TESTING FOR STRUCTURAL BREAKS IN VARIANCE WITH ADDITIVE OUTLIERS AND MEASUREMENT ERRORS

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FUNDACIÓN DE LAS CAJAS DE AHORROS
DOCUMENTO DE TRABAJO
Nº 247/2006

De conformidad con la base quinta de la convocatoria del Programa de Estímulo a la Investigación, este trabajo ha sido sometido a evaluación externa anónima de especialistas cualificados a fin de contrastar su nivel técnico.

ISBN: 84-89116-07-5

Testing for Structural Breaks in Variance with Additive Outliers and Measurement Errors[1]

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JEL Classification: C12, C15, C52
Keywords: additive outliers, variance homogeneity, CUSUM-type tests

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[1]We would like to thank Marine Carrasco, Javier Gómez-Biscarri, Uwe Hassler and seminar participants at the Humboldt University of Berlin for comments and suggestions. Any remaining error is our own. Financial support from POCTI/ FEDER (grant ref. POCTI/ECO/49266/2002) and the SEJ2005-09372/ECON project is gratefully acknowledged.
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Testing for Structural Breaks in Variance and the Effect of Additive Outliers and Measurement Errors

Abstract

This paper discusses the asymptotic and finite-sample properties of CUSUM-based tests for detecting structural breaks in volatility. Our aim is to analyze formally the effects of stochastic contamination, such as additive outliers or measurement errors. This analysis is particularly relevant for financial data, for which these tests are the most common way to detect variance breaks. In particular, we focus on the tests by Inclán and Tiao (1994) [IT] and Kokoszka and Leipus (1998, 2000) [KL], which have been intensively used on these data in the applied literature. Our results are extensible to related procedures. We show that the asymptotic distribution of the IT test can largely be affected by sample contamination, whereas the distribution of the KL test remains invariant. Furthermore, the break-point estimator of the KL test renders consistent estimates. In spite of the good large-sample properties for this test, we discuss that large additive outliers tend to generate power distortions or wrong break-date estimates in small samples.

1. Introduction

There is much evidence that economic time-series are non-stationary when observed over long enough periods of time. Police-regime shifts and other many factors may generate parameter instability in the underlying generating process, often leading to abrupt changes. While there has been an obvious interest to analyze mean shifts in variables as well as their specific sources, the recent literature in Financial Economics also concerns with instability on higher moments. A special interest has been directed towards addressing variance homogeneity, since this moment heavily characterizes the statistical properties of the economic models and their predictions. For instance, McConnell and Perez-Quirós (2000), and Sensier and van Dijk (2004), among others, find strong evidence suggesting a sharp decline in the volatility of macroeconomic variables, which has important policy implications. Also, since volatility is central to Financial theory and its related applications in risk management and investment decision-making, there is a growing interest to analyze variance stability in financial markets (see, among others, De Santis and Imrohoroglu 1997) and the effects of neglected breaks in time-series modelling (see Lamoureaux and Lastrapes 1990, Granger and Hyung 2004, Mikosch and Stărică 2004, and Hillebrand 2005).

The statistical procedures specifically designed to estimate breaks in volatility which have been mostly used in the applied literature are based on CUSUM-type procedures.[4] Into this category fall the parametric and non-parametric methods discussed in Pagan and Schwertz (1990), Phillips and Loretan (1990), Inclán and Tiao (1994), Kokoszka and Leipus (1998, 2000), Kim, Cho and Lee (2000), Sansó, Aragó and Carrión (2004), Chen, Choi and Zhou (2005), as well as several extensions of these procedures. The reason for this preference is that these tests are easily implemeted and, furthemore, most of the them are model-free and admit a fairly general class of generating processes. All the tests adopt a simmilar strategy to detect breaks, although some of them differ significantly in their basic assumptions. The ability to identify sudden changes depends critically on the characteristics of the real data and the suitability of the assumptions. For financial data, the most relevant features are the existence of time-varying volatility patterns (i.e., temporal dependences) and contaminated observations, such as outliers. Andreou and Ghysels (2002) show by Monte Carlo simulation that strongly-persistent volatility may lead to important distortions in the size of several CUSUM tests. On the other hand, the effect of contaminated observations has not been analyzed formally. Nevertheless, there are at least two sources of stochastic contamination that are worth of discussion in the framework of financial data, namely, additive outliers and measurement errors. It is well-known that returns are characterized by leptokurtic distributions, but even after accounting for this feature, extreme market movements and other unpredictable events lead to abnormally

[4]There is a small but growing literature on detecting breaks in volatility. Andreou and Ghysels (2004) and Kokoszca and Teyssière (2002) discuss alternative methods to CUSUM tests.

large observations which are considered as outliers.[5] Non-synchronous trading and thin-trading may generate measurement errors in the recorded data of individual stocks, portfolios, and market indices; see Campbell, Lo and McKinlay (1997). This problem may be particularly serious in emerging markets, for which the literature related to variance stability has paid a special attention; see Bekaert and Harvey (1997, 2000), and Aggarwal, Inclán and Leal (1999).

In this paper, we formally discuss the effects that sample contamination originating in any of these sources has on the asymptotic properties of CUSUM-type tests for detecting change points in variance. In particular, we analyze the effects of additive outliers and/or measurement errors on *i)* the asymptotic distribution, *ii)* the consistency of the turning point estimator, and *iii)* the small-sample performance of these procedures. Owing to their empirical relevance in the applied framework, the especial focus of this paper is on two tests that differ in their basic assumptions, although our conclusions are easily generalizable for related tests. On the one hand, we study the test suggested by Inclán and Tiao (1994) [IT henceforth]. In spite of being based on strong restrictions, such as normally-distributed data, this method has been intensively used on financial time-series; see, among others, Aggarwal *et al*., (1999), Morana and Beltratti (2004), Nouira *et al*. (2004), and more recently Hyung, Poon and Granger (2005). On the other hand, we also focus on the more general procedure suggested in Kokoszka and Leipus (1998, 2000) [KL henceforth]. This test is not model-specific and relies on fairly general assumptions about the underlying process. It has also been applied on several studies, see among others Andreou and Ghysels (2002, 2004) and Cuñado *et al*. (2005) for recent studies. Furthermore, other tests in this literature (e.g., Pagan and Schwert 1990, Sansó *et al*. 2004, and Chen *et al*. 2005) are strongly related to the KL test and its assumptions, so the conclusions of our analysis are straightforwardly extensible.

The results of our analysis can be summarized as follows. First, we show that the asymptotic distribution of the IT is not longer invariant if the sample includes contaminated observations. Given the characteristics of real data, our analysis predicts biases towards finding too many breaks --even in asymptotic samples, owing to the use of over-conservative critical values. By contrast, the non-parametric structure of the KL test assures invariance against the sources of contamination treated in this paper. Furthermore, we show that the KL test renders consistent estimates of the changing-point, and therefore this test proves valid under the large-sample theory. Our analysis reveals patterns which would be hard to explain in absence of a formal theoretical analysis. For instance, the distribution of the IT test is more sensitive to a small likelihood of outliers that to a large probability of

[5]Outliers are discordant observations that seem to be far beyond the process that rules most observations. In financial markets, outliers are linked to rare shocks not related to the trading process, or abnormal flows of information arrivals. A well-known example is October 19, 1987. The major US indeces fell on this day over 20%, leading to the largest one-day decline in recorded stock market history. This extreme decline still lacks explanation

them, which is exactly the relevant case for empirical applications. Similarly, the asymptotic robustness of the KL test is not obvious, since sample contamination may introduce asymptotic biases in a number of methods based in least-squares estimates.

Despite the good asymptotic performance, however, important distortions may arise in finite samples when there is a high degree of stochastic contamination. We discuss that particularly large outliers, even if they have small likelihood, are able to distort the power performance of the KL test severely. The reason is that the excess of variability originated by multiple large outliers may make it difficult to estimate correctly the location of the turning points, and hence the test tends to reject a null which is evaluated at a wrong location. Our results provide simple and straightforward reasons to explain the contradictory findings whenever the IT and KL tests are simultaneously applied. Neglected outliers bias the former test to find a large number of breaks, whereas the latter exhibits low power and would tend to find few or no breaks at all. Therefore, the techniques to deal with outliers based on deletion or bounded-influence methods, which are commonly applied in many statistical procedures, are also deserved in testing for variance stability.

The rest of the paper is organized as follows. Section 2 outlines the statistical procedures which are analyzed in this paper. Section 3 derives the asymptotic properties (asymptotic distribution and consistency of the break-point estimator) under sample contamination and provides the set of sufficient conditions to justify the results. Section 4 reports Monte Carlo experimentation, which illustrates the small-sample performance of these tests. Section 5 summarizes and concludes. Finally, a technical appendix collects the proofs of the theoretical results discussed in the paper.

2. Testing for structural breaks in volatility with unknown break date

2.1. The Inclán and Tiao Test

The IT test is as a natural extension of the CUSUM-type tests for the detection of shifts in variance. This procedure has the advantage of being easily implemented and does not require parameter estimation. However, the asymptotic distribution is derived under a set of sufficient conditions which may turn out to be too restrictive for most practical applications.

Let $\{r_t\}_{t=1}^{T}$ a sample of a real-valued stochastic process defined on a certain probability space. The test statistic of the IT test is defined as

$$IT = \sqrt{T/2}\max_{1\leq k\leq T}|D_T(k)| \tag{1}$$

where $D_T(k) = \left[\left(\sum_{t=1}^{k} r_t^2 / \sum_{t=1}^{T} r_t^2\right) - k/T\right]$, such that $D_T(0) = D_T(T) = 0$. Inclán and Tiao (1994) show that if r_t is a series of independent observations from a normal

distribution with zero mean and constant unconditional variance, then (1) converges weakly to the supremum of a standard Brownian Bridge. The test is based on the transformed series r_t^2, which are unbiased estimates of the unconditional variance of the process. Under the null hypothesis of homogenous variance over the entire sample, the test has power to detect changes in the level of the second-order moment.
This procedure is initially intended to estimate the location of a single changing point in the sample. This is estimated as

$$\hat{k} = \max_{1 \leq k \leq T} |D_T(k)|. \tag{2}$$

However, it is not difficult to define a more general procedure based on the successive computation of (1) and (2) to gain power against the alternative hypothesis of multiple breaks. Inclán and Tiao (1994) propose so-called Iterative Cumulative Sum of Squares (ICSS) method, which embeds the basic algorithm into an iterative scheme based on successive computations of the statistic at different parts of the series, which are consecutively determined after a possible change point.

2.2. The Kokoszka and Leipus Test

Kokoszka and Leipus [KL] (1998, 2000) suggested a robust method valid for detecting structural breaks under general conditions; see also Sansó *et al.* (2004). The class of errors which are allowed includes ARCH(∞) dependences, and it makes this test suitable for applications on financial data. The approach is a non-parametric extension of the IT test and, furthermore, it is also adapted to the problem of identifying multiple breaks following a sequential procedure, in the same spirit as the ICSS method.

The test statistic, say KL, is defined as

$$KL = T^{-1/2} \widehat{M}_{4,T}^{-1/2} \max_{1 \leq k \leq T} |G_T(k)| \tag{3}$$

where $G_T(k) = \left[\sum_{t=1}^{k} r_t^2 - (k/T) \sum_{t=1}^{T} r_t^2 \right]$ and $\widehat{M}_{4,T}$ is a consistent estimator of the long-run variance of $r_t^2 - E(r_t^2)$, *i.e.*, the limit of $T^{-1} E\left[\sum_{t=1}^{\infty} (r_t^2 - E(r_t^2))^2 \right]$, say $M_4 < \infty$. The break-point estimator $\hat{k}$ implied by the KL scheme is defined as

$$\hat{k} = \max_{1 \leq k \leq T} |G_T(k)|. \tag{4}$$

The computation of this statistic is straightforward and only requires a suitable method to estimate the long-run variance parameter. The authors suggest using a data-based non-parametric estimator that does not rely upon the assumption of a explicit model, and hence it is robust to model misspecification. Since the Newey and West (1987), Andrews (1991) and Davidson and de Jong (2002), among others, discuss a suitable class of non-parametric estimators which is consistent in the presence of both heteroskedasticity and autocorrelation of unknown form.

3. Asymptotic theory

Let us start by discussing a data generating process [DGP] in which the main process is perturbed with a stochastic contamination process that generates additive outliers and/or measurement errors. Our aim is to define an observable process with similar statistical properties to the financial time-series in a model-free environment. In particular, the DGP we shall consider is defined as follows:

$$r_t = \mu + \varepsilon_t + \xi_t + \mathfrak{P}_t[\lambda + \delta v_t], \ \ t = 1, \ldots, T. \tag{5}$$

In this approach, the regular component is defined through the zero-mean process ε_t ; the measurement error (ME) process ξ_t is a pure noise term that leads to the impossibility of observing the true signal ε_t ; finally, $Z_t \equiv \mathfrak{P}_t[\lambda + \delta v_t]$ is a stochastic process which generates additive outliers (AOs).[6] In the financial literature, Z_t is usually referred to as a (discrete-time) stochastic jump process. In the econometric literature, a number of papers have focused on the effects of AOs through restricted forms of this specification; see, for instance, Franses and Haldrup (1994), and van Dijk *et al*. (1999).

3.1. Assumptions

We introduce now the set of assumptions under which the asymptotic properties of the tests will be derived. We first characterize the observable series r_t and the stochastic behavior of the components ξ_t, Z_t of the contamination process in assumptions A1-A2 below. Also, we characterize the regular component ε_t to assure that sufficient conditions hold to apply a functional central limit theorem (FCLT). In particular, we assume that either A3 or A3' below applies, with condition A4 holding true in any case.

[6]Since our primary interest is in financial time series, which are nearly white noise, the distinction between additive and innovative outliers seems of little importance. We therefore focus on the effects of additive outliers only.

Assumption A1. The real-valued observable process $\{r_t, t \geq 1\}$ follows from (5).

Assumption A2. The components of the contaminating term in (5) verify:
i) The process $\{\xi_t, t \geq 1\}$ is independent of ε_t and Z_t, and $\xi_t \sim iid(0, \sigma_\xi^2)$ for some finite $\sigma_\xi \geq 0$. Also, $E(|\xi_t|^{4+\gamma}) < \infty$ for some $\gamma > 0$.
ii) The process $\{Z_t, t \geq 1\}$, $Z_t \equiv \mathfrak{P}_t[\lambda + \delta v_t]$, is independent of ε_t and ξ_t, with $\mathfrak{P}_t$ being a Bernoulli variable with support $(-1, 1, 0)$ and probabilities $\{p/2, p/2, 1-p\}$. Furthermore, $v_t \sim iid(0,1)$ such that $E(|v_t|^{4+\gamma}) < \infty$ for some $\gamma > 0$, and $0 \leq p < 1$, $0 < \lambda < \infty$, $0 < \delta < \infty$.

Assumption A3. The regular component $\{\varepsilon_t, t \geq 1\}$ verifies:
i) $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma_\varepsilon^2 < \infty$.
ii) $\sup_t E(|\varepsilon_t|^{4+\gamma}) < \infty$ for some $\gamma \geq 0$.
iii) ε_t is strong mixing with mixing numbers m_j satisfying $\sum_{j=1}^{\infty} m_j^{s/(s-2)} < \infty$ for some $s > 4$, and $\lim_{T\to\infty} E\left[T^{-1}(\sum[\varepsilon_t^2 - \sigma_\varepsilon^2])^2\right] \equiv M_4^\varepsilon$, with $0 < M_4^\varepsilon < \infty$.

Assumption A3'. Let $\mathcal{F}_t$ be the σ-field generated by $\{\varepsilon_t, Z_t, \xi_t, \varepsilon_{t-1}, Z_{t-1}, \ldots\}$. Then $\{\varepsilon_t, \mathcal{F}_t\}_{t=1}^{\infty}$ is a strictly stationary and ergodic martingale difference process and $E(|\varepsilon_t|^{4+\gamma}) < \infty$ for some $\gamma \geq 0$.

Assumption A4. ε_t is independent of $\{\xi_t, Z_t\}$.

Some comments proceed. A2 assumes that the generating process of the AOs is independent of the regular component. This seems accurate for financial returns, as it captures extreme events which are unrelated to the normal trading process but which are able to influence the observable series in a major way. Similarly, the ME component is assumed to be exogenous. Note that, since ξ_t and Z_t are bounded in probability, the regular process ε_t is not perturbed with arbitrarily large values in our analysis.

Condition A3 is fairly general and standard in the literature. It does not impose distributional restrictions and allows short-run dynamics in ε_t under mixing conditions. Into this class fall finite-order ARMA structures, and several time-varying volatility processes such as the GARCH-type and stochastic volatility models, see Carrasco and Chen (2001). The finiteness of the fourth-order moment is necessary for ensuring test consistency (see Phillips and Loretan 1990) and rules out heavily-tailed distributions for which not even this moment is well-defined. This may seem restrictive, given the characteristic degree of leptokurtosis of many financial time-series. However, the AOs allowed in A1 may generate large kurtosis in r_t even if ε_t is not extremely leptokurtic. Condition A3' may be sufficient when the series are uncorrelated but not independent, as it is often the case in financial time-series. In this case, only restrictions to ensure a FCLT for martingales are necessary. Finally, A4 is a maintained assumption in related studies, such as Franses and Haldrup (1994) and van Dijk *et al*. (1999). Its practical purpose is to allow us to analyze the effects of stochastic contamination in a model-free framework. We shall comment the effects of weakening this assumption later on.

In the sequel, we denote by ` $\Rightarrow$ ' the weak convergence of probability measures in $D[0,1]$ , the space of all right continuous real-valued functions having finite left limits on $[0,1]$ , while ` $\overset{p}{\to}$ ' is used to denote convergence in probability; $W(\tau)$ denotes a standard Wiener process on $\tau \in [0,1]$, and $W^*(\tau) = W(\tau) - rW(1)$ is a standard Brownian Bridge. Finally, $[\cdot]$ is the integer function, and $tr(\cdot)$ denotes the trace of a matrix.

3.2. Asymptotic distribution of the test statistics

In this section, we formally derive the asymptotic behavior of the distribution of the IT and KL test statistics under additive outliers and/or measurement errors. We first state two useful lemmas, whose proof is shown in the technical appendix.

Lemma 3.1. *Assume the DGP in A1 with A2, A3 or A3' and A4 holding true. Define the random vector* $\Pi_t = (\varepsilon_t^2, Z_t^2, \xi_t^2, 2\varepsilon_t Z_t, 2\varepsilon_t \xi_t, 2\xi_t Z_t)'$ *. Then, as* $T \to \infty$ *,*

$$T^{-1/2} \sum_{t=1}^{[T\tau]} (\Pi_t - E(\Pi_t)) \Rightarrow \Omega^{1/2} \mathbf{W}(\tau) \tag{6}$$

in $D[0,1]^6$ *and uniformly in* $\tau \in [0,1]$ *where* $\mathbf{W}(\tau)$ *is a multivariate standard Wiener process with diagonal covariance matrix* $\Omega = \{\omega_{ii}\}$ *such that:*

$$\begin{aligned}
\omega_{11} &= M_4^{\varepsilon} \\
\omega_{22} &= p[\lambda^4 - \lambda^2 + \delta^4 \mu_4^{v} + 2\lambda\delta^2(3\lambda + \delta\mu_3^{v}) - \delta^2] \\
\omega_{33} &= \mu_4^{\xi} - \sigma_{\xi}^2 \\
\omega_{44} &= 4\sigma_{\varepsilon}^2 p(\lambda^2 + \delta^2); \ \omega_{55} = 4\sigma_{\varepsilon}^2\sigma_{\xi}^2; \ \omega_{66} = 4\sigma_{\varepsilon}^2\sigma_{\xi}^2;
\end{aligned}$$

where $E(\varepsilon_t^2) = \sigma_\varepsilon^2$, $\mu_j^{v} = E\left(v_t^j\right)$, $\mu_j^{\xi} = E\left(v_t^j\right)$,
$M_4^{\varepsilon} = \lim_{T\to\infty} E\left(T^{-1}\left(\sum_{t=1}^{T}(\varepsilon_t^2 - \sigma_\varepsilon^2)\right)^2\right)$.

Lemma 3.2. *Denote* $\tilde{r}_t = r_t - \hat{\mu}_T$ *such that* $\hat{\mu}_T$ *is a* $\sqrt{T}$ *-consistent estimator of* μ*. Under the conditions of Lemma 3.1, the following results can be established as* $T \to \infty$ *:*

(i) $T^{-1/2} \sum_{t=1}^{[T\tau]} [\tilde{r}_t^2 - E(r_t^2)] \Rightarrow M_4^{1/2} W(\tau),$

(ii) $T^{-1/2} \sum_{t=1}^{T} [\tilde{r}_t^2 - E(r_t^2)] \Rightarrow M_4^{1/2} W(1),$

(iii) $T^{-1} \sum_{t=1}^{T} \tilde{r}_t^2 \xrightarrow{p} \sigma_\varepsilon^2 + p(\lambda^2 + \delta^2) + \sigma_\xi^2,$

for any $r \in [0,1],$ *where* $M_4 = Var(r_t^2 - E(r_t^2)) = tr(\Omega).$
Proof. See appendix.

Lemmas 3.1 and 3.2 state the convergence of the functionals involved in the IT and KL tests. With these results, it is easy to state the asymptotic distribution of the CUSUM tests, which is given as a proposition.

Theorem 3.1. *Let the IT and KL test statistics be defined as in (1) and (2) , respectively, and let* $\hat{M}_{4,T}$ *be a consistent estimator of the long-run variance parameter* M_4*. Then, as* $T \to \infty$,

$$IT \Rightarrow \frac{M_4^{1/2}}{\sqrt{2}\,(\sigma_\varepsilon^2 + p\lambda^2 + p\delta^2 + \sigma_\xi^2)} \sup_{\tau \in [0,1]} |W^*(\tau)|;$$

$$KL \Rightarrow \sup_{\tau \in [0,1]} |W^*(\tau)| \qquad (7\text{ -}8)$$

where $W^*(\tau)$ *is a standard Brownian bridge and* $M_4 = tr(\Omega)$.

Proof. Straightforward from Lemmas 3.1 and 3.2 and the continuous mapping theorem. See appendix for details.

Corollary 3.1. *Assuming the particular situation where only additive outliers contaminate the sample, then Theorem 3.1 trivially holds with* $\sigma_\xi = \mu_4^\xi = 0.$ *Similarly, assume that only measurement errors are present, i.e., the degenerate case for* $p = 0$ *, then Theorem 3.1 trivially holds by setting all parameters related to the AOs equal to zero.*

Proof. Follows straightforwardly from Lemmas 3.1, 3.2 and Theorem 3.1.

Theorem 3.1 states formally one of the theoretical results of this paper and has important implications for the empirical framework. First, the asymptotic distribution of the IT test is not invariant but it turns out to be heavily influenced by the characteristics of the contamination process. Second, the asymptotic distribution of the KL test converges to a well-defined distribution that is free of nuisance parameters, namely the supremum of a standard Brownian Bridge (SSBB). In other words, whereas the critical values from the standard distribution can be applied for the KL test even in presence of large extreme outliers or measurement errors, these are generally inadequate for the IT test.

It is interesting to comment the reasons underlying the failure of the IT test. The factor scaling its limit distribution depends on (p, λ, δ) as well as several moments of v_t and ξ_t, and it fully determines the type of departure with respect to the usual SSBB. The critical values from the correct asymptotic distribution will be larger whenever $tr(\Omega) > 2E(r_t^2)^2$, and smaller otherwise. Remarkably, this condition also determines whether the observable series r_t exhibits excess of kurtosis or not. Hence, the failure of the IT test is related to the fact that it explicitly assumes the degree of kurtosis of the normal distribution in the observable series. Naturally, this assumption can easily be violated in our context. For simplicity, set $\delta = \sigma_\xi = 0$ and assume that ε_t is i.i.d., such that $tr(\Omega) > 2E(r_t^2)^2$ reduces to $\sigma^4(\kappa - 3) + \lambda^4[p - 3p^2] > 0.$ Clearly, if ε_t is leptokurtic and/or the data include a moderate non-zero probability of extreme observations, using the SSBB as a limit distribution necessarily leads to over-conservative critical values.[7] It is remarkable that the excess of kurtosis is concave on p, so small values of p will lead the test to over-rejection, whereas large values of this parameter will cause undersizing. Given the characteristics of real data, the IT test is expected to be biased to find too many breaks in empirical applications. The situation worsens if we allow for time-varying volatility patterns in ε_t (e.g, GARCH or stochastic volatility models), because this type of dependence generates by itself excess of kurtosis. This feature is also discussed in Kim, Cho and Lin (2000), and Sansó *et al.*, (2004). The IT test should therefore be used with caution and only when the researcher is aware that the data verify the underlying assumptions, as this proves critical for the correct behavior of the procedure.

Remark 3.1: The diagonality of the asymptotic covariance matrix Ω follows directly from the assumption of independence in A4. We may weaken this condition and allow for dependences between some measurable function of ε_t, say $\mathcal{V}_1(\varepsilon_t)$, and lagged values of a function $\mathcal{V}_2(Z_t, \xi_t)$, provided conditions A3 or A3' are still fulfilled. A relevant example of $\mathcal{V}_1$, $\mathcal{V}_2$ in this context are both the quadratic or the absolute-value function so that the volatility process may be affected by outliers. Define $\bar{\Pi}_t = \Pi_t - E(\Pi_t)$ and let $\Omega^* = \lim_{T\to\infty} T^{-1} E\left(\left(\sum_{t=1}^T \bar{\Pi}_t\right)\left(\sum_{t=1}^T \bar{\Pi}_t'\right)\right)$. If A4 is replaced by the assumption that Ω^*

[7]It is not difficult to see that a similar observation would apply if the measurement error is not Gaussian.

is a finite positive definite matrix such that $\Omega^* = \Lambda\Lambda_t'$, then it follows as $T \to \infty$ that $T^{-1/2} \sum_{t=1}^{[T\tau]} \bar{\Pi}_t \Rightarrow \Lambda\, \mathbf{W}(\tau),$ with this result generalizing Lemma 3.1 in an obvious way. The diagonal elements of Ω^* are those of $\Omega,$ whereas the (non-zero) off-diagonal elements of this matrix depend specifically on the covariance structure related to $\mathcal{V}_1$ and $\mathcal{V}_2$, and hence are model-dependent. It also follows that $T^{-1/2} \sum_{t=1}^{[T\tau]} [\tilde{r}_t^2 - E(r_t^2)] \Rightarrow \mathbf{1}'\Lambda\, \mathbf{W}(\tau),$ and $T^{-1} \sum_{t=1}^{T} \tilde{r}_t^2 \overset{p}{\to} \mathbf{1}'E(\Pi_t\Pi_t')\mathbf{1},$ with $\mathbf{1}$ being a conformable vector of ones. As in Proposition 3.1, it can be shown that $IT \Rightarrow \zeta \sup_{\tau\in[0,1]}|W^*(\tau)|,$ with $\zeta = \sqrt{\mathbf{1}'\Omega^*\mathbf{1}}\left(\sqrt{2}\,(\mathbf{1}'E(\Pi_t\Pi_t')\mathbf{1})\right)^{-1},$ and again $KL \Rightarrow \sup_{\tau\in[0,1]}|W^*(\tau)|.$

Remark 3.2: A number of statistical procedures are strongly related to the KL test, since they use a HAC estiamation of the long-run variance parameter to render convergence to the distribution of the SSBB. See, for instance, Sansó *et al.* (2004). We may easily show that the asymptotic properties discussed in this paper hold for such procedures as well.

3.3. Consistency of the change-point estimator

From Theorem 3.1, applying the IT test on data exhibiting the stylized features of financial data is much likely to lead to misleading inference even in large samples. On the other hand, the KL test still converges to its standard distribution under the null hypothesis. In this section, we discuss the ability of both tests to estimate consistently the location of an unknown turning point under the alternative hypothesis. We assume that the break affects the dynamics of the variance of the regular component ε_t , as there is no practical sense in considering breaks in the contaminating structure. Also, it is necessary to characterize the magnitude of the break and its consequences, and therefore make some additional assumptions.

Let us introduce some previous notation. Denote as $0 < \tau^* < 1$ the break fraction, such that a shift in volatility occurs at time $k^* + 1,$ $k^* = [T\tau^*].$ Denote as $\varepsilon_{1t} = \{\varepsilon_t\}_{t=1}^{k^*}$ and $\varepsilon_{2t} = \{\varepsilon_t\}_{t=k^*+1}^{T}$ the pre- and post-break sub-samples of the regular component, and assume that $E(\varepsilon_{1t}^2) = \sigma_{1\varepsilon}^2$ and $E(\varepsilon_{2t}^2) = \sigma_{1\varepsilon}^2 + \Delta$, such that $0 < |\Delta| < \infty.$ Thus, we are allowing for a break in the variance of the regular component of magnitude Δ , which may be originated from a shift in the conditional or unconditional structure of the regular series. In both cases, the shift in the variance of ε_t leads to a shift of the same magnitude in the overall variance of r_t , *i.e.*, $\Delta =$ Var $(r_{t,t\leq[T\tau^*]}) -$ Var $(r_{t,t>[T\tau^*]}).$ This property allows us to test for the change in the unobservable component ε_t by using the observed series r_t instead.

The KL procedure defines an estimator of the break fraction τ^* as follows

$$\hat{\tau} : T^{-1} \max_{1 \le k \le T} \left| \sum_{t=1}^{k} r_t^2 - \frac{k}{T} \sum_{t=1}^{T} r_t^2 \right| \tag{9}$$

while the IT procedure estimates the break point as $\max_{1 \le k \le T} |D_T(k)|.$ The latter is algebraically equivalent to $\max_{1 \le k \le T} (1/T\sigma)|G_T(k)|$ for $\sigma = \sum_{t=1}^{T} r_t^2/T.$ Therefore, the break-point estimator of the IT test is the same as in the KL procedure. Our interest is to analyze if $\hat{\tau} \xrightarrow{p} \tau^*$ when the observed series $\{r_t\}$ is contaminated. We require the following condition.

Assumption A5. The sequence $\{\varepsilon_t\}_{t=1}^{\infty}$ verifies $i)$ $\sup_t E(\varepsilon_t^4) < \infty$ for all t , and $ii)$ Cov $\left(\varepsilon_t^2, \varepsilon_j^2\right) = O(\rho^{|t-j|})$ for all $1 \le t,j \le T$ and some $0 \le \rho < 1.$

Condition A5 is embedded in A3 or A3' when there are no breaks. With $i)$ we rule out shifts which dramatically change the statistical properties of the process as considered under the null, such as parameter instability leading to diverging moments up to the fourth order. Condition $ii)$ restricts the covariance structure of the time-series. Although ε_t is not stationary under parameter instability, we still require that the covariance between distant observations decay towards zero at a suitable rate. Note that condition $ii)$ holds trivially for i.i.d. series, as well as for dependent series with an ARCH (∞) characterization (see Kokoszka and Leipus 2000). Moreover, $ii)$ may be weakened considerably, as consistency can be proven under the more general restriction $\lim_{T\to\infty} T^{-2} \sum_{k=1}^{T} \sum_{i,j}^{k} Cov\left(\varepsilon_t^2, \varepsilon_j^2\right) = 0,$ which may allow for different rates of decay in the covariances. Convergence in probability for the estimator $\hat{\tau}$ is provided as a theorem below.

Theorem 3.2. *Consider the realization of a process* $\{r_t\}_{t=1}^{T}$ *as defined in A1 such that A2 and A4 hold true. Assume that the unconditional variance of the regular component shifts from* σ_ε^2 *to* $\sigma_\varepsilon^2 + \Delta,$ $0 < |\Delta| < \infty,$ *at some time* $k^* = [T\tau^*]$ *for some* $\tau^* \in (0,1)$ *such that A5 holds true. Let* $\hat{\tau}$ *given by* (9) *. Then, for an arbitrary* $\epsilon > 0$ *and some constant* C *it follows that*

$$\Pr(|\hat{\tau} - \tau^*| > \epsilon) \le \frac{C}{\epsilon^2 \Delta^2 \sqrt{T}} \tag{10}$$

and, therefore, $\hat{\tau} \xrightarrow{p} \tau^*.$

Proof. The proof for consistency uses the Hájek-Rénji inequality in Kokoszka and Leipus (2000). See appendix for details.

Theorem 3.2 shows the consistency of the estimator. The rate of convergence depends on Δ . For a fixed shift $\Delta,$ the estimation bias $|\hat{\tau} - \tau^*|$ can be shown to be $O_p(T^{-1})$, so the estimator $\hat{\tau}$ is super-consistent. This result is similar to the results in the change-point literature (the formal proof follows entirely the lines of Kokoszka and Leipus 2000 and is therefore omitted). Consistency guarantees that the KL test does not only use the correct critical values, but also that these are applied on the correct estimation of the break-date when the sample grows unbounded. It is also interesting to note that the theorem provides insight on how some of the characteristic of the DGP affect the asymptotic bias. The numerator on the right side of (11) is influenced by the degree of serial dependence, the time of the break, and the excess of variability generated by the contaminating process (see Appendix for details). The latter is particularly important in our context, becuase it implies that the contamination process makes it more difficult to locate the breaks position correctly. Intuitively, the break-date must be inferred by observing $r_t,$ which conveys noisy (contaminated) information about the break time. More accurate testing could be carried out if the `clean' series ε_t could be observed instead. The estimation bias increases with these terms, and reduces with the factors appearing in the denominator of (11), namely the sample size and/or the magnitude of the shift. The tension between these factors totally disappears as $T \to \infty,$ but the characteristics of data may drive to potential distortions when the sample is small.

Remark 3.3: If we allow for dependences between the lagged values of the contamination process and a measurable function ε_t^2 , as in Remark 3.1, consistency will hold if we additionally require in A5 that Cov $(\varepsilon_t^2, x_j) = O(\rho^{|t-j|})$ for all $1 \leq t,j \leq T$, and $x_j = \left\{Z_j^2, \xi_j^2\right\}.$ More generally, consistency holds under the high-level assumption lim $_{T\to\infty} T^{-2} \sum_{k=1}^{T} \sum_{i,j}^{k} Cov\left(r_t^2, r_j^2\right) = 0.$

4. Finite-sample behavior

In this section we turn to the small-sample properties of the IT and KL tests discussed theoretically in the previous section. Our main goal is to analyze whether the asymptotic theory accurately characterize the small-sample properties of these tests in applications with a finite number of observations. Given its particular relevance and for the sake of saving space, we center our attention on the effects of additive outliers.

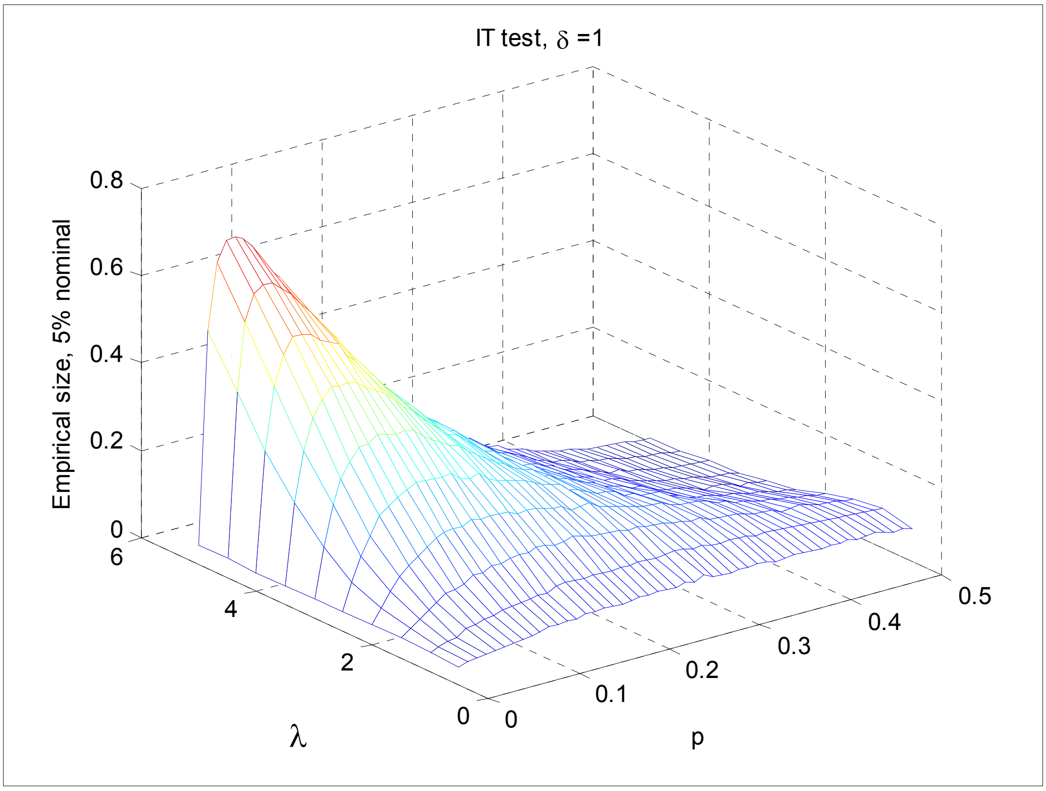
There are several reasons to concern about the small-sample performance of these procedures. First, serial dependence is known to slow down the speed of convergence of Kolmogorov-Smirnov type tests. Time-series with strongly persistent volatility may require much more observations in order to make accurate inference; see, for instance, the departures observed in the experimental analysis of Andreou and Ghysels (2002).

Similarly, outliers may interfere with the small-sample properties of the tests. Finally, consistency is purely an asymptotic property. As seen in the theoretical analysis, the characteristics of the contamination process may have pervasive influences in the ability to estimate correctly the break-fraction in a finite sample. We design several experiments to address possible small-sample size departures when the regular component is $A)$ an i.i.d. process, $B)$ it follows a time-varying volatility process, and to address $C)$ consistency of the turning point estimator in small-samples.

A) Empirical size: additive outliers and independent observations.

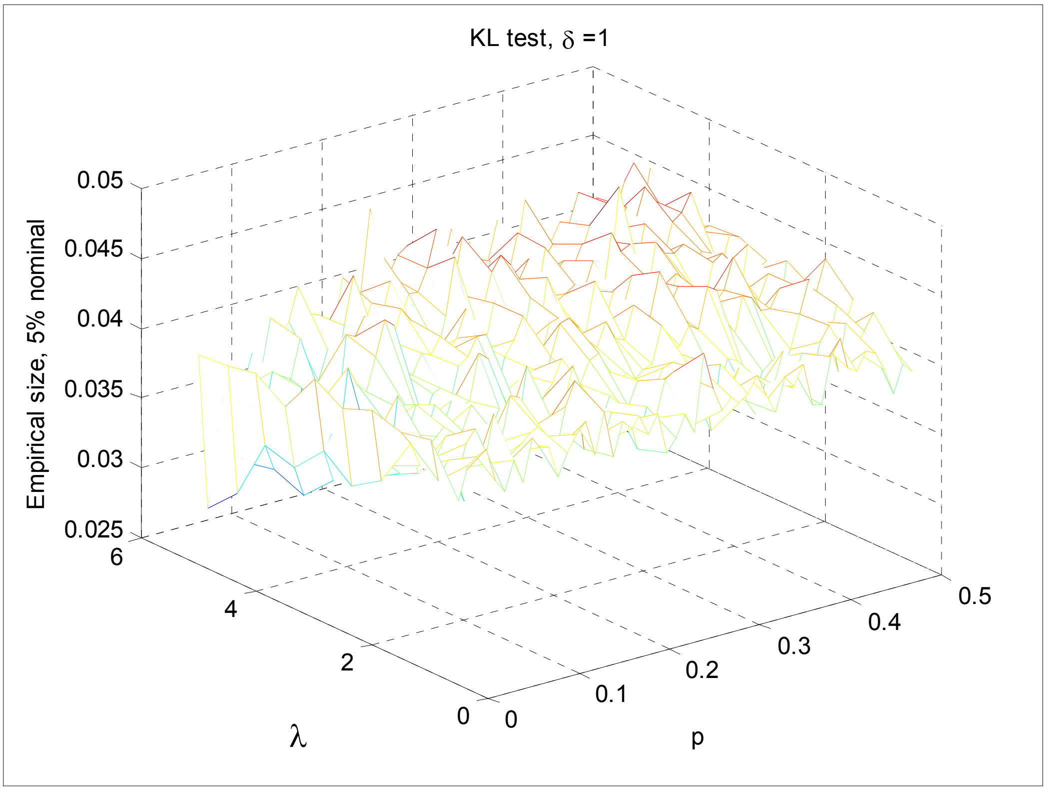
We first analyze the possibility of size distortions due to additive outliers by assuming that the regular component ε_t is i.i.d. We generate simulated paths for $r_t = \sigma\eta_t + \mathfrak{P}_t[\lambda + \delta v_t]$ with $\eta_t, v_t \sim iidN(0,1)$. The Bernoulli variable $\mathfrak{P}_t$ takes values $(-1,1,0)$ given the grid of probabilities $p = \{0, 0.01, \ldots, 0.50\}$, with increments of 0.01. We set $\lambda = \{0.5, 1, \ldots, 5\}$, with increments of 0.50 . In this way, we cover a wide set of realistic values for empirical purposes. We initially set $\delta = \sigma = 1$. The sample size is $T = 1000$ and we repeat the simulation process 25000 times for any combination of the analyzed values. The IT and KL tests are computed using the simulated series, and the corresponding statistics compared with the 95% percentile from the supremum of a Brownian bridge. The rejection rates of the null hypothesis are plotted in Figures 1 and 2.

Figure 1. Empirical size of the Inclán-Tiao test (5% nominal size) with outlier-contaminated data.



The data $\{r_t\}$ are simulated from $r_t = \sigma\eta_t + D_t[\lambda + \delta v_t]$, $\eta_t, v_t \sim Niid(0,1)$, $D_t = \{\pm 1, 0\}$ with probabilities $\{p/2, 1-p\}$. The probability of outliers is plotted on the x -axis, and the expected size of the outlier, λ, is plotted on the y -axis. We use 15,000 simulations for samples of length 1000 and $\delta = 1$. The test statistic is tested with the critical values from the supremum of a Brownian bridge under the null of variance homogeneity. The experimental proportion of rejections are plotted on the vertical axis.

Figure 2. Empirical size of the Kokoszka-Leipus test (5% nominal size) with outlier-contaminated data.



The data $\{r_t\}$ are simulated from $r_t = \sigma\eta_t + D_t[\lambda + \delta v_t]$, $\eta_t, v_t \sim Niid(0,1)$, $D_t = \{\pm 1, 0\}$ with probabilities $\{p/2, 1-p\}$. The probability of outliers is plotted on the x-axis, and the expected size of the outlier λ is plotted on the y-axis. The long-run variance of $r_t^2 - E(r_t^2)$ is computed through the Newey-West estimator with Bartlett kernel and bandwidth $h = \left[4(100/T)^{2/9}\right]$. We use 15,000 simulations for samples of length T=1000 and $\delta = 1$. The test statistic is tested with the critical values from the supremum of a Brownian bridge under the null of variance homogeneity. The experimental proportion of rejections are plotted on the vertical axis.

As discussed in the theoretical section, the distribution of the IT is strongly affected by the parameters that characterize the dynamics of the outlier process. We observe that this type of heterogeneity may lead to large departures from the nominal size, especially for small yet non-zero values of p together with large values of λ, which is precisely the type of process that may be expected in real data. These values generate excess of kurtosis and lead the IT test to over-reject. On the other hand, relatively large values of p lead to undersized tests. This may seem surprising, but from Theorem 3.1 we note that large values of p may lead to $tr(\Omega) < 2E(r_t^2)^2$, thus undersizing the test.[8] However, it seems unlikely that such a degree of heterogeneity can be observed in practice. Although not reported here, in order to save space, the distortion in size is amplified as δ increases (since kurtosis depends on this parameter as well). By contrast, the KL test shows a flat, uniform distribution for the empirical rate of rejection which does not depend on nuisance parameters. For an i.i.d. regular component, there are no significant distortions in small samples when the KL test is applied. The test shows a slightly negative bias which is observable even if $p = 0$, and therefore attributable to finite-sample bias.

B) Empirical size: additive outliers and time-varying volatility.

We now turn our attention to the case in which the regular component shows time-varying volatility patterns. Given its empirical relevance, we assume that ε_t follows a GARCH(1,1)-type model,

$$\begin{aligned} r_t &= \sigma_t \eta_t + Z_t;\ \ \eta_t \sim iidN(0,1) \\ \sigma_t^2 &= \omega + \alpha \psi_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \tag{11}$$

with $\omega > 0, \quad \alpha, \beta > 0$. There are two possible specifications, depending on whether outliers only affect the level of the series (*level outliers*), or affect both the level and the variance (*volatility outliers*); see Hotta and Tsay (1998). In the first case σ_t^2 is independent of $Z_t,$ and therefore $\psi_t = \sigma_t \eta_t$. Thus, the returns result from the convolution of a jump-process and a standard GARCH model. This is the case studied in the theoretical section, and the DGP considered in standard empirical applications. Alternatively, if σ_t^2 is also perturbed by AOs, it follows that $\psi_t = r_t$ and then the GARCH model includes jumps, a far more complex type of non-linear volatility process. We shall analyze both possibilities in our simulations.

First, consider the level-outlier case with AOs affecting only the conditional mean. We normalize the unconditional variance to unity by setting $\omega = (1 - \alpha - \beta),$ and set different values for (α, β). In particular, we consider the same DGPs as in Andreou and

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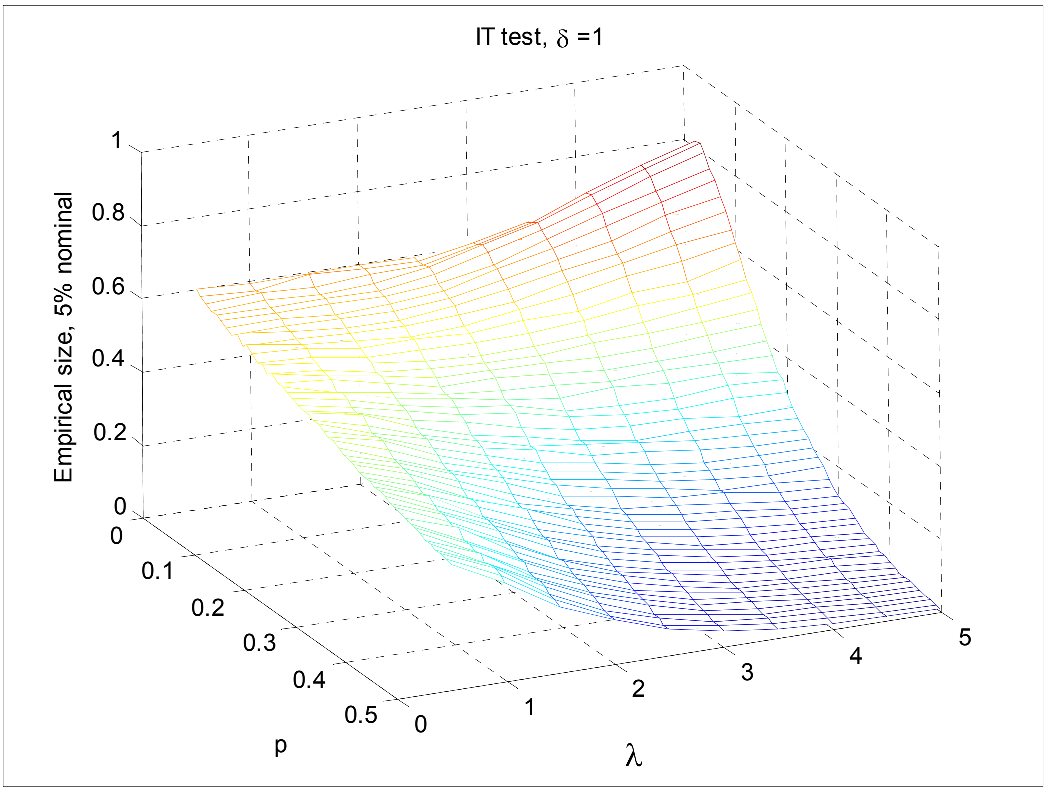
[8]For instance, if ε_t is an i.i.d. Gaussian series ($\kappa = 3$) and we set $\delta = 0$ for simplicity, then the condition $tr(\Omega) > 2E(r_t^2)^2$ reduces to $p < 1/3.$ A probability of outliers exceeding this threshold leads to undersized tests when using the standard limit distribution of the IT test.

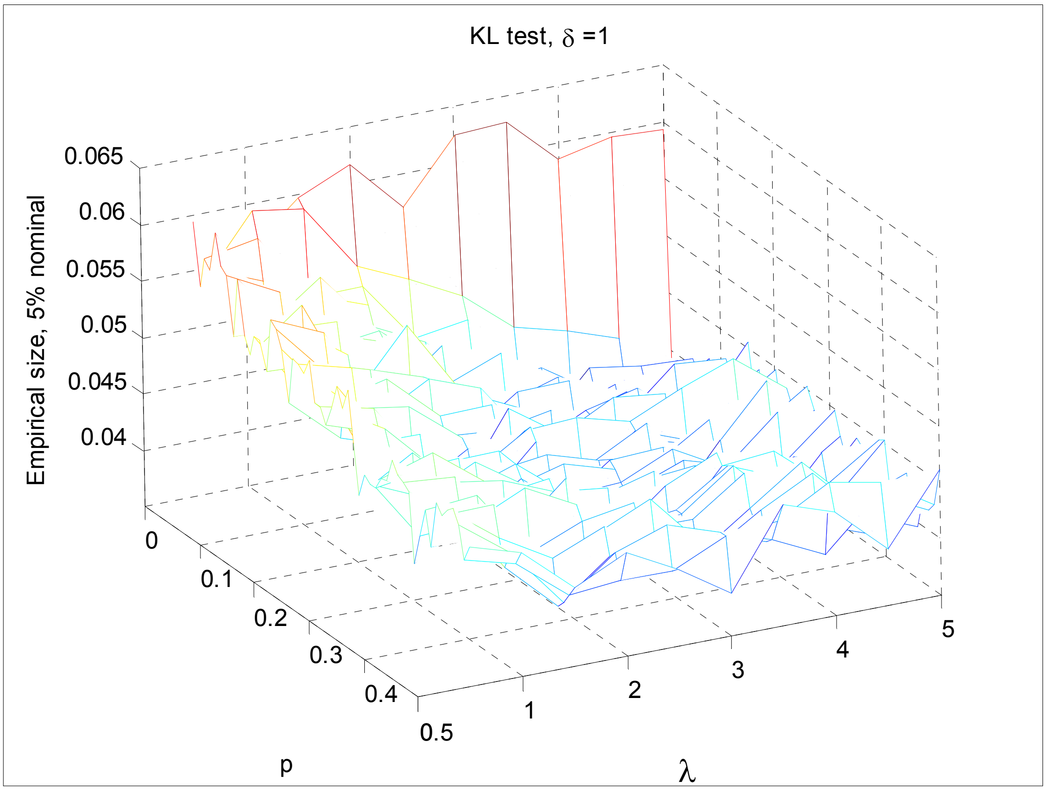
Ghysels (2002) to make comparisons with the results therein. These are a low persistent ($\alpha = 0.10, \beta = 0.50$) [GARCH1] and a highly-persistent GARCH model ($\alpha = 0.10, \beta = 0.80$) [GARCH2]. Simulations are performed as in experiment A), with the long-run variance paramete being computed for the KL test by using a Newey-West estimator with Bartlett kernel and a deterministic bandwidth selection procedure. The rate of rejections of the null hypothesis in this experiment are shown in Figures 3, 4 and 5.

Figure 3. Empirical size of the Inclán-Tiao test (5% nominal size) with GARCH [GARCH2] errors and outliers.



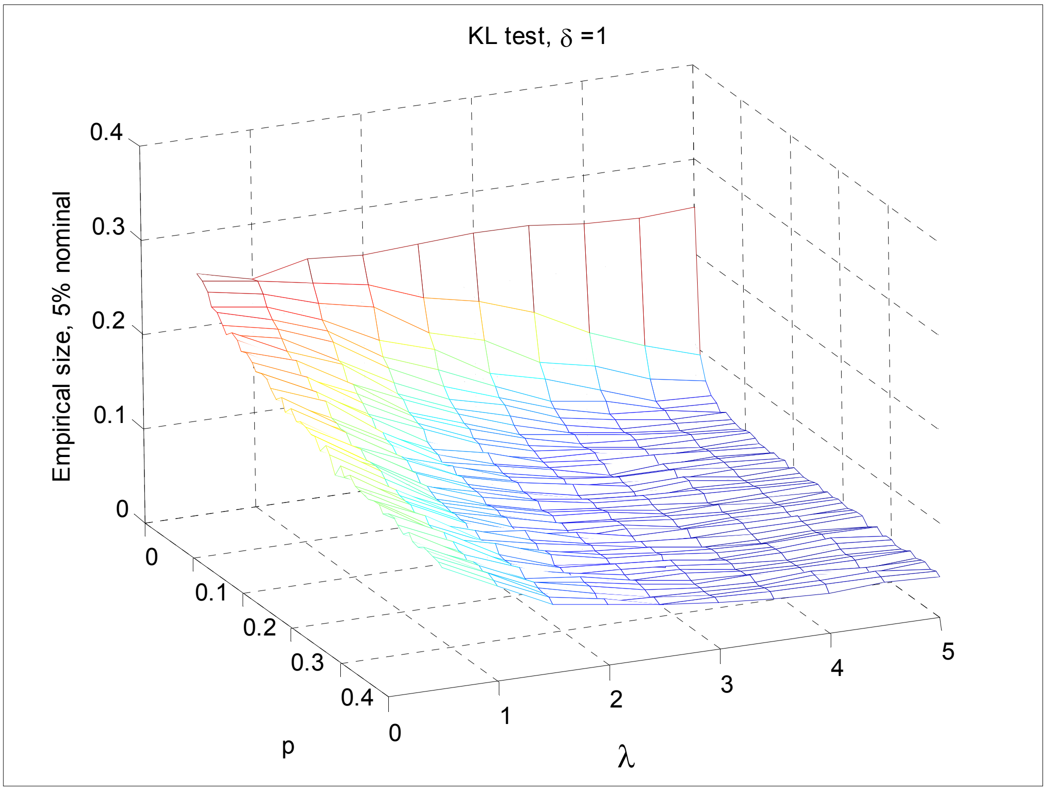
The data $\{r_t\}$ are simulated from $r_t = \sigma_t \eta_t + D_t[\lambda + \delta v_t]$, $\eta_t, v_t \sim Niid(0,1)$, $D_t = \{\pm 1, 0\}$ with probabilities $\{p/2, (1-p)\}$. The conditional volatility σ_t^2 follows a GARCH(1,1) process with parameters $(\alpha, \beta) = (0.1, 0.8)$. The probability of outliers is plotted on the y-axis, and the expected size of the outlier λ is plotted on the x-axis. The long-run variance of $r_t^2 - E(r_t^2)$ is computed through the Newey-West estimator with Bartlett kernel and bandwidth $h = \left[4(100/T)^{2/9}\right]$. We use 15,000 simulations for samples of length T=1000 and $\delta = 1$. The test statistic is tested with the critical values from the supremum of a Brownian bridge under the null of variance homogeneity. The experimental proportion of rejections are plotted on the vertical axis.

Figure 4. Empirical size of the Kokoszka-Leipus test (5% nominal size) with GARCH errors [GARCH1] and outliers.



The data $\{r_t\}$ are simulated from $r_t = \sigma\eta_t + D_t[\lambda + \delta v_t]$, $\eta_t, v_t \sim Niid(0,1)$, $D_t = \{\pm 1, 0\}$ with probabilities $\{p/2, (1-p)\}$. The probability of observing outliers is plotted on the y-axis, and the expected size of the outlier λ is plotted on the x-axis. The conditional volatility σ_t^2 follows a GARCH(1,1) process with parameters $(\alpha, \beta) = (0.1, 0.5)$. The long-run variance of $r_t^2 - (Er_t^2)$ is computed through the Newey-West estimator with Bartlett kernel and bandwidth $h = \left[4(100/T)^{2/9}\right]$. We use 15,000 simulations for samples of length T=1000 and $\delta = 1$. The test statistic is tested with the critical values from the supremum of a Brownian bridge under the null of variance homogeneity. The experimental proportion of rejections are plotted on the vertical axis.

Figure 5. Empirical size of the Kokoszka-Leipus test (5% nominal size) with GARCH errors [GARCH2] and outliers.



The data $\{r_t\}$ are simulated from $r_t = \sigma\eta_t + D_t[\lambda + \delta v_t]$, $\eta_t, v_t \sim Niid(0,1)$, $D_t = \{\pm 1, 0\}$ with probabilities $\{p/2, (1-p)\}$. The probability of outliers, p, is plotted on the y-axis, and the expected size of the outlier λ is plotted on the x-axis. The conditional volatility σ_t^2 follows a GARCH(1,1) process with parameters $(\alpha, \beta) = (0.1, 0.8)$. The long-run variance of $r_t^2 - (Er_t^2)$ is computed through the Newey-West estimator with Bartlett kernel and bandwidth $h = \left[4(100/T)^{2/9}\right]$. We use 15,000 simulations for samples of length T=1000 and $\delta = 1$. The test statistic is tested with the critical values from the supremum of a Brownian bridge under the variance homogeneity null hypothesis. The experimental proportion of rejections are plotted on the vertical axis.

Figure 3 shows the empirical size for the IT test for the GARCH2 model.[9] As expected, the empirical analysis confirms the results of the theoretical section. First, GARCH-type dependences generate excess of kurtosis leading to size departures in the IT test even if $p = 0$. When $p > 0$, rare extreme events (low p and large λ) considerably increase the total kurtosis of the distribution, leading to even larger size distortions. Figure 4 and 5 show the empirical size for the KL tests given GARCH1 and GARCH2 models, respectively. We can observe that a different degree of persistence generates has different effects. Remarkably, in absence of outliers, the KL test suffers of important small-sample size distortions for large values of $\alpha + \beta,$ as reported in Andreou and Ghysels (2002). This finite-sample distortion (recall that the test would display correct critical values asymptotically) is related to the fact that the estimator of the long-run variance parameter is known to suffer of large small-sample biases for strongly persistent data. In contrast to the case of the IT test, the existence of AOs does not worsen the behavior of the KL test respect to the case $p = 0$. The observed departures are solely attributable to patterns of depedence in the volatility process.

When outliers affect the both the conditional mean and the variance, i.e., setting $\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta\sigma_{t-1}^2$ in $(\text{ref. GARCH}),$ the departures from the nominal size of the IT test are even larger than before. For instance, in the case of the GARCH2 model we do not observe empirical sizes inferior to 50%. The excess of kurtosis is now ever greater, as the kurtosis of r_t increase as a result of the positive correlation between ε_t^2 and the lags of Z_t^2 . On the other hand, there are there are meaningful differences from the preceding case in the case of the KL test. As in the case of i.i.d. observations, the empirical rate of rejection seems almost totally invariant to the driving parameters of the jump process.

Therefore, in the context of stongly-persistent volatility patterns and additive outliers, the IT test is severely distorted and expected to provide unreliable inference. Similarly and despite the large-sample theory, the KL test can show large size departures when the volatility is strongly-persistent in a finite sample. It should be stressed that such distortions are attributable to the long-run dependence in volatility and not to additive outliers. Size departures are related to the poor performance of HAC-type estimators, which are known to suffer of important small-sample biases under serial depedence; since ε_t^2 is strongly correlated as $\alpha + \beta$ tends to one, more and more observations would be necessary to remove the finite-sample bias in the estimator, and hence eliminate size distortion. In the limit, if the researcher might dispose of an arbitrarily long sample, these distortions will end up disappearing completely, which nevertheless is not the case for the IT test.

[9]We omit presentation of results for the GARCH1 model in order to save space, since these are not qualitatively different from the results presented in the text. However, these are available upon request.

C) Finite-sample properties: consistency of the break-point estimator

As in Chen, Choi and Zhou (2005), the aim of our last experiment is to evaluate the average size of the estimation bias $(\hat{\tau} - \tau^*)$, and the corresponding standard error of the break-point estimate (efficiency) in a finite sample. We set the turning-point fractions $\tau^* = \{0.25, 0.50, 075\}$ and consider $\sigma_1 = 1$ as well as several values for Δ. As in the previous experiments, we repeat the simulations with a DGP in which the disturbances are i.i.d., and follow the GARCH1 and GARCH2 models. For the sake of conciseness, we summarize the results of this experiment for different values of (λ, p), for a relatively large shift $\Delta = 0.50$, and for $\delta = 1$ in Table 1. The table shows the average value of $\hat{\tau}$ and the standard error of the estimator in a sample of $T = 1000$.

First, it should be noticed that the testing procedures are consistent in detecting structural breaks, since increasing T and/or the magnitude of the shift makes the estimation bias and the standard error of $\hat{\tau}$ decrease (we do not show the results for different values of T or Δ for the shake of saving space). Nevertheless, for a finite value of T, AOs may lead to meaningful distortions. The extent of finite-size distortions depends on the number of available observations T, the shift-magnitude Δ, and the relative position of the break τ^*. For instance, note in Table 1 that even in absence of outliers, $\hat{\tau}$ may show an incresing skewness as the true change-point moves to the right. This is common to analogous CUSUM-type estimators, see also . Serial depedence also interferes with the ability of the test to identify the break location, and strongly-persistent GARCH patterns increase the estimation bias and deteriorate its efficiency. When allowing for additive outliers, we observe slightly larger distortions for moderate values of λ and p. However, the properties of $\hat{\tau}$ prove particularly sensitive to the size of outliers, since large values of λ are able to bias $\hat{\tau}$ towards 1/2 (as is the case when no break is present in the sample) and considerably increase the standard error of the estimates.

Table 1. Average Estmation, E(τ), and standard errors (x100) of τ.

	i.i.d						GARCH1						GARCH2					
	τ						τ						τ					
	0.250		0.500		0.750		0.250		0.500		0.750		0.250		0.500		0.750	
(λ,p)	E(τ)	s.e.	E(τ)	s.e.	E(τ)	s.e.	E(τ)	s.e.	E(τ)	s.e.	E(τ)	s.e.	E(τ)	s.e.	E(τ)	s.e.	E(τ)	s.e.
(0,0)	0.289	0.050	0.509	0.020	0.745	0.020	0.306	0.070	0.514	0.030	0.742	0.030	0.358	0.120	0.530	0.060	0.728	0.080
(0,0,10)	0.293	0.060	0.509	0.020	0.743	0.030	0.309	0.080	0.515	0.030	0.740	0.040	0.361	0.120	0.530	0.060	0.728	0.080
(0,0.25)	0.297	0.070	0.510	0.020	0.739	0.040	0.312	0.080	0.515	0.030	0.736	0.040	0.362	0.130	0.530	0.060	0.724	0.090
(0,0.50)	0.304	0.080	0.512	0.030	0.734	0.040	0.319	0.090	0.519	0.040	0.733	0.050	0.367	0.130	0.530	0.060	0.720	0.090
(1,0)	0.289	0.050	0.509	0.020	0.745	0.020	0.307	0.070	0.514	0.030	0.742	0.030	0.358	0.120	0.530	0.050	0.730	0.080
(1,0,10)	0.297	0.060	0.501	0.020	0.739	0.030	0.314	0.080	0.515	0.030	0.736	0.040	0.361	0.120	0.530	0.060	0.723	0.090
(1,0.25)	0.306	0.080	0.510	0.030	0.734	0.050	0.322	0.090	0.516	0.040	0.729	0.060	0.367	0.130	0.530	0.060	0.717	0.100
(1,0.50)	0.320	0.100	0.513	0.040	0.721	0.070	0.333	0.110	0.518	0.050	0.719	0.080	0.375	0.140	0.532	0.070	0.707	0.110
(2.5,0)	0.289	0.050	0.509	0.020	0.746	0.020	0.307	0.080	0.514	0.030	0.742	0.030	0.357	0.120	0.528	0.060	0.729	0.080
(2.5,0,10)	0.320	0.100	0.510	0.050	0.714	0.080	0.333	0.110	0.515	0.050	0.710	0.090	0.378	0.140	0.531	0.080	0.701	0.120
(2.5,0.25)	0.351	0.140	0.514	0.080	0.683	0.120	0.367	0.140	0.517	0.080	0.682	0.130	0.394	0.160	0.529	0.100	0.670	0.150
(2.5,0.50)	0.391	0.170	0.518	0.110	0.655	0.160	0.396	0.170	0.524	0.110	0.656	0.160	0.420	0.170	0.536	0.120	0.647	0.180
(5,0)	0.289	0.060	0.509	0.020	0.746	0.020	0.307	0.080	0.514	0.030	0.742	0.030	0.358	0.120	0.530	0.060	0.729	0.080
(5,0,10)	0.412	0.190	0.506	0.140	0.603	0.190	0.415	0.190	0.509	0.140	0.604	0.190	0.431	0.190	0.519	0.150	0.601	0.200
(5,0.25)	0.455	0.200	0.514	0.170	0.566	0.210	0.459	0.200	0.513	0.170	0.569	0.210	0.465	0.200	0.522	0.180	0.567	0.210
(5,0.50)	0.477	0.210	0.523	0.180	0.556	0.220	0.485	0.210	0.523	0.190	0.561	0.220	0.488	0.210	0.526	0.190	0.556	0.220

From this experiment, we observe that relatively large additive outliers may generate large biases in finite samples, even if they occur with a small probability. The reason is that the additional variability generated by multiple outliers is able to mask the true position of a shift. Intuitively, whereas the IT test will tend to find spurious breaks from using over-conservative critical values, the KL test may be biased towards non-detection in this context because the correct critical values may be applied on wrongly estimated turning-points, thereby rejecting the null. Furthermore, even if the test were able to reject the null, it would be likely the case that the estimation of the break-point is inaccurate.

It is interesting to deep into this idea through further experimentation. Consider the most favorable case for the KL test in which ε_t is i.i.d., and asssume that a large single shift increases the unconditional variance from 1 to 1.5 in a sample of 1000 observations. We consider the same relative location for this break as before, namely $\tau^* = \{0.25, 0.50, 0.75\}$, and apply the KL test. In absence of outliers, the average value of $\hat{\tau}$ is always in the neighborhood of τ^*, and the probability of rejecting the false null hypothesis is nearly 100% in using the 5% asymptotic nominal size. On the other hand, if the series is contaminated with AOs (e.g, setting $\lambda = 5$ and $p = 0.10)$ the average value of $\hat{\tau}$ turns out to be heavily biased, $\{0.41, 0.51, 0.41\}$. Most importantly, the probability of rejection the false null of homogenous variance collapses dramatically towards $\{21.1\%, 41.2\%, 25.7\%\}$. Note that in the most favourable case for the KL test in which the break occurs in half of the sample, the power reduction nearly reaches 60% with respect to the no-outliers case. Moreover, when we analyze the cases in which the test is able to reject, the conditioned distribution of $\hat{\tau}$ given τ^* and a previous rejection is heavily skewed, showing expectations which overwhemingly differ from the true values. These biases are entirely attributable to a finite-sample effect, and vanish as the sample size increases.

Finally, it has been argued that using the absolute value of the returns as a proxy for volatility instead of the squares may enhance results if one suspects that outliers contaminate the sample. We replicate the previous analysis by considering this proxy of volatility. Although this strategy is able to reduce the bias of $\hat{\tau},$ the enhancement is very conservative and far from being satisfactory in the cases in which the results based on squared-series are distorted.

5. Conclusions

In this paper, we have analyzed the size properties of CUSUM-type tests for detecting structural breaks in variance when the series of interest include some of the most relevant features that characterize financial data. Our especial focus has been on additive outliers, which prove able to generate large distortions of the test proposed by Inclán and Tiao (1994). This test was originally intended for Gaussian i.i.d. series and, therefore, considerable caution should be exercised when it is applied to financial data. On the other hand, the asymptotic distribution of the procedure of Kokoszka and Leipus (2000) is consistent against a wide class of errors provided the conditions for the central limit theorem for dependent and heterogenous data hold. From our analysis, we show that the distribution of the test statistic is robust against additive outliers and measurement errors in asymptotic samples. Furthermore, the estimator of the break point is shown to be consistent. Unfortunately, in the case of small samples, the characteristics of the outlier generating process may lead to strong distortions in dating the break and misleading conclusions.

A question of empirical relevance is what to do with extreme anomalous observations in order to perform a given statistical procedure, such as testing for variance homogeneity. Although outliers do not have significant effects on the asymptotic distribution of the KL test, they can generate large distortions in finite samples, as the test shows poor ability to date unbiasedly the break position. This evidence makes it necessary to resort to the use of robust procedures, such as deleting or control methods. Bai (1995, 1998) early proposed to use robustified procedures in the context of regressions with structural changes, see also the bounded-influence estimators proposed in Fiteni (2004). These methods can largely outperform least-squares based estimators under possible contaminated distributions. Since the KL test strongly builds on least-squares estimates, the use of bounded-influence estimators may provide further improvements in the small-sample performance of this test. This is an interesting question which is left for future research.

Appendix

Proof of Lemma 3.1.

Consider the process defined in A1. Without loss of generality, set $\mu = 0$ and note that

$$\begin{aligned} r_t^2 &= \varepsilon_t^2 + Z_t^2 + \xi_t^2 + 2\varepsilon_t Z_t + 2\varepsilon_t \xi_t + 2\xi_t Z_t \\ &= \mathbf{1}'\Pi_t \end{aligned}$$

with the vector Π_t being trivially defined and $\mathbf{1}$ is a conformable vector of ones. Furthermore,

$$\begin{aligned} E(r_t^2) &= E(\varepsilon_t^2) + E(Z_t^2) + E(\xi_t^2) \\ &= E(\mathbf{1}'\Pi_t) = \mathbf{1}'E(\Pi_t) \end{aligned}$$

where from the assumptions presented for the processes involved the expected values of the cross products is zero.
Hence, we can write

$$\begin{aligned} \sum_{t=1}^{[T\tau]}[r_t^2 - E(r_t^2)] &= \sum_{t=1}^{[T\tau]}(\varepsilon_t^2 - \sigma_\varepsilon^2) + \sum_{t=1}^{[T\tau]}[Z_t^2 - E(Z_t^2)] + \sum_{t=1}^{[T\tau]}(\xi_t^2 - \sigma_\xi^2) \\ &\quad + 2\sum_{t=1}^{[T\tau]} Z_t\varepsilon_t + 2\sum_{t=1}^{[T\tau]} \xi_t\varepsilon_t + 2\sum_{t=1}^{[T\tau]} \xi_t Z_t \\ &= I_1 + I_2 + I_3 + I_4 + I_5 + I_6 \\ &= \sum_{t=1}^{[T\tau]} \mathbf{1}'\Pi_t - \mathbf{1}'E(\Pi_t) = \sum_{t=1}^{[T\tau]} \mathbf{1}'[\Pi_t - E(\Pi_t)] \end{aligned}$$

where it is easy to show that these terms are uncorrelated because of the i.i.d. property of $\{Z_t, \xi_t\}$ and the independence with $\{\varepsilon_t\}$. Under conditions A3 or A3', all the terms, I_j, $j = 1, \ldots, 6,$ involved satisfy a functional central limit theorem (FCLT) and it can be shown that $I_j = O_p\left(\sqrt{T}\right)$. More specifically, the functionals $I_1, I_4, I_5,$ satisfy a FCLT for mixing sequences (martingale differences) under A3 (A3'), while I_4, I_5, I_6 verify directly the FCLT from Donsker's lemma under A2; see White (2000) for an overview. Therefore, it is straightforward to show that under these assumptions, as $T \to \infty$,

$$\frac{1}{\sqrt{T}} I_j \Rightarrow \sqrt{\omega_{jj}}\, W(\tau)$$

where $\omega_{jj} \equiv \lim_{T\to\infty} T^{-1} E\left(\left\{\sum_{t=1}^{T} [\pi_{jt} - E(\pi_{jt})]\right\}^2\right),$ with π_{jt} being the j -th element of Π_t.

Finally, since the I_j terms are uncorrelated, it follows from the Gaussian properties of the

Wiener process that

$$T^{-1/2}\sum_{t=1}^{[T\tau]}[\Pi_t - E(\Pi_t)] \Rightarrow \Omega^{1/2}\mathbf{W}(r),$$

where $\mathbf{W}(r)$ is a 6-dimensional Wiener process and the matrix $\Omega = \{\omega_{ii}\}$ is diagonal with elements defined in Lemma 3.1. The joint convergence follows straightforwardly from stacking the individual components into a vector. The Cramer-Rao device completes the proof. ■

Proof of Lemma 3.2.

Let **1** be the a vector of ones in R^6 . Since

$$T^{-1/2}\sum_{t=1}^{[T\tau]}[\tilde{r}_t^2 - E(r_t^2)] = T^{-1/2}\sum_{t=1}^{[T\tau]}\mathbf{1}'[\Pi_t - E(\Pi_t)] + o_p(1)O_p(1)$$

it follows from standard probabilistic convergence that the limit distribution of the functional converges weakly to the distribution of $\mathbf{1}'\mathbf{W}(\tau) = W(\tau)$, a standard Wiener process, with scalar variance $\mathbf{1}'\Omega\mathbf{1} = tr(\Omega).$ This yields the required result. Part (ii) of the lemma is immediate by considering the particular case $\tau = 1$, and part (iii) follows from applying an argument of weak law of large numbers, as for Lemma 3.1; see White (2000). ■

Proof of Theorem 3.1.

We first show the convergence of the IT test. Observe that $D_T(k)$ can be rewritten as

$$C_T(T)^{-1}[C_T(k) - (k/T)C_T(T)] = C_T(T)^{-1}G_T(k) + o_p(1)$$

where $C_T(k) = \sum_{t=1}^{k} r_t^2$. Therefore,

$$T^{-1/2}D_T(k) = [C_T(T)/T]^{-1}[T^{-1/2}G_T(k)] + o_p(1)$$

From i) and ii) of Lemma 3.2 it follows that

$$\begin{aligned} T^{-1/2}[C_T(k) - (k/T)C_T(T)] &\Rightarrow \sqrt{M_4}\,W(\tau) - \sqrt{M_4}\,\tau W(1) \\ &= \sqrt{M_4}\,W^*(\tau) \end{aligned}$$

with

$$M_4 \equiv \lim_{T\to\infty} T^{-1} E\left(\left\{ \sum_{t=1}^{T} [\pi_{jt} - E(\pi_{jt})] \right\}^2 \right) = tr(\Omega)$$

and from Lemma 3.2iii),

$$C_T(T)/T \overset{p}{\to} Var(r_t) = \sigma_\varepsilon^2 + p(\lambda^2 + \delta^2) + \sigma_\xi^2.$$

From this result and the continuos mapping theorem, it follows that

$$\max_{1\le k\le T} \sqrt{T/2}\,|D_T(k)| \Rightarrow \frac{M_4^{1/2}}{\sqrt{2}\,(\sigma_\varepsilon^2 + p\lambda^2 + p\delta^2 + \sigma_\xi^2)} \sup_{\tau\in[0,1]} |W^*(\tau)|.$$

In case of the KL test, assume that $\widehat{M}_{4,T}$ is a consistent estimator of M_4. From Lemma 3.2 and the continouos mapping theorem it follows straightforwardly

$$T^{-1/2}\widehat{M}_{4,T}^{-1/2} \max_{1\le k\le T} |G_T(k)| = \max_{1\le k\le T} \left| T^{-1/2}\widehat{M}_{4,T}^{-1/2} C_T(k) - T^{-1/2}\widehat{M}_{4,T}^{-1/2} (k/T) C_T(T) \right|$$

$$\Rightarrow \sup_{\tau\in[0,1]} |W^*(\tau)|.$$

This completes the proof. ■

Proof of Theorem 3.2.

First observe that the estimator $\hat{k}$ is the solution of the objective function

$$\max_{1\le k\le T} \left| \sum_{t=1}^{k} r_t^2 - \frac{k}{T} \sum_{t=1}^{T} r_t^2 \right|$$

which is algebraically equivalent to

$$\max_{1\le k\le T} \frac{k(T-k)}{T} \left| \frac{1}{k} \sum_{t=1}^{k} r_t^2 - \frac{1}{T-k} \sum_{t=k+1}^{T} r_t^2 \right| \equiv \max_{1\le k\le T} |R_k|$$

where $\sum_{t=1}^{k} r_t^2/k$ and $\sum_{t=k+1}^{T} r_t^2/(T-k)$ are the least-squares estimators of the unconditional variance of the first k and last $T-k$ observations. Therefore, for any $\tau = [k/T]$ and for $k^* = [\tau^* T]$ we note

$$E(R_k) = \Delta\tau(1-\tau^*)\mathbf{1}_{k\le k^*} + \Delta\tau^*(1-\tau)\mathbf{1}_{k>k^*}$$

and

$$E(R_{k^*}) = \Delta\tau^*(1-\tau^*)$$

which implies that

$$|E(R_{k^*})| - |E(R_k)| = |\Delta|(\tau^* - \tau)(1-\tau)\mathbf{1}_{k\le k^*} + |\Delta|\tau^*(\tau - \tau^*)\mathbf{1}_{k>k^*}$$

with $\mathbf{1}_{(\cdot)}$ being an indicator function, and hence

$$|E(R_{k^*})| - |E(R_k)| \geq |\Delta||\tau^* - \tau| \min\{\tau^*, 1 - \tau^*\}.$$

Based on this result, it can be shown that

$$|\Delta||\tau^* - \tau| \min\{\tau^*, 1 - \tau^*\} \leq 2 \max_{1 \leq k \leq T} |R_k - E(R_k)|$$

or, equivalent,

$$|\tau^* - \tau| \leq \frac{1}{T} \max_{1 \leq k \leq T} \frac{4 \sum_{t=1}^{k} \left| r_t^2 - E(r_t^2) \right|}{|\Delta| \min\{\tau^*, 1 - \tau^*\}}.$$

From the Hájek-Rényi type inequality shown in Theorem 4.1 in Kokoszka and Leipus (2000), it follows that

$$\epsilon^2 \Pr\left(\frac{1}{T} \max_{1 \leq k \leq T} \sum_{t=1}^{k} \left| r_t^2 - E(r_t^2) \right| > \epsilon \right) \leq \frac{2}{T^2} \sum_{k=1}^{T-1} \left(Var(r_{k+1}^2) \left(\sum_{i,j=1}^{k} Cov\left(r_i^2, r_j^2\right) \right) \right)^{1/2} + \frac{1}{T^2} \sum_{k=0}^{T-1} Var(r_{k+1}^2).$$

Recall that $r_t^2 = 1'\Pi_t,$ with Π_t defined in the proof of Lemma 3.1. Since $\{Z_t, \xi_t\}$ is an i.i.d. sequence which is independent of ε_t^2, and Cov $\left(\varepsilon_i^2, \varepsilon_j^2\right) = O(\rho^{|t-j|}),$ it follows that $Cov\left(r_i^2, r_j^2\right)$ has finite upper bounds that decay exponentially. Note that, from the independence of $Z_t,$ it follows that

$$Cov\left(r_i^2, r_j^2\right) = Cov\left(\varepsilon_i^2, \varepsilon_j^2\right) + Var(Z_i)\mathbf{1}_{i=j} + Var(\xi_i)\mathbf{1}_{i=j}.$$

For $i = j,$ $0 \leq Cov\left(r_i^2, r_j^2\right) \leq \pi,$ with $\pi = C_1^* + Var(Z_i) + Var(\xi_i) < \infty$ for some constant $C_1^* > 0$. Note that, from the Cauchy-Schwartz inequality, it follows straightforwardly that $C_1^* \leq \sup_t E(\varepsilon_t^4)$. For $i \neq j$, $Cov\left(r_i^2, r_j^2\right) = Cov\left(\varepsilon_i^2, \varepsilon_j^2\right)$ and therefore $0 \leq Cov\left(r_i^2, r_j^2\right) \leq C_1^* \rho^{|i-j|}$ for some $0 \leq \rho < 1.$ Note that ρ rules the temporal structure of the correlations as a function of the parametric specification of the model (see Kokoszka and Leipus (2000) for a discussion in the case of an ARCH (∞) model).

Hence, it follows that $0 \leq Cov\left(r_i^2, r_j^2\right) \leq \pi\rho^{|i-j|}$ uniformly for $1 \leq i,j \leq T.$ Denote $\sigma_4^* = \max_{1<t<T} Var(\varepsilon_t^2).$ Then,

$$\epsilon^2 \Pr\left(\frac{1}{T} \max_{1 \le k \le T} \sum_{t=1}^{k} \left| r_t^2 - E(r_t^2) \right| > \epsilon \right) \le \frac{2\sqrt{\sigma_4^*}}{T^2} \sum_{k=1}^{T-1} \left(\sum_{i,j=1}^{k} Cov\left(r_i^2, r_j^2\right) \right)^{1/2} + \frac{\sigma_4^*}{T}$$

$$\le \frac{2\sqrt{\sigma_4^* \pi}}{T^2} \sum_{k=1}^{T-1} \left(\sum_{i,j=1}^{k} \rho^{|i-j|} \right)^{1/2} + \frac{\sigma_4^*}{T}$$

$$\le \frac{2\sqrt{\sigma_4^* \pi}}{T^2 \sqrt{(1-\rho)}} \sum_{k=1}^{T-1} k^{1/2} + \frac{\sigma_4^*}{T}$$

$$\le \frac{K_1}{\sqrt{T}} + \frac{\sigma_4^*}{T} \le \frac{K_2}{\sqrt{T}}$$

where $K_1 = (4/3)\sqrt{\sigma_4^* \pi/(1-\rho)}$ and for some $K_2 > K_1 > 0$. Finally,

$$\Pr(|\tau^* - \tau| > \epsilon) \le \Pr\left(\frac{1}{T} \max_{1 \le k \le T} \sum_{t=1}^{k} \left| r_t^2 - E(r_t^2) \right| > \frac{\epsilon |\Delta| \min\{\tau^*, 1-\tau^*\}}{4} \right)$$

$$\le \frac{16 K_2}{\epsilon^2 \Delta^2 (\min\{\tau^*, 1-\tau^*\})^2 \sqrt{T}} \le \frac{C}{\epsilon^2 \Delta^2 \sqrt{T}}.$$

This completes the proof .■

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