009	0.8872	0.6775	0.0008
$(0.20, 0.99)$	–	0.7802	0.3458	0.0009	0.9077	0.5835	0.0003

Summary of the empirical size at a 5% nominal level of the nonparametric CUSUM test with Bartlett kernel HAC estimator and bandwidth detemined under the l_T^d (deterministic), l_T^r (random) and l_T^b (inconsistent) rules shown in Figures 1-7, and the theoretical population $\mathcal{M}_2$ (values presented for the region in which this moment is well-defined). The driving parameters of the DGPs are $(\theta_1, \theta_2) = (\alpha, \phi)$ and $(\theta_1, \theta_2) = (\sigma_v, \xi)$ for the GARCH(1,1) and the SV model, respectively.

Conclusion

In this paper, we have discussed the extent of the finite sample size distortions which may occur when addressing variance homogeneity in financial and economic variables through nonparametric HAC-based CUSUM tests. Our analysis is linked to the extant lines of recent work related to the performance of these tests when applied to financial data; see, *e.g.*, Andreou and Ghysels (2002, 2003), Sansó *et al.*, (2004) and Rodrigues and Rubia (2006), and provides further evidence about the empirical problems related to LRV estimation through HAC-type estimators in different fields of applied econometrics; see, also Phillips and Sul (2003), Sul, Phillips and Choi (2005), Müller (2004, 2005), and Bandi and Russel (2005).

HAC-type estimator are the key for CUSUM tests to gain invariance against a wide number of empirically relevant DGPs without having to specify any particular parametric relation. Provided regular conditions, these tests exhibit correct asymptotic properties and, furthermore, deliver reliable results in small samples for many types of DGPs. However, if the actual process is strongly conditional heteroskedastic, as is usually observed in high-frequency financial data, nonparametric CUSUM tests may lose the ability to reject at the correct asymptotic nominal size as a direct consequence of biases in the HAC estimator. When non-consistent estimators are used, the problem traslates into larger power distortions.

The practical implication of the poor small-sample performance of the HAC estimator is the lack of ability to detect whether a volatility process seemingly persistent is originated by long-range dependence or by structural breaks. This problem is a finite-sample analoge of the size departures that would asymptotically occur under long-range dependence, namely, spurious rejection and bias to identify break-points around the middle of the sample (see, for instance, Kuan and Hsu 1998, Granger and Hyung 2004). A strongly persistent volatility process observed over a finite sample would reproduce these features and, although the empirical size would tend to its nominal level asymptotically, large biases would likely arise in small samples. Whereas there are some theoretical doubts about the suitabilty of long-memory models, a strictly stationary GARCH or stochastic volatility model with enough persistence and leptokurtosis suffices to break down the properties of the CUSUM testing in small samples.

This result is important for empirical applications, because suggests strong practical limitations of the nonparametric CUSUM strategy owing to stylized characteristics which are present in financial data. It advises using CUSUM tests on data recorded at a lower basis, such as monthly data, for which temporal dependence in volatility is considerably weaker due to aggregation, and for which both the asymptotic and finite sample performance may be better suited. High-frequency stock returns are far from being an ideal underlying series, since they are particularly noisy and show extreme movements as a consequence of the information flow and the occurence of events. As a result, it is not unfrequent to observe transient excesses of volatility, which may be spuriously identified as structural breaks by the testing procedure (e.g., the market crash in october 1987), or mask true breaks. But, even in absence of such an irregular behavior, high-frequency returns always exhibit a strong degree of persistence and excess of kurtosis, which jeopardize the correct performance of the HAC-based testing.

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Appendix: Technical Proofs

Proof of Proposition 2.1. For any $0 \leq \tau \leq 1,$ define the partial sum process $S_{s,[T\tau]} = \sum_{t=1}^{[T\tau]} X_{t,s}^c.$ Under Assumption $\mathcal{A}$, the following FCLT applies uniformly on τ and as $T \to \infty,$ $T^{-1/2} S_s^{[T\tau]} \overset{d}{\to} \mathcal{M}_s\, W(\tau)$. Write $k = [\tau T],$ and note that the functional $\mathcal{U}_T(\tau) = T^{-1/2}(S_{s,[T\tau]} - ([T\tau]/T)S_{s,[T\tau]})$ is a stochastic element of $D[0,1]$. From the FCLT, $\mathcal{U}_T(\tau) \overset{d}{\to} \mathcal{M}_s(W(\tau) - \tau W(1)) = \mathcal{M}_s W^*(\tau).$ Since $\mathcal{U}_T(\tau) = G_T(k),$ and since it has been assumed that $\widehat{\mathcal{M}}_{T,s} \overset{p}{\to} \mathcal{M}_s,$ then $\mathcal{R}_{T,s}(k) \to W^*(\tau)$, and the continuous mapping theorem (CMT) completes the proof.

Proof of Proposition 2.2. Since $E(r_t|\mathcal{F}_{t-1}) = 0$ and sup $_tE(|r_t|^{4+\delta}) < \infty$ for some $\delta > 0$, it follows a) that for every $\varepsilon > 0$ and $\delta > 0$ there exists a constant T_0 such that when $T > T_0,$ $\Pr\left(|\hat{k}_{T,s} - k_0| > T\delta\right) < \varepsilon,$ and b) a positive, finite constant B such that for every $A > 0$ and $m > 0$ $\Pr\left(\sup_{m \leq k \leq T} \frac{1}{m}|\sum_{t=1}^{m} X_{t,s}^c| > A\right) \leq B/(A^2 m),$ see lemmas A.1 and A.2 in Chen *et al*., (2005). The proof follows the arguments in Proposition 3 in Bai (1994) along with results a) and b).

Proof of Proposition 2.3. The proof of part $\mathrm{i)}$ requires showing $\widehat{\mathcal{M}}_{T,s} \overset{p}{\to} \mathcal{M}_s$, which follows from restrictions ii) and iv) in Assumption $\mathcal{A}$, see Giraitis *et al*., (2003, Thm 3.1). The FCLT completes the proof. For part $\mathrm{ii)}$, note that $\mathcal{R}_{T,s}(k) = \hat{\mathcal{M}}_{T,s}^{-1} T^{-1/2} G_{T,s}(k),$ where from lemma 2.1 $T^{-1/2} G_{T,s}(k) \overset{d}{\to} \mathcal{M}_s W^*(\tau)$. The proof simply requires $\hat{\mathcal{M}}_{T,s}^2 \overset{d}{\to} \mathcal{M}_s^2 Q_\omega(b),$ for $Q_\omega(b)$ being a functional that depends on the kernel function and the ratio $b = l_T/T.$ For simplicity, but no loss of generality, we provide the proof for the Bartlett kernel HAC and $l_T/T = 1.$ The proof for different kernel functions, or a general $0 < b < 1$, follows from restriction ii) in Assumption $\mathcal{A}$ and Kiefer and Vogelsang (2005, Thm 1). For Bartlett weights and $l_T = T$ we have $\widehat{\mathcal{M}}_{T,s}^2 = T^{-1}\sum_{i=1}^{T}\sum_{j=1}^{T} \hat{X}_{i,s}^c(1 - |i-j|/T)\hat{X}_{j,s}^c,$ with $\hat{X}_{t,s}^c = |r_t|^s - T^{-1}\sum_{t=1}^{T}|r_t|^s$. Direct algebra as in Kiefer and Vogelsang (2002) shows that $\widehat{\mathcal{M}}_{T,s}^2 = 2T^{-1}\sum_{k=1}^{T}\left(\sum_{t=1}^{k} T^{-1/2}\hat{X}_{t,s}^c\right)^2.$ Because $\sum_{t=1}^{k}\hat{X}_{t,s}^c = \left(\sum_{t=1}^{k} X_{t,s} - k\hat{\sigma}_s^2\right) \coloneqq G_{T,s}(k)$ for $\hat{\sigma}_s^2 = T^{-1}\sum_{t=1}^{T} X_{t,s},$ it follows inmediately from the FCLT and by applying the CMT that $\left(\sum_{t=1}^{k} T^{-1/2}\hat{X}_{t,s}^c\right)^2 \overset{d}{\to} \mathcal{M}_s^2 W^*(\tau)^2$. Finally, by noting the weak convergence of integrals to sums, $\int_0^1 \mathcal{M}_s^2 W^*(\tau)^2 d\tau = \sum_{k=1}^{T} \mathcal{M}_s^2 \int_{(i-1)/T}^{i/T} W^*(\tau)^2 d\tau = T^{-2}\sum_{k=1}^{T}\left(\sum_{t=1}^{k} G_{T,s}(k)\right)^2,$ and therefore $\widehat{\mathcal{M}}_{T,s}^2 \overset{d}{\to} 2\int_0^1 \mathcal{M}_s^2 W^*(\tau)^2 d\tau$. Finally, the CMT shows the weak convergence of $\mathcal{R}_{T,s}(k)$ to $W^*(\tau)\left(2\int_0^1 W^*(\tau)^2 d\tau\right)^{-1/2}.$

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