MEASUREMENT AND ANALYSIS OF THE SPANISH STOCK EXCHANGE USING THE LYAPUNOV EXPONENT WITH DIGITAL TECHNOLOGY

Salvador Rojí Ferrari
Ana Gonzalez Marcos

FUNDACIÓN DE LAS CAJAS DE AHORROS
DOCUMENTO DE TRABAJO
Nº 246/2006

De conformidad con la base quinta de la convocatoria del Programa de Estímulo a la Investigación, este trabajo ha sido sometido a evaluación externa anónima de especialistas cualificados a fin de contrastar su nivel técnico.

ISBN: 84-89116-07-5

Measurement and analysis of the Spanish Stock Exchange using the Lyapunov exponent with digital technology

Salvador Rojí Ferrari srojifer@ccee.ucm.es U. Complutense de Madrid
Ana Gonzalez Marcos agonmar@tfo.upm.es U. Politécnica de Madrid

Abstract

This work tries to find some kind of non linear dependency on the Spanish Stock Exchange by using a metric test, the Lyapunov exponents. Although, there have been some studies that have tried to find out chaotic determinism by using this methodology, the results have not been very conclusive. These studies have used an analogical technology. Our goal is to find those exponents through the use of a digital technology that we have developed to avoid some of the adjustments and tune-ups previously required. To do so, we need to adapt some functions and data to financial time series. The result is not different than the one found in other Stock Exchange time series, namely, there is a chaotic signal in the Spanish Stock Exchange, albeit at high dimensions, and therefore, not feasible for obtaining risk-adjusted extra returns in an efficient financial market like the Spanish one.

Keywords

Non linearity, chaotic determinism, financial time series, digital technology

1. Introduction

Nowadays, there are many manuals that study non linear time series, including deterministic chaos and complexity structures within those time series. The methods that study these series can be broken down in classical statistics, metric, topological, and other methods (R/S analysis, 1/f noise, fractional Brownian motion, integrated fractional, series, statistical mechanics (SMFM), and the family of ARCH models, more related to classical statistics).

The metric methods measure the distances between points within an strange atractor, whereas the topological ones study the organization of the points within a strange atractor. Both groups of methods are specifically designed to study deterministic chaos. The most important topological methods are the recurrence tests and the nearest neighbours, whereas the most important metric tests are the correlation, capacity, information, and fractal dimensions, Kolmogorov entropy, and our focus of this paper, the methodology of the Lyapunov exponents. While the dimension gives us an estimation of the complexity of the system, the entropy and the exponents compute the level of chaos in the dynamical system. Although there are some packages of analogical software designed to try to calculate the exponents, their parameters require excessive tune-up. They are also very sensitive to noise. We expect to refine the procedure through the use of a digital technology that overcomes these problems. Therefore, the goal of this paper is to use a digital technology to find out the level of chaos in the Spanish stock exchange time series. This new approach, nevertheless, implies to carry out some adaptations.

Lyapunov exponents measure the mean exponential rate of divergence (or convergence) of two adjacent orbits (Fig. 1). They represent one of the basic characteristics of deterministic chaos: sensitivity to initial conditions. There are as many exponents as degrees of freedom. A negative exponent means that the trajectories converge in the same direction. This means that the signal has a mean reversion behaviour, leading toward a fixed point. If it were positive, then, the divergence become wider, confirming the chaotic behaviour hypothesis. The negative exponent compress the system, while the positive one expands it (its sum is identical to the Kolmogorov entropy, which gives information about the reduction of the system predictability). In this way, only one positive exponent can show sensitivity to initial conditions, showing a “direction”. Therefore. the long term behaviour of any initial conditions specified with uncertainty can not be expected. In reality, this means that the behaviour is chaotic. If the result is infinite or goes to infinite, then, we have a random signal (assuming the corrections have been done correctly). If the result is null, then, the sequence is stable (periodic).

2. Background

Farmer y Sidorowich (1988) relate the exponents to the speculative bubbles, denying the use of financial models based on rational expectations. Therefore, the exponents measure, through the higher exponent, the amount of instability in the time series. Another more classical method is based on the probability distribution.. As early as Mandelbrot, in 1963, observed that the frequency distribution of daily stock exchange returns has thick tails, assuming that the reason for this anomaly was speculative bubbles (Fama and French, 1996; Brock and Malliaris, 1991; Shiller, 1989). Nevertheless, others (Flood, Hodrick, 1986) think that the excessive time series volatility is due to a poor specification of the model.

In either case, the Lyapunov exponent (and the correlation dimension) has been used mainly in the field of Physics applied to the study of chaos. Their use, in financial or economical time series, requires an adaptation (Isham, 1993). Notwithstanding, this adaptation has generated high correlation dimensions, which means that the structure of the financial time series is closer to randomness than to deterministic chaos, and, as Brock and Sayers suggested (1988), this kind of tests to detect the existence of chaos in the time series is very weak. Gilmore(1992) points out that using this type of filters or adaptations affects not only the correlation dimension, but also the Lyapunov exponent. Due to little evidence of deterministic chaos at low dimension levels in the financial time series (Brock, 2000), other tests have been developed with the intention to discover and measure their level of nonlinearity (whiteness and random diagnostic). In this work, and following to Brock and Sayers (1988), we will increase the dimension in order to try to get the signal of the exponent.

The economic consequence of a certain deterministic chaos in the stock exchange time series will depend on the level of complexity of the dynamical system or the degree of their positive exponents. Nevertheless, many studies (see Brock, 2000) have already shown that, after considering trading costs, the capacity of obtaining risk-adjusted extra returns in efficient financial markets by using any chaos model is null. Therefore, research in this field is mainly academic and speculative. Chaos models, like other "*quanta models*", are just tools that try to prove or disapprove the Efficient Market Hypothesis more than trying to forecast short-term future stock exchange returns.

In Spain, there are some research papers on deterministic chaos in financial time series (Blasco de las Heras, et al, 1995; Fernández Rodríguez y Martí 1995; and Olmeda & Pérez ,1995). They found, depending on the model, internal structures and non-linearity. Blasco et al. (1995), using the algorithm of Wolf et al (1985) to the IGBM (Madrid Stock Exchange General Index), and covering the period 1980-1993, showed exponents higher than zero in all cases studied, but without reaching any conclusion about a possible implicit determinism in the series due to the instability of the correlation dimension. They applied the standard methodology to different sets based in a sensitivity analysis, producing some results that support the weak presence of deterministic chaos. On the other hand, Bajo et al (1992) express the idea of chaos existence in exchange rate series. However, we must be cautious about the trustworthiness of the results because the number of samples and data is too small.

Brock, back in 1986, warned about the possibility of estimating artificially low dimensions due to the use of small economic and financial time series. Tata y Vassiliscos (1993) used the Lyapunov exponents on the NYSE covering over a century, with more than 29,000 observations, being the longest economic series ever used without finding any track of chaotic behaviour. However, when the series were divided in small samples, they found positive exponents, converging to zero when the number of observations in each sample is higher. They concluded that the positive results are consequence of using not enough data.

Brock and Sayers (1988) also concluded that, using financial time series of high frequency, chaos is "extremely improbable" due to the frictionless, and lack of measurement errors. For Jensen (1987), there is an indivisibility of the observed data that could alter any underlying chaotic process. Besides, when changes are very small, transaction costs made the chaotic mechanism very difficult to detect. Regarding the series frequency spectrum, it is convenient to check other kind of higher amplitude, because the daily ones are too noisy, although if the amplitude were too low, stationarity could ensued.

There are many ways to define the exponent (Lorenz, 1993; Gilmore, 1992; Dechert y Gencay, 1992) being the one of Wolf et al. (1985) the most used: "Giving a continuous dynamic system in a n-dimension space phase, the infinitesimal sphere of the initial conditions moves in an n-dimension ellipsoid object due to the deforming nature of the flux".

The exponent *i* of a dimension on terms of the extension of the ellipsoid principal axis

$$\lambda_i = lim_{t\to\infty} \frac{1}{t} \log_2 \frac{p_i(t)}{p_i(0)}$$

where the exponents λ_i are classify from high to small. As the ellipsoid orientation changes continuously, the direction associate to the exponent can not be defined. The linear relation of the ellipsoid grows at the rate $2^{\lambda i\ t}$, the area defined by the two axes grows at the rate $2^{(\lambda_1+\lambda_2)\ t}$, the volume defined by the three axis grows at the rate $2^{(\lambda_1+\lambda_2+\lambda_3)\ t}$, and so on. This property generates another definition of the exponent spectrum: the sum of the first *j* exponents by the exponential growth rate.

An attractor of a dissipative system with at least a positive exponent is chaotic, and the sign (+/-) gives information about the quality of the dynamical system. The greater exponent will define the horizon of prediction further of what it is possible to predict.

Many researches have proposed algorithms to estimate the higher exponent, but they have been considered of little reliability, at least for small sets, and difficult to implement and compute. Other simpler methods, though, like the delay method are based on analogical methodology (Sato el al., 1987).

Our paper has adapted some of the techniques applied to evaluate analogical sequences to digital form. To do so, we have developed several programs to analyse different aspects of the sequence (neighbour search, distance measurement between neighbour processes, and finally, the Lyapunov coefficients. This process of subdivisions makes the development, checking, and implementation processes easier, avoiding the continuous use of commands to execute the subprograms. To do so, we have developed another "execute program" using Matlab, and whose parameters are the only functional data needed. This program is able to execute other subprograms: accumulate, cluster, configure, digitalize, statistic, lysp, series, and neighbour.

There are two basic methods to calculate the coefficients, and the selection of the most suitable depends on the knowledge about the signal generating equation. If it is known, the log of the derivative module of the equation that produces the signal for all range of starting values is calculated. If it is not known, then, the procedures have to be carried out directly on the whole data series using the mean of many starting points. The result, though, is similar to the one obtained with the derivative system. This second method, used in this paper, could, nevertheless, suffer some kind of upward bias due to the fact that the coefficients are calculated in a noisy system. Also, although we do not have a precise knowledge of the dimension of the financial time series (stock exchange returns and exchange rate changes), many research papers (Brock, 2000) have shown that the complexity or the underling dimension of the time series, is six, which it is much higher than previously thought.

Our goal is to find those exponents through the use of a digital technology that we have developed to avoid some of the adjustments and tune-ups previously required. To do so, we need to adapt some functions and data to financial time series.

3. Empirical Results

Starting with a dimensional set between five and seven, a system of starting point averages is used, from which a point and its neighbours are randomly chosen in such a way that its distance be inferior than a specific threshold ε. The difference between the following points of the series is calculated, using as the raw material the difference averages. Subsequently, the rate growth of the logarithm of the difference between steps is also calculated. But, for digital signals, as it is our case study, the calculation of the neighbours must be carried out for several points or dimensions *n*, with the intention to eliminate the risk that the sequence is not around such point and its neighbour. This process must be carried out especially with digitalized signals. The reason is that during the process of digitalisation, errors can be incurred. If the results are similar for some consecutive values, then, the conclusion is that we have being using correct values. But, on the contrary, if there is an increase in the coefficient values, then, we have being using erroneous values.

Another procedure can be carried out by finding the average of several starting points. However, for digitalized values, the difference with respect to a neighbour point could be zero, so that its logarithm comes near ($-\infty$) and, therefore, generating spurious results. To avoid this situation, a very high value is used (-100, -1000). No neighbour is produced if distances lower than the minimum difference between two sequence points are used. Finally, the factor starting point could be the key. The reason is that the final result could generate large changes; therefore, the operation is carried out using several and different starting points, and subsequently finding the geometric average among all of them. Due to all these conditions, it is advisable to carry out a sensitivity analysis, changing different values and study the results with the intention to delimit the measure process.

The program used in this paper is based in two parameters, the signal to be analized, and a value or mode to configure the Lyapunov coefficients. The total setup is made up of the following variables and vectors: The first number tells us about the kind of signal, analogical (1) or digital (0). The second value shows the maximum size of the signal used (to calculate the Lyapunov exponents require, depending of the author, a lot of information). For Eckmann and Ruelle (see Rosenstein et al 1992), the size should be N>10 ^{D} , being D the dimension). The third value represents the number of starting points needed to produce the average, being 50 by default. The four value specifies the minimum distance between neighbours , normally, greater than 0.001. Finally, the fifth value indicates the number of steps that the distance of differences is calculated for, generally, 30 or 50. Our paper calculates the maximum exponent as the slope of a linear regression model. Following the least square procedure, we adjust a straight line to the curve of a graph in which the logarithm of the divergence is set along the vertical axis, and the data returns are set along the horizontal axis. After a short review, we found out that the results are quite similar, independently of the volume used in the second parameter.

In a first phase, we use analogical non chaotic and chaotic time series produced by three standard equations with the intention of comparing graphically the different figures produced according to different procedures. Subsequently , we follow the same process, but this time using digital signals, and finding out the Lyapunov exponents. This experiment allows us to check, on one hand, if we can detect the chaotic signals as such, obtaining the expected results and, on the other hand, it allows us to see the way our system works with digital signals. In other words, if the process of digitalisation is correct.

Testing with analogical and digital signals

I) Non Chaotic signals

We run the program "to analyze" using the time series produced by the sinusoidal equations:

$$\text{Sin }(x*\pi/8)$$

And the additive sinusoidal

$$\sin(x*8)+\sin(x*\ \pi/100) + \sin(x*\pi/4)$$

II) Chaotic signals

Using the logistic equation

$$X_{n+1} = A X_n (1 - X_n)$$

with parameters A=4 and 0.1 as the starting value X.

Signals were digitalized for these calculations in 2^{16} levels. In other words, we used 16 bits, producing a distance between consecutive values of $\frac{1}{2^{16}}$, using the remaining values as if they were analogical signals.

We have calculated 15,000 values for each series, from which we use the first 200 just to draw the graphs. We can see, through an analysis of the frequency domain that the sinusoidal profiles belongs thoroughly to a sine signal (fig. 2) and to a set made of the sum of three sine signals of different frequencies (fig. 3), representing expected and deterministic periodic functions. This clear-cut prediction can be seen in their corresponding Lyapunov coefficient graphs, with a naught value (the slope) for the sinusoidal signal (fig.4), and a negative value for the set signal. This negative exponent means that there is an approximation of trajectories in the same direction. In other words, the signal is convergent and not chaotic.

To verify our model, we have used the logistic equation, which is a one dimensional non linear difference feedback system. By changing the key variable, it can generate stable, deterministic, chaotic or totally random behaviour, and at some specific range variable, what apparently looks like a random time series, it has really hidden inside a deterministic chaotic dynamic. We can see that it generates a curve (fig.7) with a slope, defining the Lyapunov exponent, of 0.6914, very close to the theoretical values. The results are very similar to the ones obtained by Rosenstein et al (1992) in his work about exponents on small sets, in numerical as well as graphical representation. Previously, they have conducted a sensitivity analysis about known chaotic generators, among them, the logistic equation. They used the methodology of Sato et al (1987) to several dimensions (0-5) and time series, calculating for each combination a range of exponents between 0.675 and 0.686, very similar to the 0.6914 found in our work. The small difference could be the result of not using the sixth dimension, which, according to Brock (2000), it should be the most useful.

After checking out that the program responded correctly to the digitalized signals, a sensitivity analysis procedure is carried out. The intention is, depending on the number of bits used, to find out the limitations of the configuration values. We have found the following effects:

1) Reduction of the distance between neighbours. This distance must be in line with the digitalisation of the signal. The reason is that the process of digitalisation requires a minimum distance below which there are not differences. For instance, if we digitalize using 8 bits, then, distances under 1/256 can not be found, due to the fact that a very short distance will produce, as a result, neighbours totally similar to base values.

2) Reduction of the number of starting points. If we do not pick up the right number of points, the result could be wrong. There is a correct number of points depending on any specific series.

3) Variation of the dimension. The best way to find out if the dimension is the right one consists in switching up and down the parameter and, from a specific value on, the result becomes stable.

4) The distance between neighbours. This distance should be sufficiently small. The reason is that the sequences might not have enough margin to diverge and, therefore, unable to be measured. On the other hand, if we use a too small a value, we might not find any neighbours, specially in the case of a digital signal.

5) The number of digital bits. We have observed that the results are quite poor if we use just 8 bits, but we obtain a much better representation by increasing it to 16 and 32.

The dataset used covers the period between 1986 and 2003, both years included. The raw material is the daily return of stocks defined as the logarithm of price differences. We just report the very well documented fact that the unconditional distribution of many financial series are leptokurtic, and the skew coefficients also differ from the ones corresponding to an normal distribution. According to Brock et al, (1991), this difference procedure do not affect the dynamical traits of a chaotic time series. Also, the augmented Dickey-Fuller test failed to reject the null hypothesis of a unit root in the series (see Olmeda and Perez, 1995).

The result can be observed in a graph, with the numerical estimations attached. The four series used are made up of two private companies, Telefónica (phone company) and Acciona (a construction and conglomerate firm), and two indices, the IGBM, (Madrid Stock Exchange General Index) with around 120 stocks, , not corrected for dividends or splits, and the Ibex 35, not corrected for splits (although it was created in January 1990, we have adapted it backwards).

The three key variables that produce information on the Lyapunov exponents are: the dimension, in our study, 3, 5, 6 and 7, the number of starting points used to calculate the mean (20 and 50), and the distance between neighbours (0,01, 0,005, and 0,001), giving within brackets the number of stock companies used in the sensitivity analysis. The total combination of variables to carry out the sensitivity analysis of the four series is 432, plus their attached graphs. Due to this problem, we include in our analysis only those combinations that produce new results. In short, we put aside persistent negative results with the intention of avoiding drawing too many useless graphs. For instance, the distance 0.001 does not produce any kind of information, therefore, we reject amplifying the sensitivity analysis with such parameter beyond a minimum requirement.

When changing the number of base values used to calculate the mean, from 50 to 20, within a minimum distance framework between neighbours of the average level, the Lyapunov exponent of Acciona changes from naught to positive. We speculate with the idea that, by being Acciona a relatively volatile stock, and by using too many stocks to calculate the mean within a short distance framework, our method will not be able to find the exponent. Notwithstanding, the signal is recorded when the minimum distance between neighbours goes up to 0.01. Telefónica, on the other hand, shows the exponents at dimensions beyond five, independently of the number of starting points used, although, the signal is lost when the minimum distance between neighbours is too short (0,001) (fig.8-18). This last result happens to be similar for all series. Therefore, we can assume that the coefficients signal requires a distance longer that this minimum. The IGBM shows, in a range of dimensions (3-6), a lack of signal, even within the framework of the largest minimum distant of 0,01. However, the Ibex 35, at dimension six, considered to be the most reliable one, shows signals of exponents not only for the minimum distant but for the intermediate one as well.

4. Conclusions

Using a digital methodology, we confirm the results found in other stock exchange time series, namely, that there is a chaotic signal in the Spanish Stock Exchange, albeit at high dimensions. However, when we split the series between single and collective ones, such signals can be obtained in dimension five for single series, and dimension six for the collective. This result could be due to a weak signal. A hypothesis is that the variations in the 120 stocks that make up the IGBM, and in the 35 of the Ibex compensate for each other through internal correlations, In doing so, the chaotic signal could be harder to find, requiring, thus, higher dimensions than the ones required for individual stocks.

As we have previously mentioned it, when we increase data frequency, short memory tends to disappear, emerging, notwithstanding, a kind of autoregressive process. Its significance, by using quarterly data, however, is almost zero, surging, on the other hand, a process of long memory. Blasco de las Heras and Santamaría (1994a) already reported about the existence of such memory, questioning, along most researchers, the veracity of their results in their search of chaotic behaviour in stock exchange time series. Following the advise of Brock (1986), the sampling raw material should be based on monthly or quarterly data, and covering as many decades as possible. In our case, it becomes specially worrisome, and the reason is that the times series data of the Madrid Stock Exchange is not only short but somehow noisy, and the low frequency needed could make the time series unfortunately even shorter, with not use for further analysis.

Finally, the searching of chaotic signals before the initiation of the Madrid Stock Exchange General Index, and the Ibex 35 in 1986 and 1990 respectively, would have been of little use due to their not so transparent and efficient nature. In this fashion, we find ourselves with the discovery, by using analogical and digital technology, chaotic signals in very high dimensions, without the capacity to really verify if it is the result of an authentic chaotic or just a noisy process of some kind.

Graphs

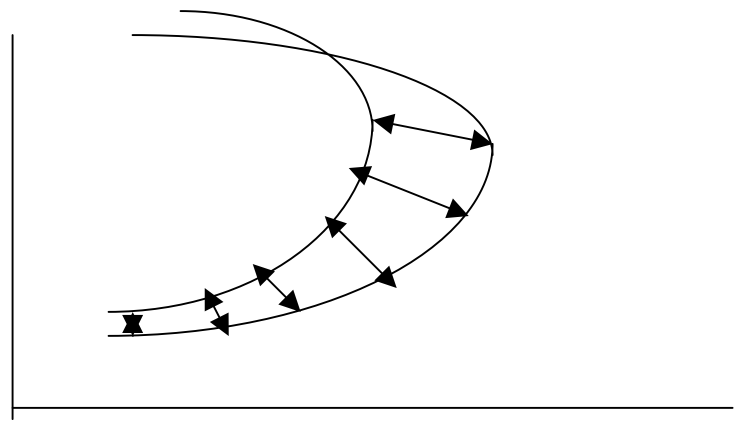


Figure 1. Two neighbour trajectories in an atractor showing the divergence trait across time and associated to a chaotic behaviour.

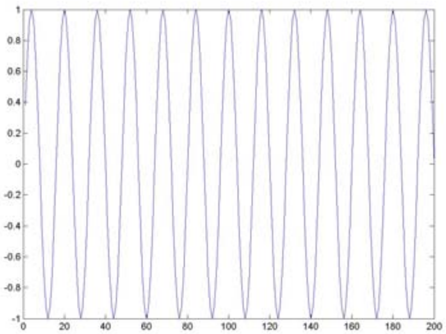


Fig 2: First sinusoidal.

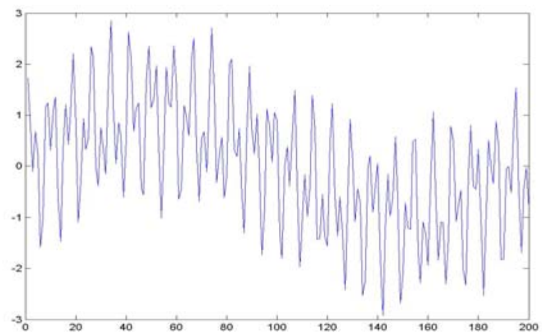


Fig 3: Sum of sinusoidals

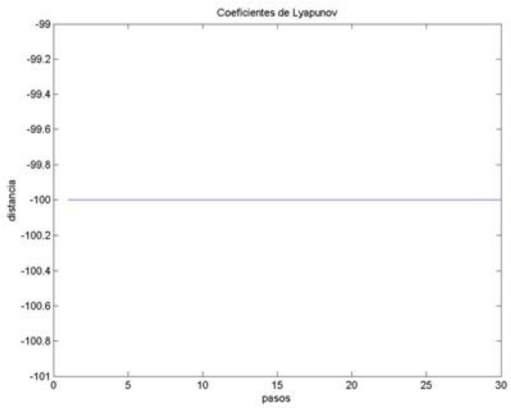


Fig 4: Sinusoidal Lyapunov coefficients

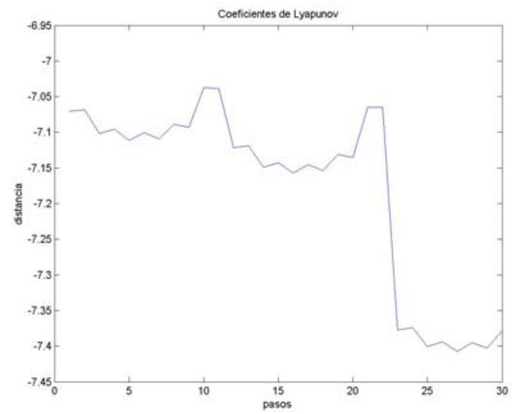


Fig 5 : Sinusoidal sum of Lyapunov coefficients

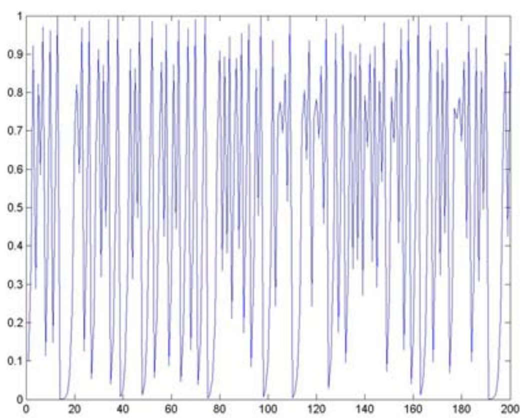


Fig 6: Logístic function

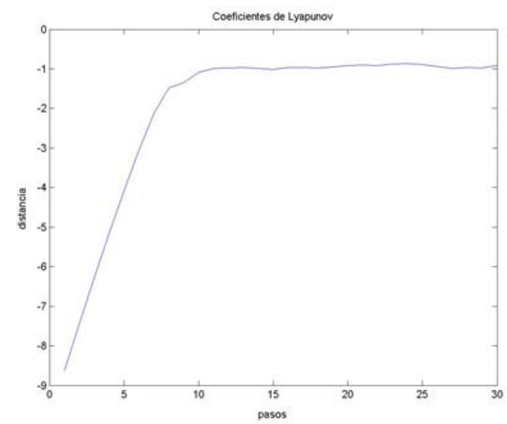


Fig 7 : Coefficients of the logistic function

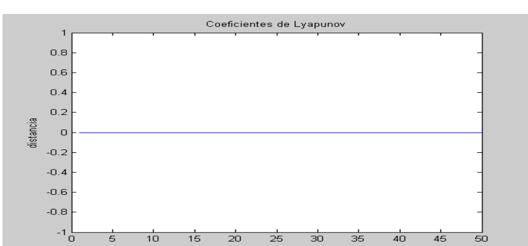


Fig. 8 dimen=[3];IGBM
lya=[1 25000 50 0.01 50];

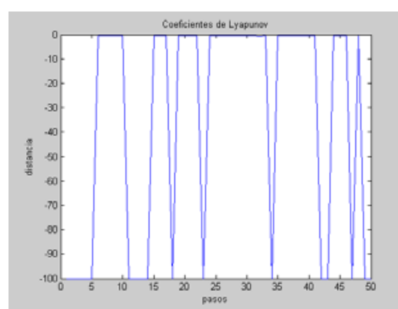


Fig. 9 dimen=[6]; IGBM
lya=[1 25000 50 0.001 50]

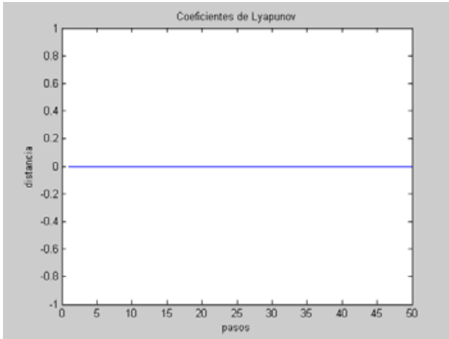


Fig. 10 dimen=[6]; Tel.
lya=[1 25000 20 0.001 50];

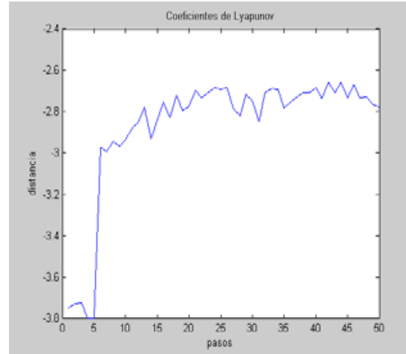


Fig. 11 dimen=[6]; Tel.
lya=[1 25000 50 0.01 50];

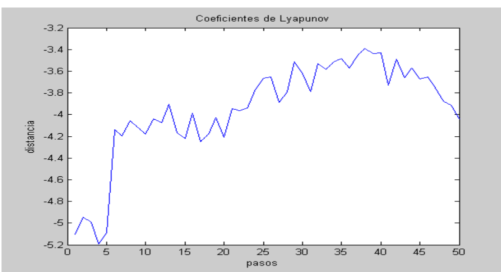


Fig. 12 dimen [6]; lya=[1 25000 20 0.01 50];

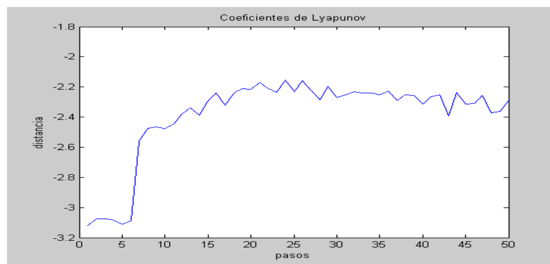


Fig. 13 dimen [7] lya=[1 25000 50 0.01 50];

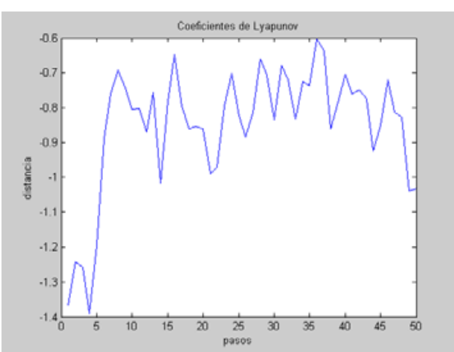


Fig. 14 dimen [6]; lya=[1 25000 50 0.01 50];

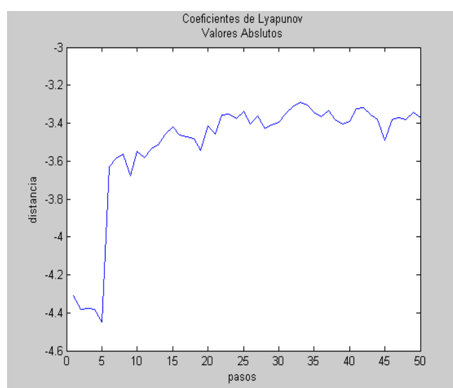
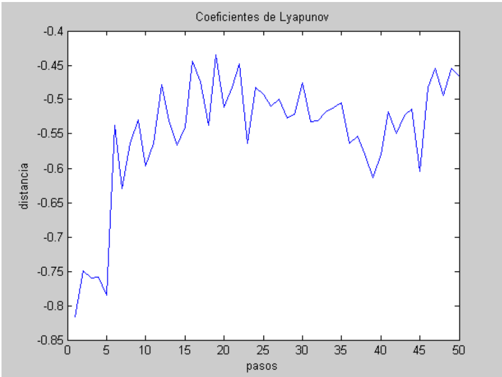


Fig. 15 dimen [6] lya=[1 25000 50 0.01 50];



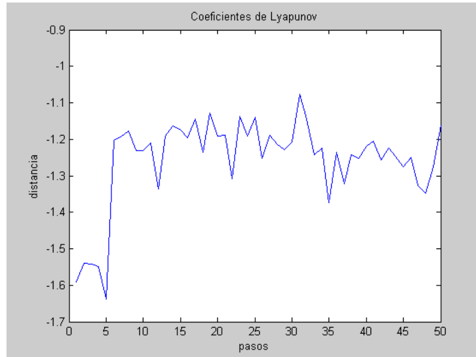


Fig 16 dimen=[6]; lya=[1 25000 50 0.01 50];

Fig 17 dimen [6]; lya=[1 25000 50 0.005 50];

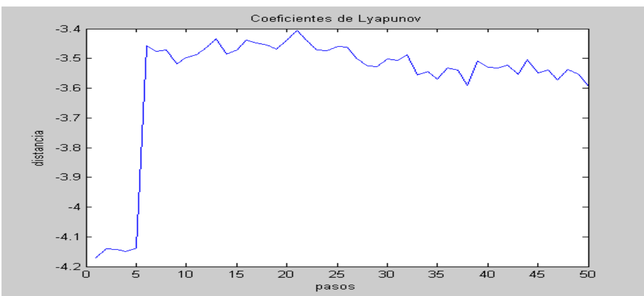


Fig. 18 dimen lya=[1 25000 50 0.01 50];

Bibliography

Bajo, O. Fernández , F, Sosvilla-Rivero, S. (1992): *Volatilidad y predecibilidad en las series del tipo de cambio peseta-dólar: Un enfoque basado en el caos determinista,* "Revista Española de Economía", monográfico "Mercados Financieros Españoles", 91-109.

Blasco de las Heras N., del Río, C., Santamaria, R., (1994a): *Memoria a largo plazo en el mercado de valores español: Una aproximación a través del análisis R/S,* "Investigaciones Económicas", 18, 571-583.

Blasco de las Heras N., del Río, C., Santamaria, R., (1994b) *¿Son las rentabilidades de las acciones series fraccionalmente integradas?. Resultados en el mercado de valores español,* "Revista de economía aplicada", 6, 5-28.

Blasco de las Heras N., del Río, C., Santamaría, R., (1995): *Teoría del caos y mercado de capitales: Un estudio exploratorio del IGBM 1980-1993,* "ICE", 746, págs. 151-167.

Brock, W (1986): *Distinguishing Random and Deterministic Systems: Abridged Version,* "Journal of Economic Theory", 40, 168-195.

Brock, W. (2000): *Whither Nonlinear?,* Journal of Economic Dynamics and Control, 24.

Brock, W. y Sayers, C. (1988): *Is the Business Cycle Characterized by Deterministic Chaos?,* "Journal of Monetary Economics", 22, 71-90.

Brock, W., y Malliaris, A. (1991): *Differencial Equations, Stability and Chaos in Dynamic Economics,* North-Holland, Amsterdam.

Brock, W., Hsieh, D., y LeBaron, B. (1991): *Nonlinear Dinamics, Chaos, and Instability: Statistical Theory,* MIT Press, Cambridge, Mass.

Dechert, W. y Gencay, R. (1992): *Lyapunov Exponents as a Nonparametric Diagnostic for Stability Análisis,* "Journal of Applied Econometrics", 7, 41-60.

Cootner, P. (1964): *The Random Character of Stock Prices,* Cambridge, Mass, The MIT Press.

Fama, E. y French, K, (1996): *Multifactor Erxplanations of Asset Pricing Anomalies,* "Journal of Finance", 51, 55-84.

Farmer, J., y Sidorowich, J., (1988): *Can New Approaches to Nonlinear Modeling Improve Economic Forecasts*?, 99-115, en Anderson, P., Arrow, K., y Pines, D. (1988): *The Economy as an Evolving Complex System,* Addison-Wesley, Ca.

Fernandez, F., y Martí, J. (1995): *Distinción entre el caos y azar en series ruidosas mediante predicciones locales,* "Estadística Española", 37, 287-304.

Flood, R. y Hodrick, R. (1986): *Assert Price volatility, Bubbles, and Process Switching,* "Journal of Finance", 41, 831-842.

Gilmore, C. (1992): *Financial Markets and the Theory of Chaos, non published thesis, Dre*xel University, EE UU.

Greene, M. y Fielitz, B. (1977): *Long-Term Dependence in Common Stock Returns,* "Journal of Financial Economics", 4, 339-349.

Isham, V. (1993): *Statistical Aspects of Chaos: A Review,* en Barndorff-Nelsen, O., Jensen, J., y Kendall, W., *Networks and Chaos. Statistical and Probabilistic Aspects,* Chapman &v Hall, RU.

Jensen, R. (1987): *Classical Chaos,* "American Scientist", 75, 168-181.

Lee, T., White, H., y Granger, C. (1989): *Testing for Negleted Nonlinearity in Time Series Models: A Comparison of Neural Network Methods and Alternative Tests,* University of California, San Diego.
Lo, A., (1991): *Long-Term Memory in Stock Market Prices,* "Econometrica", 59, 1279-1313.
Lo, A., y Mackinlay, A., (1988): *Stock Market Prices do not Follow Random Walks: Evidence from a Simple Specification Test,* "Review of Financial Studies", 3, 175-205.
Lorenz , H. (1993): *Nonlinear Dynamical Economics and Chaotic Motion,* Springer-Verlag.
Mandelbrot, B. (1963): *The Variation of Certain Speculative Prices,* "Journal of Finance", 36, included in the Cootner edition, chapter 15.
Olmeda, I., Perez, J. (1995): *Non-Linear Dynamics and Chaos in the Spanish Stock Market,* "Investigaciones Económicas", 19, 217-248.
Peters, E. (1993): *Fractal Market Analysis,* Wiley & sons.
Rosenstein, M., Collins, J., De Luca, C., (1992): *A Practical Method for Calculating Largest Lyapunov Exponents from Small Data Sets,* no available pages (paper published in internet).
Sato, S., Sano, M., Sawada, Y. (1987): *Practical Methods of Measuring the Generalized Dimension ant the Largest Lyapunov Exponent in High Dimensional Chaotic Systems,* Prog. Theor. Phys 77, I, non available pages.
Shiller, R. (1989): *Market Volatility,* Cambridge, Mass., The MIT Press.
Tata, F., y Vassilicos, Ch., (1993): *Is There Chaos in Economic Time Series? A Study of the Stock and the Foreign Exchange Markets,* en Crilly, A., Earnshaw, R., Jones, H., *Applications of Fractals and Chaos,* Springer-Verlag.
Wales, D.L. (1991): *Calculating the Rate Loss of Information from Chaotic Time Series by Forecasting,* Nature 350, 485.
Wolf, A.. Swift, J., Swinney, H., y Vastano, J. (1985): *Determining Lyapunov Exponents form Time Series,* "Physica", 16D, 285-317.

FUNDACIÓN DE LAS CAJAS DE AHORROS

DOCUMENTOS DE TRABAJO

Últimos números publicados

159/2000 Participación privada en la construcción y explotación de carreteras de peaje
Ginés de Rus, Manuel Romero y Lourdes Trujillo

160/2000 Errores y posibles soluciones en la aplicación del *Value at Risk*
Mariano González Sánchez

161/2000 Tax neutrality on saving assets. The spahish case before and after the tax reform
Cristina Ruza y de Paz-Curbera

162/2000 Private rates of return to human capital in Spain: new evidence
F. Barceinas, J. Oliver-Alonso, J.L. Raymond y J.L. Roig-Sabaté

163/2000 El control interno del riesgo. Una propuesta de sistema de límites
riesgo neutral
Mariano González Sánchez

164/2001 La evolución de las políticas de gasto de las Administraciones Públicas en los años 90
Alfonso Utrilla de la Hoz y Carmen Pérez Esparrells

165/2001 Bank cost efficiency and output specification
Emili Tortosa-Ausina

166/2001 Recent trends in Spanish income distribution: A robust picture of falling income inequality
Josep Oliver-Alonso, Xavier Ramos y José Luis Raymond-Bara

167/2001 Efectos redistributivos y sobre el bienestar social del tratamiento de las cargas familiares en el nuevo IRPF
Nuria Badenes Plá, Julio López Laborda, Jorge Onrubia Fernández

168/2001 The Effects of Bank Debt on Financial Structure of Small and Medium Firms in some European Countries
Mónica Melle-Hernández

169/2001 La política de cohesión de la UE ampliada: la perspectiva de España
Ismael Sanz Labrador

170/2002 Riesgo de liquidez de Mercado
Mariano González Sánchez

171/2002 Los costes de administración para el afiliado en los sistemas de pensiones basados en cuentas de capitalización individual: medida y comparación internacional.
José Enrique Devesa Carpio, Rosa Rodríguez Barrera, Carlos Vidal Meliá

172/2002 La encuesta continua de presupuestos familiares (1985-1996): descripción, representatividad y propuestas de metodología para la explotación de la información de los ingresos y el gasto.
Llorenc Pou, Joaquín Alegre

173/2002 Modelos paramétricos y no paramétricos en problemas de concesión de tarjetas de credito.
Rosa Puertas, María Bonilla, Ignacio Olmeda

174/2002 Mercado único, comercio intra-industrial y costes de ajuste en las manufacturas españolas.
José Vicente Blanes Cristóbal

175/2003 La Administración tributaria en España. Un análisis de la gestión a través de los ingresos y de los gastos.
Juan de Dios Jiménez Aguilera, Pedro Enrique Barrilao González

176/2003 The Falling Share of Cash Payments in Spain.
Santiago Carbó Valverde, Rafael López del Paso, David B. Humphrey
Publicado en "Moneda y Crédito" nº 217, pags. 167-189.

177/2003 Effects of ATMs and Electronic Payments on Banking Costs: The Spanish Case.
Santiago Carbó Valverde, Rafael López del Paso, David B. Humphrey

178/2003 Factors explaining the interest margin in the banking sectors of the European Union.
Joaquín Maudos y Juan Fernández Guevara

179/2003 Los planes de stock options para directivos y consejeros y su valoración por el mercado de valores en España.
Mónica Melle Hernández

180/2003 Ownership and Performance in Europe and US Banking – A comparison of Commercial, Co-operative & Savings Banks.
Yener Altunbas, Santiago Carbó y Phil Molyneux

181/2003 The Euro effect on the integration of the European stock markets.
Mónica Melle Hernández

182/2004 In search of complementarity in the innovation strategy: international R&D and external knowledge acquisition.
Bruno Cassiman, Reinhilde Veugelers

183/2004 Fijación de precios en el sector público: una aplicación para el servicio municipal de suministro de agua.
Mª Ángeles García Valiñas

184/2004 Estimación de la economía sumergida es España: un modelo estructural de variables latentes.
Ángel Alañón Pardo, Miguel Gómez de Antonio

185/2004 Causas políticas y consecuencias sociales de la corrupción.
Joan Oriol Prats Cabrera

186/2004 Loan bankers' decisions and sensitivity to the audit report using the belief revision model.
Andrés Guiral Contreras and José A. Gonzalo Angulo

187/2004 El modelo de Black, Derman y Toy en la práctica. Aplicación al mercado español.
Marta Tolentino García-Abadillo y Antonio Díaz Pérez

188/2004 Does market competition make banks perform well?.
Mónica Melle

189/2004 Efficiency differences among banks: external, technical, internal, and managerial
Santiago Carbó Valverde, David B. Humphrey y Rafael López del Paso

190/2004 Una aproximación al análisis de los costes de la esquizofrenia en españa: los modelos jerárquicos bayesianos
F. J. Vázquez-Polo, M. A. Negrín, J. M. Cavasés, E. Sánchez y grupo RIRAG

191/2004 Environmental proactivity and business performance: an empirical analysis
Javier González-Benito y Óscar González-Benito

192/2004 Economic risk to beneficiaries in notional defined contribution accounts (NDCs)
Carlos Vidal-Meliá, Inmaculada Domínguez-Fabian y José Enrique Devesa-Carpio

193/2004 Sources of efficiency gains in port reform: non parametric malmquist decomposition tfp index for Mexico
Antonio Estache, Beatriz Tovar de la Fé y Lourdes Trujillo

194/2004 Persistencia de resultados en los fondos de inversión españoles
Alfredo Ciriaco Fernández y Rafael Santamaría Aquilué

195/2005 El modelo de revisión de creencias como aproximación psicológica a la formación del juicio del auditor sobre la gestión continuada
Andrés Guiral Contreras y Francisco Esteso Sánchez

196/2005 La nueva financiación sanitaria en España: descentralización y prospectiva
David Cantarero Prieto

197/2005 A cointegration analysis of the Long-Run supply response of Spanish agriculture to the common agricultural policy
José A. Mendez, Ricardo Mora y Carlos San Juan

198/2005 ¿Refleja la estructura temporal de los tipos de interés del mercado español preferencia por la liquidez?
Magdalena Massot Perelló y Juan M. Nave

199/2005 Análisis de impacto de los Fondos Estructurales Europeos recibidos por una economía regional: Un enfoque a través de Matrices de Contabilidad Social
M. Carmen Lima y M. Alejandro Cardenete

200/2005 Does the development of non-cash payments affect monetary policy transmission?
Santiago Carbó Valverde y Rafael López del Paso

201/2005 Firm and time varying technical and allocative efficiency: an application for port cargo handling firms
Ana Rodríguez-Álvarez, Beatriz Tovar de la Fe y Lourdes Trujillo

202/2005 Contractual complexity in strategic alliances
Jeffrey J. Reuer y Africa Ariño

203/2005 Factores determinantes de la evolución del empleo en las empresas adquiridas por opa
Nuria Alcalde Fradejas y Inés Pérez-Soba Aguilar

204/2005 Nonlinear Forecasting in Economics: a comparison between Comprehension Approach versus Learning Approach. An Application to Spanish Time Series
Elena Olmedo, Juan M. Valderas, Ricardo Gimeno and Lorenzo Escot

205/2005	Precio de la tierra con presión urbana: un modelo para España Esther Decimavilla, Carlos San Juan y Stefan Sperlich
206/2005	Interregional migration in Spain: a semiparametric analysis Adolfo Maza y José Villaverde
207/2005	Productivity growth in European banking Carmen Murillo-Melchor, José Manuel Pastor y Emili Tortosa-Ausina
208/2005	Explaining Bank Cost Efficiency in Europe: Environmental and Productivity Influences. Santiago Carbó Valverde, David B. Humphrey y Rafael López del Paso
209/2005	La elasticidad de sustitución intertemporal con preferencias no separables intratemporalmente: los casos de Alemania, España y Francia. Elena Márquez de la Cruz, Ana R. Martínez Cañete y Inés Pérez-Soba Aguilar
210/2005	Contribución de los efectos tamaño, book-to-market y momentum a la valoración de activos: el caso español. Begoña Font-Belaire y Alfredo Juan Grau-Grau
211/2005	Permanent income, convergence and inequality among countries José M. Pastor and Lorenzo Serrano
212/2005	The Latin Model of Welfare: Do 'Insertion Contracts' Reduce Long-Term Dependence? Luis Ayala and Magdalena Rodríguez
213/2005	The effect of geographic expansion on the productivity of Spanish savings banks Manuel Illueca, José M. Pastor and Emili Tortosa-Ausina
214/2005	Dynamic network interconnection under consumer switching costs Ángel Luis López Rodríguez
215/2005	La influencia del entorno socioeconómico en la realización de estudios universitarios: una aproximación al caso español en la década de los noventa Marta Rahona López
216/2005	The valuation of spanish ipos: efficiency analysis Susana Álvarez Otero
217/2005	On the generation of a regular multi-input multi-output technology using parametric output distance functions Sergio Perelman and Daniel Santin
218/2005	La gobernanza de los procesos parlamentarios: la organización industrial del congreso de los diputados en España Gonzalo Caballero Miguez
219/2005	Determinants of bank market structure: Efficiency and political economy variables Francisco González
220/2005	Agresividad de las órdenes introducidas en el mercado español: estrategias, determinantes y medidas de performance David Abad Díaz

221/2005 Tendencia post-anuncio de resultados contables: evidencia para el mercado español
Carlos Forner Rodríguez, Joaquín Marhuenda Fructuoso y Sonia Sanabria García

222/2005 Human capital accumulation and geography: empirical evidence in the European Union
Jesús López-Rodríguez, J. Andrés Faíña y Jose Lopez Rodríguez

223/2005 Auditors' Forecasting in Going Concern Decisions: Framing, Confidence and Information Processing
Waymond Rodgers and Andrés Guiral

224/2005 The effect of Structural Fund spending on the Galician region: an assessment of the 1994-1999 and 2000-2006 Galician CSFs
José Ramón Cancelo de la Torre, J. Andrés Faíña and Jesús López-Rodríguez

225/2005 The effects of ownership structure and board composition on the audit committee activity: Spanish evidence
Carlos Fernández Méndez and Rubén Arrondo García

226/2005 Cross-country determinants of bank income smoothing by managing loan loss provisions
Ana Rosa Fonseca and Francisco González

227/2005 Incumplimiento fiscal en el irpf (1993-2000): un análisis de sus factores determinantes
Alejandro Estellér Moré

228/2005 Region versus Industry effects: volatility transmission
Pilar Soriano Felipe and Francisco J. Climent Diranzo

229/2005 Concurrent Engineering: The Moderating Effect Of Uncertainty On New Product Development Success
Daniel Vázquez-Bustelo and Sandra Valle

230/2005 On zero lower bound traps: a framework for the analysis of monetary policy in the 'age' of central banks
Alfonso Palacio-Vera

231/2005 Reconciling Sustainability and Discounting in Cost Benefit Analysis: a methodological proposal
M. Carmen Almansa Sáez and Javier Calatrava Requena

232/2005 Can The Excess Of Liquidity Affect The Effectiveness Of The European Monetary Policy?
Santiago Carbó Valverde and Rafael López del Paso

233/2005 Inheritance Taxes In The Eu Fiscal Systems: The Present Situation And Future Perspectives.
Miguel Angel Barberán Lahuerta

234/2006 Bank Ownership And Informativeness Of Earnings.
Víctor M. González

235/2006 Developing A Predictive Method: A Comparative Study Of The Partial Least Squares Vs Maximum Likelihood Techniques.
Waymond Rodgers, Paul Pavlou and Andres Guiral.

236/2006 Using Compromise Programming for Macroeconomic Policy Making in a General Equilibrium Framework: Theory and Application to the Spanish Economy.
Francisco J. André, M. Alejandro Cardenete y Carlos Romero.

237/2006 Bank Market Power And Sme Financing Constraints.
Santiago Carbó-Valverde, Francisco Rodríguez-Fernández y Gregory F. Udell.

238/2006 Trade Effects Of Monetary Agreements: Evidence For Oecd Countries.
Salvador Gil-Pareja, Rafael Llorca-Vivero y José Antonio Martínez-Serrano.

239/2006 The Quality Of Institutions: A Genetic Programming Approach.
Marcos Álvarez-Díaz y Gonzalo Caballero Miguez.

240/2006 La interacción entre el éxito competitivo y las condiciones del mercado doméstico como determinantes de la decisión de exportación en las Pymes.
Francisco García Pérez.

241/2006 Una estimación de la depreciación del capital humano por sectores, por ocupación y en el tiempo.
Inés P. Murillo.

242/2006 Consumption And Leisure Externalities, Economic Growth And Equilibrium Efficiency.
Manuel A. Gómez.

243/2006 Measuring efficiency in education: an analysis of different approaches for incorporating non-discretionary inputs.
Jose Manuel Cordero-Ferrera, Francisco Pedraja-Chaparro y Javier Salinas-Jiménez

244/2006 Did The European Exchange-Rate Mechanism Contribute To The Integration Of Peripheral Countries?.
Salvador Gil-Pareja, Rafael Llorca-Vivero y José Antonio Martínez-Serrano

245/2006 Intergenerational Health Mobility: An Empirical Approach Based On The Echp.
Marta Pascual and David Cantarero

246/2006 Measurement and analysis of the Spanish Stock Exchange using the Lyapunov exponent with digital technology.
Salvador Rojí Ferrari and Ana Gonzalez Marcos